inter•noise 2001

The Hague, Holland

The 2001 International Congress and Exhibition
on Noise Control Engineering
The Hague, The Netherlands, 2001 August 27-30

Vibrational Power Flow Considerations Arising From Multi-Dimensional Isolators

Rajendra Singh and Seungbo Kim
The Ohio State University
Columbus, OH 43210-1107, USA

Abstract

Much of the vibration isolation research has focused on uni-directional behavior of the system. For many real-life problems, the role of rotational and shear stiffness components must be understood and a multi-dimensional formulation needs to be developed, especially at higher frequencies. Consequently, characterization and modeling of vibration isolators is increasingly becoming more important in mid and high frequency regimes where very few methods are known to exist. This paper presents a new experimental identification method that yields frequency-dependent multi-dimensional dynamic stiffnesses of an isolator. The proposed identification method is applied to one practical rubber isolator, and experimental results are successfully compared with data measured on commercial equipment for axial motions. The effects of multi-dimensional isolator on vibration power transmitted to the receiver structure are analytically investigated. Rigid body and Timoshenko beam models are employed to describe source and path respectively. Infinite beam is used to represent a compliant foundation. Also, linear, time-invariant system assumption is made.

1. Introduction

Vibration isolators are often characterized as discrete elastic elements, with or without viscous or hysteritic damping [1]. The compressional stiffness term is typically used to develop isolation system models though the transverse (shear) and rotational components could be a significant contributor to the vibration transmission [2]. Additionally, at higher frequencies, inertial or standing wave effects occur within the isolator. Nonetheless, the isolators are still modeled by many researchers in terms of spectrally-invariant discrete stiffness elements without any cross-axis coupling terms [2]. Such descriptions are clearly inadequate at higher frequencies. Consequently, one must adopt the distributed parameter approach. Only a few articles have examined the elastomeric devices using the continuous vibration system theories [3-4].

Experimental methods must be adopted to dynamically characterize stiffnesses of rubber, hydraulic, air and metallic isolators since they invariably exhibit frequency-dependent properties and are sensitive to mean loads and dynamic excitation levels. Historically, characterization methods have focused on axial or compressional stiffness. Also, in most studies only the lower frequency range has been considered [1-2], and consequently the direct measurement of dynamic stiffness on commercial machines is typically limited to lower frequencies. Several approximate identification methods for transfer stiffness of resilient elements have been proposed at higher frequencies and have been refined for lower frequencies [5]. See references [4, 6] for a detailed literature review and a list of relevant articles.

Overall, an appropriate characterization method for the measurement of multi-dimensional stiffnesses of an isolator has yet to be proposed. The underlying measurement and estimation issues are even more difficult as the frequency increases [1, 5]. Also, no prior article has examined the shear deformation and rotary inertia effects of an isolator. In this article, we propose a new dynamic characterization method that should be valid over low, mid and high frequency regimes, based on a new isolator stiffness matrix formulation. Further, we examine the influence of isolator parameters on the behavior of an isolation system using a continuous system model. Problem formulation is conceptually shown in Figure 1 in the context of source, path (isolator) and receiver. The scope is limited to a linear time-invariant (LTI) system, and the effects of preload, etc. are not considered.

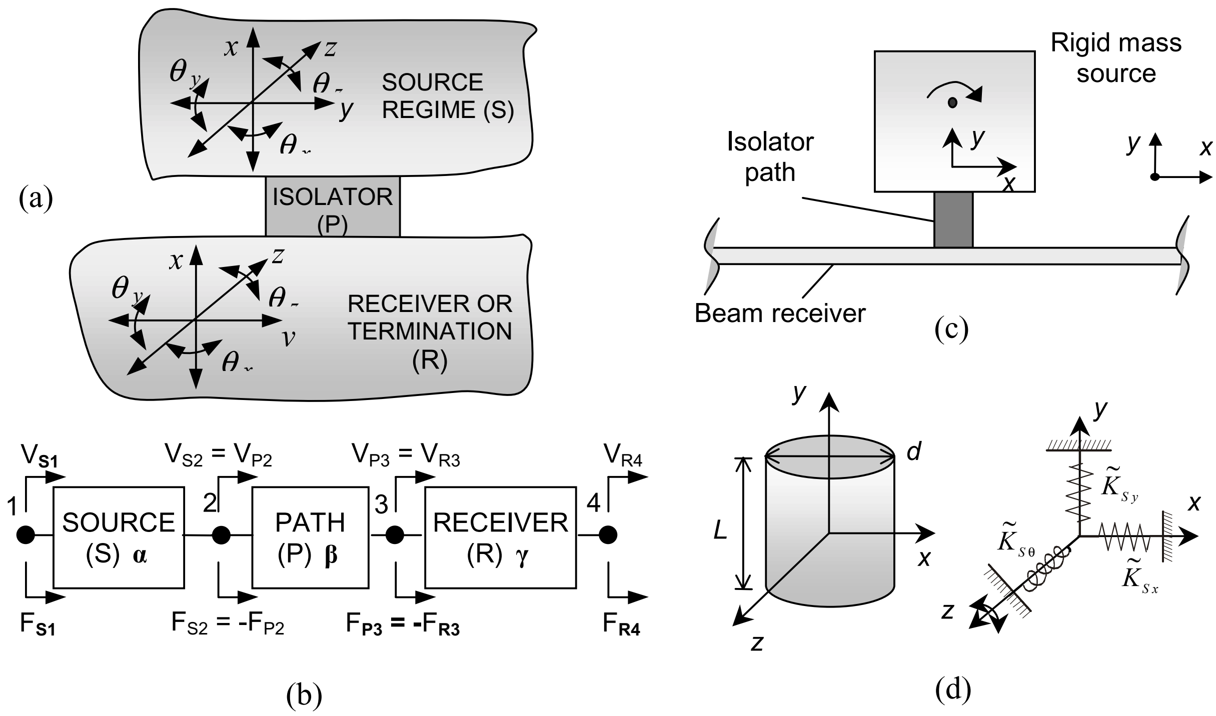

Figure 1. Problem formulation. (a) Isolator is depicted as a multi-dimensional path for any practical problem; (b) source-path-receiver system and their mobility matrices $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. Here F and V are vectors; (c) Vibration transmission via multi-dimensional motions of an isolator to a beam receiver; (d) a cylindrical isolator with static stiffness components.

2. identification of mobility matrix of an isolator

Multi-dimensional mobility matrix of a vibration isolator can be identified using a mobility synthesis formulation. Figure 2(a) shows a schematic that is used for experimental work. Multi-degree of freedom connections at both ends of an isolator are modeled since the information at both ends of a sub-system is needed. Two masses are attached to an isolator as shown in Figure 2(a) and the synthesized mobility for the overall system is formulated first.

Then the mobility matrix of an isolator is reformulated given the synthesized formulation. Finally, the mobility matrix of an isolator can be obtained by substituting the synthesized mobility matrix with measured mobility matrix for the combined system. It is possible to measure all elements of the multi-dimensional mobility matrix from the fact that an application of force with an offset from the reference point results in both force and moment simultaneously. Mobility model for one rubber isolator is identified using the proposed procedure. Two masses are attached to the ends of each isolator and the combined system of Figure 2(a) is suspended to simulate free boundaries. The reciprocity principle has been applied throughout the synthesis procedure since small inconsistencies or noise in frequency response function measurements can significantly contaminate results via the numerical inversion process that is essential to the entire procedure. Typical results for transfer stiffnesses are shown in Figures 3 and 4 for an isolator example of Figure 2(b). Note that the experimental validations are conducted using the MTS 831.50 machine (up to 1000 Hz). The MTS method employs the blocked end boundary and only the transfer stiffness is measured. First, stiffness modulus and loss angle in axial direction are shown in Figures 12. Static stiffness in axial motion is 5 N/mm for the isolator. Identified results from the proposed experiment are given from 0 to 2000 Hz. Predictions for zero preload are compared with the MTS test results that are obtained under three different preloads, up to 1 kHz. Excellent agreements are observed between results based on the proposed identification method and experimental data yield by the MTS machine. Similar results are seen for two additional mounts [4].

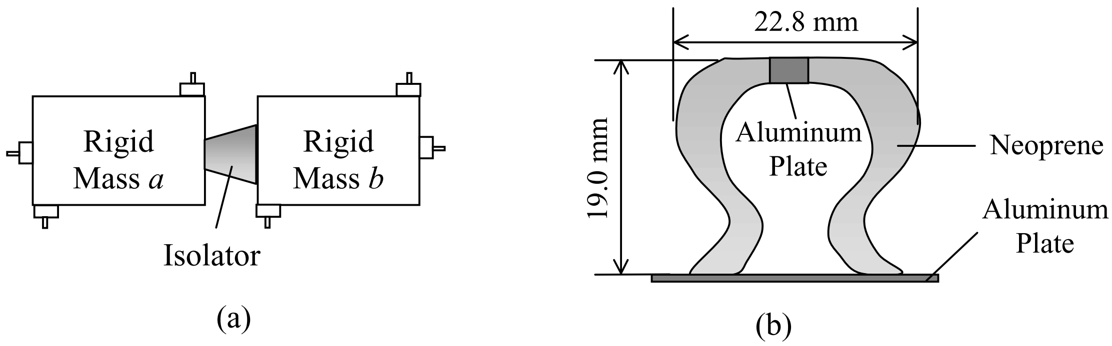

Figure 2. Proposed identification method. (a) Simplified scheme to identify mobilities of an isolator (b) practical rubber isolator used for experimental studies.

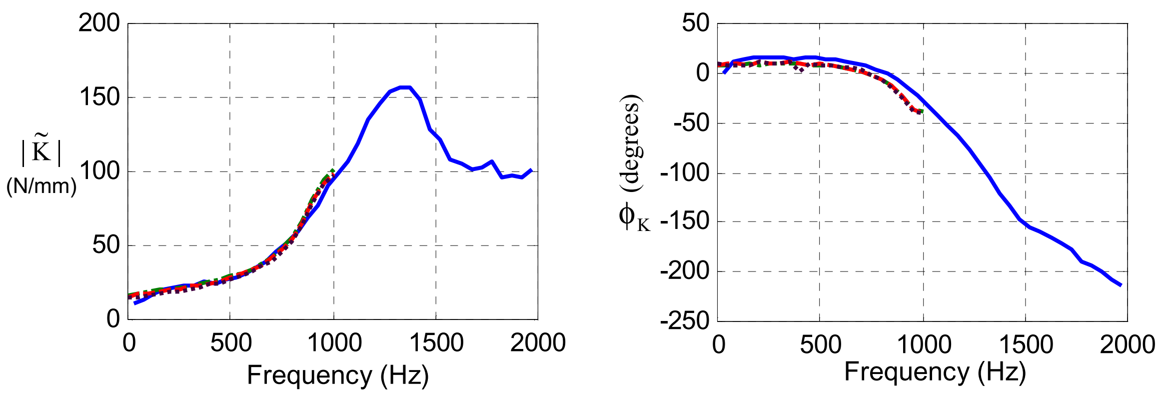

Figure 3. Comparison of axial dynamic stiffnesses for isolator. (a) Dynamic stiffness modulus; (b) loss angle. Key: —— , Mobility model; –·–· , Measured (MTS): f_{mean} = 3 N; -----, Measured (MTS): f_{mean} = 12 N; ··········, Measured (MTS): f_{mean} = 23 N.

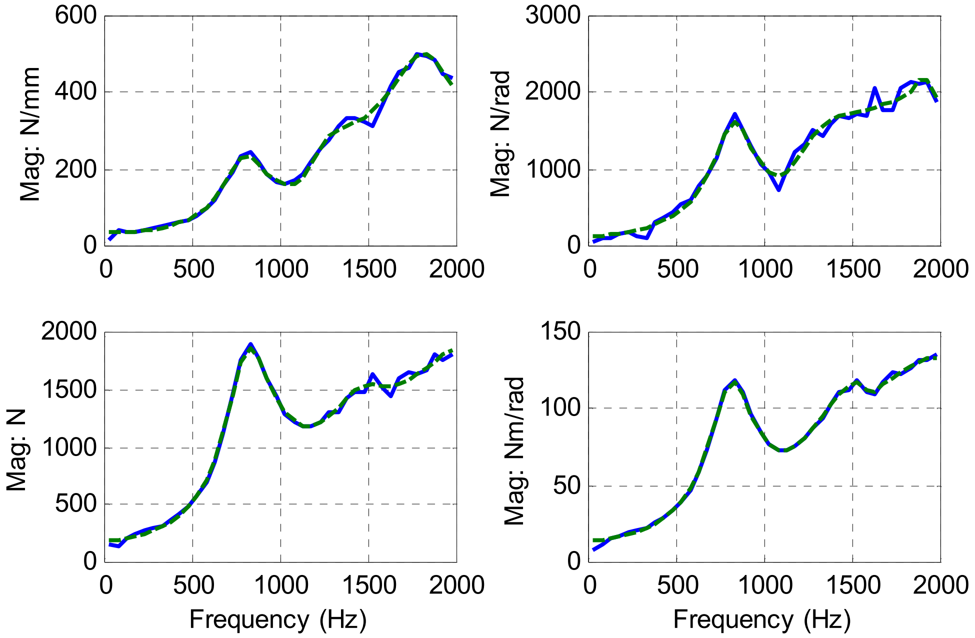

Figure 4. Transfer stiffnesses in flexural motion for isolator, as extracted using the identification scheme. (a) Lateral stiffness modulus; (b) coupling stiffness modulus; (c) coupling stiffness modulus; (d) rotational stiffness. Key: ——, Experimental result; -----, curve fit.

3. Vibration power transmitted to an infinite beam receiver

The vibrational behavior is examined for an isolation system (Figure 1c) with an infinite beam receiver. Harmonic excitation is applied at the mass center of a cubic rigid body with a mass of 1 kg and a length of 50 mm. Circular isolator is shown in Figure 1(d) along with vibration components transmitted through the path. The isolator is modeled using the Timoshenko beam theory (flexural motion) and the wave equation (longitudinal motion). Detailed mathematical treatment is given in the recent journal article we wrote [6]. Note that the coupling mobility does not exist for an infinite beam receiver. However, a coupling arises because the motion (or force) in shear direction of an isolator is coupled with the longitudinal direction of a receiver beam. The Young's modulus, shear modulus and mass density of a rubber isolator of for this study are 1.62 MPa, 5 MPa and 1000 kg/m^3 respectively. Also, the circular cylindrical isolator has the length of 30 mm and the radius of 12 mm. Material properties of a receiver beam having a thickness of 10 mm and a width of 100 mm are 6.688×10^4 MPa, 2723 kg/m^3 and 0.001 for Young's modulus, mass density and loss factor respectively. In order to understand the effect of isolator properties, it is useful to examine the static stiffnesses (K_S) of an isolator. It should be noted that flexural stiffnesses have to be dealt with in a matrix form since there exist coupling terms between lateral (shear x) and rotational (θ) stiffnesses. Further, note that G (or E) is common to all stiffness terms. Highly damped material with a loss factor of 0.3 is used for this isolator so that overall frequency-dependent characteristics are observed without the influence of isolator resonances. Results for Γ are shown in Figure 5(a) for a variation in G values for an isolator when a harmonic moment is applied to the mass center of a rigid body source. It is observed in Figure 5(a) that Γ rises due to an increase in G as the frequency increases. Additionally, the Γ spectra of

Figure 5(a) decrease as the frequency increases. The power efficiency is also shown in Figure 5(b) when a harmonic force (f_y) is applied to the mass center of a source. In this case, only axial stiffness of the mount affects vibration power transmission. Like the moment application case, Γ grows with G as the frequency increases. Also, note that the Γ spectra of Figure 5(b) for the axial power transmission are close to unity at low frequencies unlike the one of Figure 5(a) for the flexural power transmission. Normalized power components with respect to the total actual power transmitted to the receiver beam are also shown in Figures 5(c-d) for the shear modulus variation. As discussed previously, axial and coupling power components do not exist in this case and therefore the sum of the normalized lateral (shear direction of mount) and rotational power components is equal to unity. Overall, the lateral power component is larger than the rotational component. It is shown in Figure 5(c-d) that the lateral component dominates.

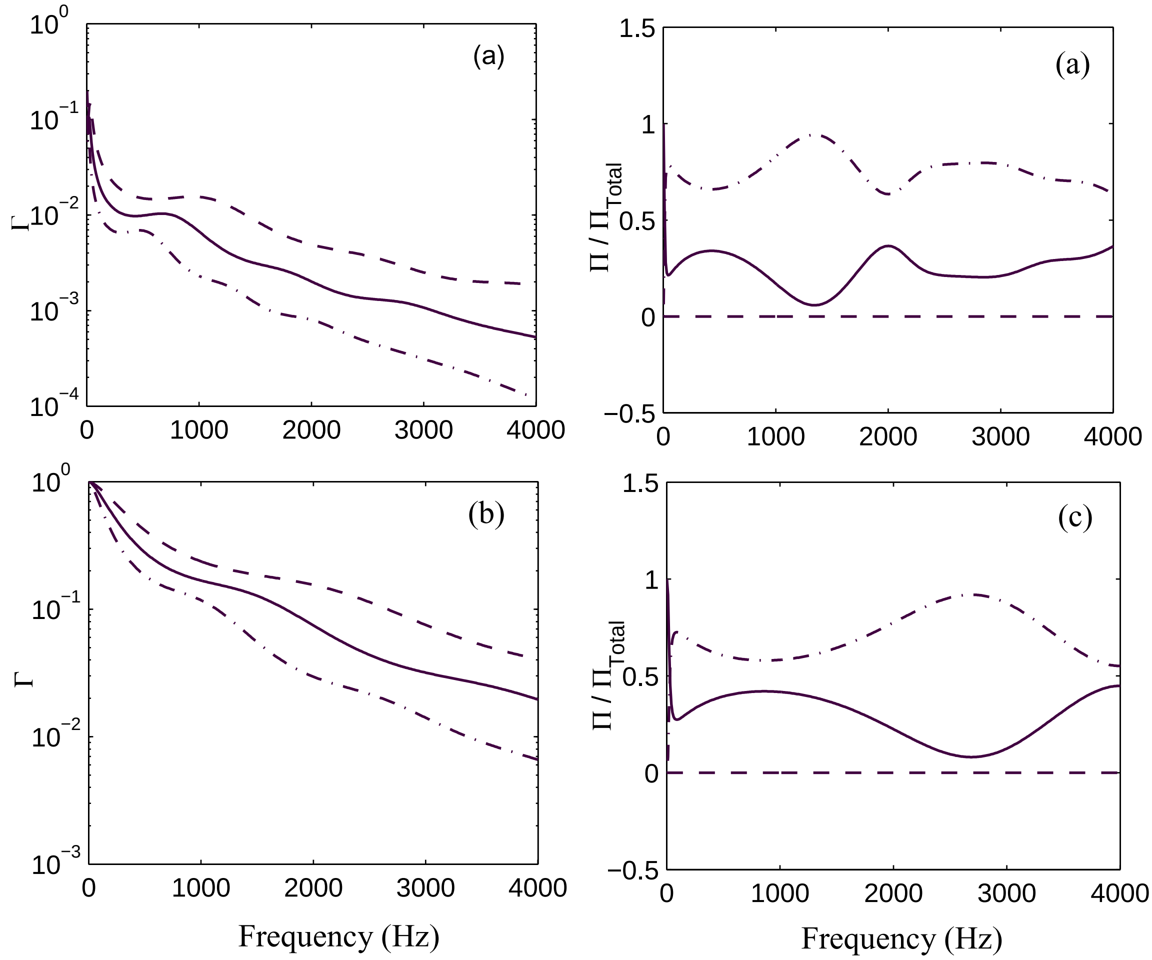

Figure 5. Effect of shear modulus G of an isolator on efficiency (Γ) with an infinite beam receiver. (a) For a Timoshenko beam isolator model given moment excitation; (b) given force excitation f_y. Key: –·–·– , 0.5G; ——— , G; ------ , 2G. (c) normalized vibration power transmission given moment excitation 0.5G; (d) normalized vibration power transmission given moment excitation 2G. Key: ------, axial (y); ———, rotational (θ); –·–·–, lateral (x).

Conclusion

A new characterization method has been proposed for the identification of multi-dimensional frequency-dependent transfer stiffnesses of an isolator. Our method uses a physical system that consists of two inertial elements and an isolator. Further, refined multi-dimensional mobility synthesis and decomposition procedures have been formulated. Results of the proposed scheme compare well with test data for one practical isolator in axial motions on a commercial machine up to 1 kHz. Another main contribution of this paper is the application of continuous system theory to an elastomeric isolator and the examination of associated flexural and longitudinal motions of the source-path-receiver system. Two different frequency response characteristics of an elastomeric isolator are predicted by the Timoshenko beam theory. The second type solution, that has been previously believed to occur at extremely high frequencies (say around 80 kHz) for metallic structures and therefore not of interest in structural dynamics, takes place at relatively low frequencies (say around 1.5 kHz) for a rubberlike material. The behavior of a typical vibration isolation system has been examined using the power transmitted to an infinite beam receiver, when excited by a harmonic moment or force at the source. Parametric study of isolator properties on the transmission measures has been conducted using the Timoshenko beam isolator model and an infinite beam receiver. The vibration power efficiency for an isolation system with an infinite beam structure increase with frequency as the isolator shear modulus is increased. Future work is required to quantify the vibration source in terms of power transmission. Proper interpretation of various vibration isolation measures for a multi-dimensional system, such as power efficiency and effectiveness, must also be sought over a broad range of frequencies. Finally, nonlinear effects of an isolator should be examined.

Acknowledgements

The General Motors Corporation (Noise and Vibration Center) and the Goodyear Tire and Rubber Company (Transportation Molded Products) are gratefully acknowledged for supporting this research.

References

1. T. JEONG and R. SINGH 2000 *Journal of Sound and Vibration*, Inclusion of Measured Frequency- and Amplitude-Dependent Mount Properties in Vehicle or Machinery Models (in press).
2. M. A. SANDERSON 1996 *Journal of Sound and Vibration*, **198**(2), 171-191. Vibration Isolation: Moments and Rotations Included.
3. P. GARDONIO, S. J. ELLIOTT and R. J. PINNINGTON 1997 *Journal of Sound and Vibration*, **207**(1), 61-93. Active Isolation of Structural Vibration on a Multiple-Degree-of-Freedom System, Part I: The Dynamics of the System.
4. S. KIM and R. SINGH 2000 *Journal of Sound and Vibration*, Multi-Dimensional Characterization of Vibration Isolators over a Wide Range of Frequencies (in press).
5. D. J. THOMPSON, W. J. VAN VLIET and J. W. VERHEIJ 1998 *Journal of Sound and Vibration*, **213**(1), 169-188. Development of the Indirect Method for Measuring the High Frequency Dynamic Stiffness of Resilient Isolator.
6. S. KIM and R. SINGH 2001 *submitted to the Journal of Sound and Vibration*, Vibration Transmission Through an Isolator Modeled by Continuous System Theory.