

Design and analysis of supporting structure with smart struts for active vibration isolation

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ABSTRACT

This research investigates a supporting structure with smart struts under crude vibration, in the aspect of their modeling and analysis. Particularly, in case of most rotorcrafts, structure-borne noise and vibration transmitted from the gearbox contains multiple spectral elements and higher frequencies, which include gear mesh frequencies and their side bands. In order to manage this issue, many research efforts have devoted to active smart struts which have tunable stiffness such that higher level of attenuation is possible. However, present techniques on the active control are restricted mostly to the control of a single or multiple sinusoids and thus these are not applicable to manage modulated and multi-spectral signals. Therefore, enhanced control algorithms are required in order to achieve simultaneous attenuation of gear mesh frequencies and their side bands. Proposed algorithms are employing two nonlinear methods and one model-based technique. Their performances are verified by comparing with conventional algorithms. Moreover, these algorithms are implemented to exhibit whether they are feasible to narrowband or broadband control through experiments with a single smart strut. Novel methodologies are expected to be applied to several active vibration and noise control practices such as vehicles and other engineering structures.

Keywords: Smart Struts, Active Control, Multi-spectral Controller, Least Mean Squares, Model Predictive Sliding Mode Control

1. INTRODUCTION

The motivation of this research is the rotorcraft cabin noise excitation at gear mesh frequencies and side bands ranging from 500 Hz to 4 kHz. These are mostly high level structure-borne noise and vibration from the gearbox transmitted to the cabin through these supporting struts. However, if active struts are utilized for the supporting struts of the gearbox, stiffness and damping can be tuned and therefore higher attenuation is possible. Thus, the scope of this research is to examine gear noise reduction concepts based on smart materials and devices with novel design of active struts and enhanced control algorithms.

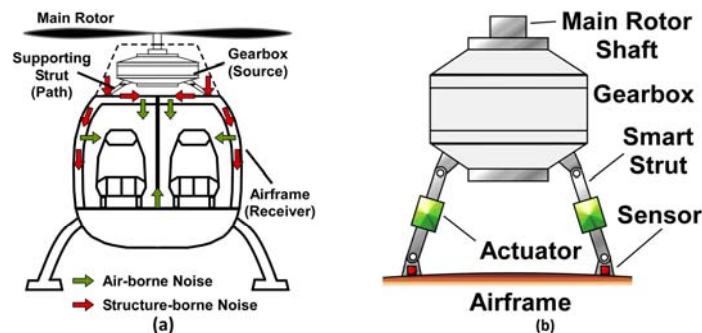


Figure 1 Gearbox induced noise in cabin; (a) Structure-borne and air-borne noise paths, (b) Concept of active gearbox struts

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There have been many research efforts on the active vibration control, particularly relevant to the application of active struts to various vehicles and engineering structures. Loewy¹ originated various different concepts of passive struts containing dampers and springs to attenuate noise and vibration. Kawaguchi et al.² proposed an active vibration reduction system applying hydraulic actuators inside struts of the gearbox. Sutton et al.³ investigated a helicopter gearbox strut with three magnetostrictive actuators attached on the surface to observe the capability of active vibration reduction. Millott et al.⁴ suggested an active control system for canceling noise with gear mesh frequencies from the helicopter employing proof-mass actuators. Baz⁵ and Asiri et al.⁶⁻⁷ examined periodic structures and their application in active control. Gemblert et al.⁸, Maier et al.⁹, and Hoffmann et al.¹⁰ established a method for the active vibration isolation by implementing active struts with multi-layer piezoelectric stacks and utilized filtered-x LMS algorithm to fulfill mitigation at the fundamental gear mesh frequency. However, while it is possible to attenuate one primary gear mesh frequency significantly, frequency components in broadband still remain at similar level. Previous research is limited to manage a single frequency and thus more studies are required on multi-spectral control techniques for simultaneous reduction of side bands and higher harmonics of gear mesh frequencies.

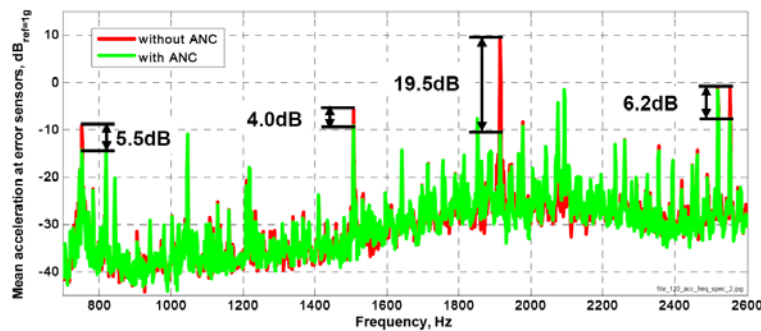


Figure 2 Reduction levels at gear mesh frequencies with active noise control (from reference [10])

2. PROBLEM STATEMENT

In recent times, smart material systems have been employed to the active vibration and noise control, which is one of the most representative research areas on vibration attenuation of vehicles and engineering structures, interruption of radiating sound, etc. In order to mitigate vibration and unwanted noise from the target plant, active force application is utilized for this approach with numerous kinds of actuators using hydraulics, voice coils, and piezoelectrics. Many research attempts have been concentrated on better algorithms for active control systems and among those, the least mean squares (LMS) algorithm is the most popular and broadly accepted algorithm. It is a kind of feedforward control and has advantages such as simplicity, plain implementation, and noticeable performance. However, it is not appropriate to manage a variety of complex signals such as signals with gear mesh frequencies, modulated signals, etc. Especially, although primary frequencies of a targeted noise are commonly targeted, its side bands could not be focused and the remainders will show nearly the same level of noise.

The main objective of this work is to propose novel adaptive filtering algorithms with improved least mean squares algorithms employing sliding mode control (SMC) for faster and effective convergence, stability, and adaptiveness, and model predictive sliding mode control (MPSMC) for more robust and optimized tracking performance against signals with complex spectra. With supports of the model-based controller design and a nonlinear control methodology, it is expected that the model-based controller covers effective management of multi-spectral frequency components the nonlinear controller covers uncertainties, nonlinearities, and other unexpected situations.

There are three performance criteria for this research. The proposed algorithms should be able to control signals with complex frequency spectra such as frequency modulated signal with better tracking performance and faster convergence. Next, it should be stable with existence of secondary path dynamics. And finally, it should have robustness or adaptiveness with uncertainty in the reference signal or the system model in the case of model-based control.

Proof of the proposed techniques is verified with numerical simulations and experiments for several cases. In addition, in order to check feasibility in real system, a single active strut with a piezoelectric actuator is prepared and three novel algorithms and a conventional algorithm are tested with a sample multi-spectral signal.

Adaptive filtering systems are generally applied for active noise and vibration control, by performing numerical works to gain desired pattern of secondary path signals, which are utilized for reducing or cancelling the output of target systems. In those systems, recursive algorithms are employed to update the coefficients of the transversal filter automatically at every time step and among these algorithms, least mean squares (LMS) algorithm is the most popular and broadly accepted feedforward control, since its formulation is relatively simple and shows good performance¹¹. Filter coefficients are regulated using the error signal, which is the difference between the filter output and the unwanted signal, and the reference signal with frequency elements targeted at the current sampling time. Equation 1 shows the LMS algorithm

$$\hat{w}_{k+1} = \hat{w}_k + \mu u_k \cdot e_k \quad (1)$$

where \hat{w}_k , u_k , and e_k are a vector of the filter coefficients, a reference vector, and an error vector, respectively. Also, μ indicates a parameter that determines the rate of convergence and the stability. The key process on the derivation of the algorithm is minimizing the mean squared value of the error assisted by the steepest descent concept. This quadratic form of the cost function guarantees that the minima always exist such that the adaptive filtering system works properly. Figure 3 shows the schematic of the adaptive filtering systems.

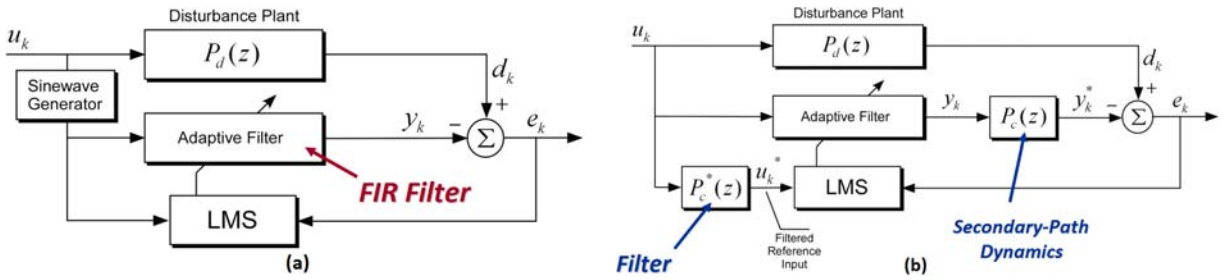


Figure 3 Adaptive filtering system with (a) LMS and (b) FX-LMS

On the other hand, the system tends to become unstable or converge to an erroneous state in real experimental situations because of the effects from system's secondary path induced from actuators, sensors, filters, data acquisition systems, amplifiers, etc. The filtered-X least mean squares (FX-LMS) algorithm can manage this by employing filtered reference signal with the system model of the secondary-path, which is identified in advance. The filter $P_c^*(z)$ has the equal amount of time delay or phase shift with the secondary-path $P_c(z)$, which is the primary contribution. In addition, the adaptive filtering system with LMS algorithm can be modified to a system with multiple transversal filters aligned in parallel, in order to deal with multiple frequencies more effectively. The same number of adaptive filters with the number of spectral contents of an unwanted signal is employed and the canceling signal is generated by adding all the filter outputs¹². Despite several advantages of the LMS, it has critical defects such as misadjustment and immoderate mean squared error that intensify linearly according to an unwanted signal's magnitude¹³. Additionally, the simplifying assumptions in the process of deriving the LMS algorithm result in noisy and discrepant tracking¹⁴.

3. SLIDING MODE LEAST MEAN SQUARES ALGORITHMS

Three novel enhanced algorithms for adaptive filtering systems, which are based on the conventional LMS algorithm, are proposed in this section. Model-based and nonlinear control methodologies are applied for developing new controllers such that predominant performance can be observed with regards to faster convergence, adaptivity, stability, and feasibility to manage complex signals. Two sliding mode methods and one model-based method are introduced and their formulations and characteristics are discussed.

3.1 Sliding mode LMS method 1

It is observed that there are several drawbacks of the LMS algorithm such as excessive mean squared error, misadjustment, and noisy tracking performance. In order to overcome those and expect better results, the sliding mode control (SMC) is employed to create a modified LMS algorithm, which is named as the sliding mode LMS method 1 (SM-LMS 1), for faster, more precise, and more optimal convergence. SMC is one of the most popular nonlinear controllers in modern control field. The location of the system states are regulated to change between both sides of a sliding surface with discontinuous inputs and finally arrives on it, which is called sliding mode, and the system shows expected system characteristics¹⁵. Thus, a sliding surface is defined in the error state plane as described in Figure 4 and SM-LMS 1 employs the squared sliding mode as a cost function, while a cost function for the LMS is the squared error between the unwanted signal and the filter output. It is anticipated that unnecessary movement of the states in transient would be attenuated by more effective coefficient changes of the filters. The key feature is the use of nonlinear controllers, which can handle disturbances, uncertainties, and of course, nonlinearities of the system.

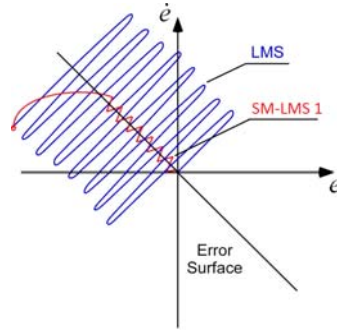


Figure 4 Concept of sliding mode LMS method 1 (SM-LMS 1)

At first, equation (2) shows the error signal e_k defined as a difference between the filter output y_k , which can be calculated with the filter coefficients w_k and the reference signal u_k , and the unwanted signal d_k . Then, sliding mode s_k is chosen in the error state plane and a cost function ξ is defined as an expected value of squared sliding mode, as described in equation (3).

$$e_k = d_k - y_k \text{ where } y_k = \sum_{j=0}^N w_{j,k} \cdot u_{k-j} \quad (2)$$

$$s_k = \alpha e_k + e_{k-1}, \quad \xi = E[s_k^2] \quad (3)$$

From equation (2) and (3), the cost function ξ can be expanded as equation (3).

$$\begin{aligned} \xi = \alpha^2 \{ & E[d_k^2] - 2w_k^T P_k + w_k^T R_k w_k \} + \{ E[d_{k-1}^2] - 2w_{k-1}^T P_{1,k} + w_{k-1}^T R_{1,k} w_{k-1} \} \\ & + 2\alpha \{ E[d_k \cdot d_{k-1}] - w_{k-1}^T P_{2,k} - w_k^T P_{3,k} + w_k^T R_{2,k} w_{k-1} \} \end{aligned} \quad (3)$$

Next, equation (3) is partially differentiated with vector w_k to obtain the gradient of the cost function ξ as presented in equation (4). Equation (5) defines the cross correlation vectors P and the auto correlation matrices R .

$$\nabla \xi = -2\alpha^2 P_k + 2\alpha^2 R_k w_k - 2\alpha P_{3,k} + 2\alpha R_{2,k} w_{k-1} \quad (4)$$

$$\text{where } P_k = E[u_k \cdot d_k], \quad P_{3,k} = E[u_k \cdot d_{k-1}], \quad R_k = E[u_k \cdot u_k^T], \quad R_{2,k} = E[u_k \cdot u_{k-1}^T] \quad (5)$$

It is assumed that the expected values of the squared sliding mode, the auto correlations, and the cross correlations can be substituted with direct estimations at the current time step for lessening the calculation burden of the algorithm.

$$\hat{\xi} = s_k^2 \text{ and } \hat{P}_k = u_k \cdot d_k, \hat{P}_{3,k} = u_k \cdot d_{k-1}, \hat{R}_k = u_k \cdot u_k^T, \hat{R}_{2,k} = u_k \cdot u_{k-1}^T \quad (6)$$

From equation (4) and (6), direct estimation of the gradient $\hat{\zeta}$ is derived as in equation (7).

$$\nabla \hat{\xi} = -2\alpha^2 \cdot u_k \cdot e_k - 2\alpha \cdot u_k \cdot e_{k-1} \quad (7)$$

Finally, the steepest descent method¹¹ in equation 8 is utilized for the formulation regulating the change of the filter coefficients. Integrating it with equation (7), the sliding mode LMS method 1 is derived.

$$w_{k+1} = w_k - \frac{\mu}{2} \nabla \hat{\xi} \rightarrow \hat{w}_{k+1} = \hat{w}_k + \mu \cdot u_k \cdot \left\{ \alpha^2 \cdot e_k + \alpha \cdot e_{k-1} \right\} \quad (8)$$

This new approach is investigated with a simple simulation example. The disturbance plant is assumed to be a second order system with $\omega_n = 20 \text{ Hz}$ and $\zeta = 0.1$ and a single frequency sinusoidal input at 100 Hz is utilized as a reference input and also as an input to the disturbance plant.

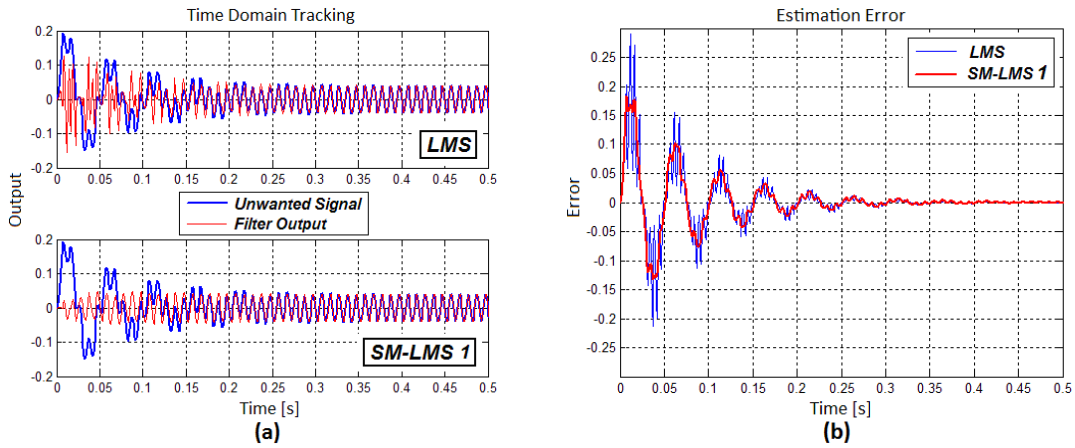


Figure 5 LMS vs. SM-LMS 1 results; (a) Time domain tracking and (b) Estimation error

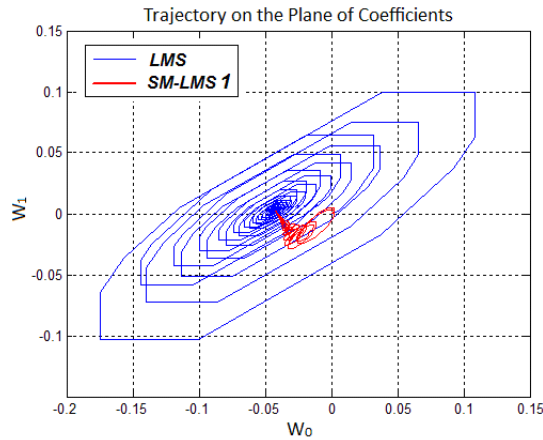


Figure 6 LMS vs. SM-LMS 1 result; Trajectory on the plane of coefficients

Figure 5 (a) compares time domain tracking performance for both cases of LMS and SM-LMS 1. The LMS shows worse tracking in transient state when the signal experiences hasty changes of direction, while SM-LMS 1 reduces the fluctuations significantly. This can be verified in the comparison of estimation errors in figure 5 (b) and the trajectory on

the plane of coefficients in figure 6. The SM-LMS 1 demonstrates invariable and forthright trajectory to the steady state such that the change of the filter coefficients is more effective and faster, without unnecessary motion of the system. Note that the filter coefficients from the both algorithms approach to the same values in the steady state, which addresses that the sliding mode control applied to the SM-LMS 1 does not alter the original system characteristics, but purely cooperate with the LMS for better performances.

3.2 Sliding mode LMS method 2

Even though the SM-LMS 1 shows better performance than the conventional LMS in terms of the rate of convergence and the effectiveness, the overall system is still a kind of feedforward control system. Thus, its ability is limited for dealing with noises, disturbances unexpected, several uncertainties, and time-variant system dynamics. A state-of-the-art structure of the adaptive filtering system with a feedback loop is proposed and the sliding mode controller is utilized as a feedback controller, which is entitled as the sliding mode LMS method 2 (SM-LMS 2). It is expected that the feedback control is covering the limitation of the feedforward control in the conventional control systems. Figure 7 shows the proposed system structure. A modified filter output y_k is generated by subtracting the output of the feedback controller $y_{C,k}$ with the primary filter output $y_{F,k}$. Then, it is utilized as an input to the secondary path $P_c(z)$. Equation (9) describes the compensating signal from the feedback loop.

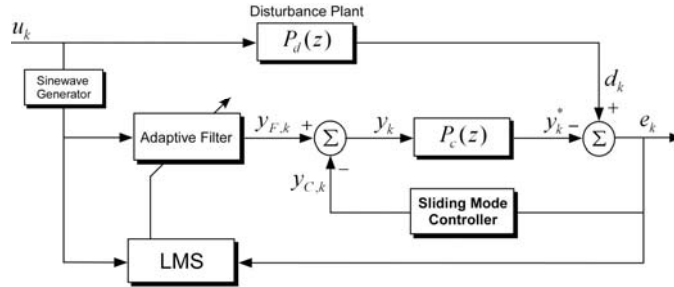


Figure 7 Sliding mode LMS method 2 (SM-LMS 2)

A sliding mode s_k is selected in the error state plane and an appropriate parameter M is selected to ensure the existence of the sliding mode.

$$y_{C,k} = -M \operatorname{sgn}(s_k) \text{ where } s_k = \alpha e_k + e_{k-1} \quad (9)$$

When the sliding mode control is applied to a real system, the chattering phenomenon should be managed properly in order to prevent damage or failure of actuators. A boundary layer method²⁰ is commonly implemented and the discontinuous input in equation (9) is changed to the continuous input as in equation (10). On the other hand, some degradation of the control system's performance should be considered since the real sliding mode does not happen because of the continuous input.

$$y_{C,k} = \begin{cases} -M \operatorname{sign}(s_k) & \text{if } s > \varepsilon \\ -\frac{M}{\varepsilon} \cdot s_k & \text{if } s \leq \varepsilon \end{cases} \quad (10)$$

The SM-LMS 2 is also examined with some simulation examples. It is assumed that the same conditions in the previous section are applied to these examples, in terms of the plant, the input signal, and the filter. With a single sinusoidal signal, it shows similar trend as the SM-LMS 1 with respect to the rate of convergence and the effectiveness. In addition, SM-LMS 2 has several other prevalent characteristics. If the effect from the secondary path dynamics cannot be ignored, the filtered reference signal is required as discussed in the previous section. The SM-LMS 2 can deal with this issue without the filtered reference signal, which requires off-line system identification. Additionally, in case that the control system utilizes a mis-tuned reference signal, the tracking performance of the LMS becomes extremely bad. However, the SM-LMS 2 shows relatively good results and thus it can be considered more adaptive with unexpected change in environments.

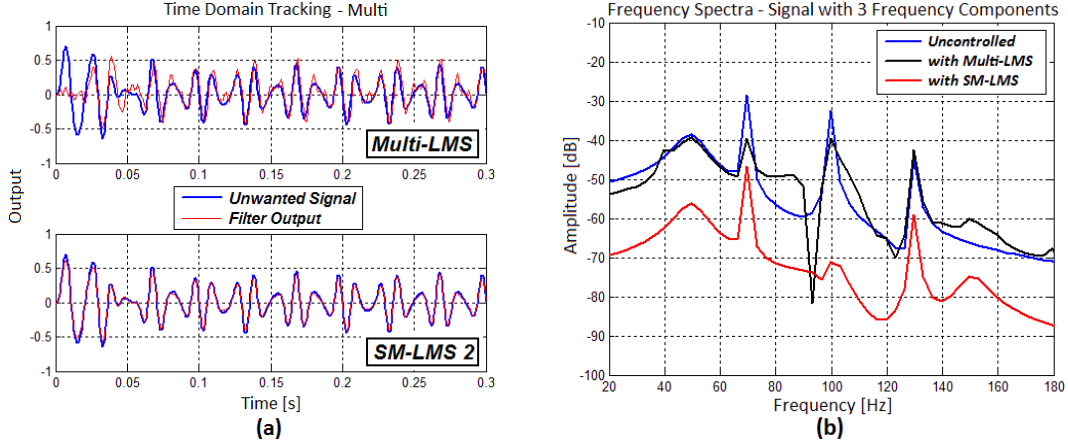


Figure 8 LMS vs. SM-LMS 2 results; (a) Time domain tracking and (b) Frequency spectra

Moreover, the performance of the SM-LMS 2 is evaluated with multi-spectral signals by comparing with the multi-channel LMS. Even if the multi-channel LMS achieves some amount of attenuation at the fundamental peaks at 70 Hz, 100 Hz, and 130 Hz, spillover in other frequency range degrades the results. However, the SM-LMS 2 can accomplish broadband mitigations.

4. MODEL PREDICTIVE SLIDING MODE LMS

Lastly, a model-based and nonlinear controller is employed to propose an enhanced LMS algorithm. This concept is basically the same as the SM-LMS 2, with the exception of the controller in the feedback loop. The model predictive sliding mode control (MPSMC) is utilized in the feedback loop, which has prevalent characteristics for nonlinear systems and signals with complex spectra, and it is entitled as the model predictive sliding mode LMS (MPSM-LMS). The key attribute of the model-based control is that it can design the effective trajectory of the system states supported by the pre-identified model, which is utilized for designing the controller. Figure 9 shows the block diagram of the MPSM-LMS system.

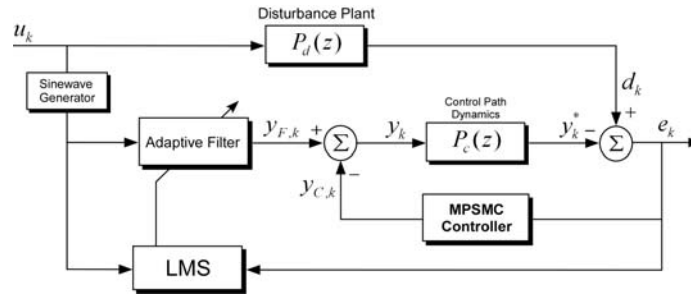


Figure 9 Model predictive sliding mode LMS (MPSM-LMS)

Model predictive sliding mode control is an enhanced, novel, and robust nonlinear model-based control technique and it is originated from the basic structure of the sliding mode control with the support of the model predictive control (MPC)^{16,17}. It is established as a way to prevail the disadvantages of the SMC, which includes the chattering and the saturation phenomena. Since real systems are not able to change its actuating direction at extremely high or infinite frequency, ideal sliding mode cannot be fulfilled and the chattering phenomenon would take place causing system's instability, fatigue, and failure. Moreover, the sliding mode should be enforced after one sampling instant in case of discrete sliding mode control and it engenders the saturation effect on the controller, which results in system's mis-operation or destabilization¹⁸. Thus, the idea of MPSMC is to move the system states to force the sliding mode with optimal

trajectory, compared with discontinuous behavior of the SMC, and finally become to have an expected system characteristics. For this optimal trajectory, the model predictive control (MPC) is introduced, a class of model-based control approaches, and a process model for the given system is implemented to predict the future plant outputs up to a pre-defined sampling instant¹⁹. This technique makes the system states follow the reference as proximate as possible by generating optimized control inputs with past, present, and future inputs and outputs. The sliding surface is selected first and a prediction equation for the sliding mode is derived, while a prediction equation for the system states is derived in case of the model predictive control. Figure 10 shows the concept of the MPSMC.

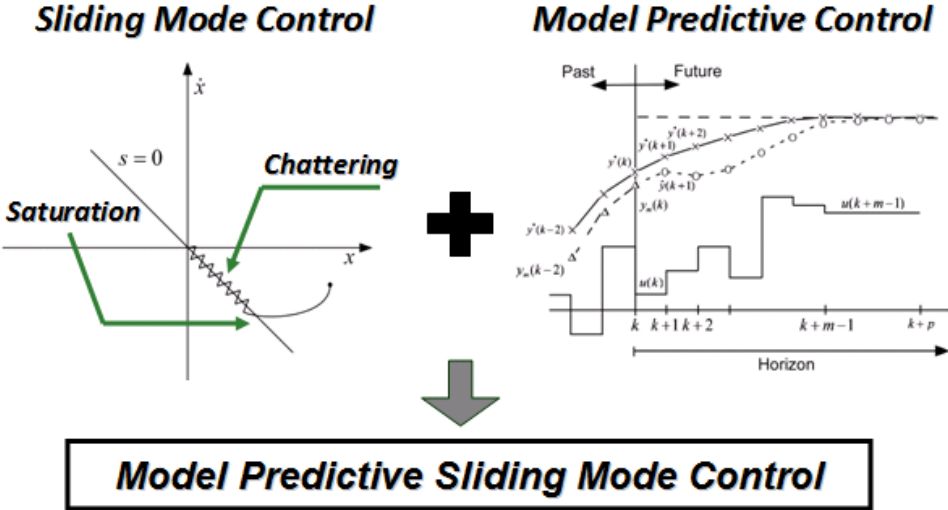


Figure 10 Concept of the model predictive sliding mode control

In the concept of MPSM-LMS, the sliding mode control, as a nonlinear control technique, is conducive to handling unexpected disturbances, nonlinearities, and uncertainties of several factors, whereas the model predictive sliding mode control, as a model-based control methodology, contributes to optimized control for multi-spectral signals assisted by a process model. It is not trivial to obtain a system model of the disturbance plant $P_d(z)$ practically. However, if the model can be identified very roughly, the designed MPSMC works properly. Through an identical manner with the SM-LMS 2, a modified filter output is utilized for an input to the secondary path dynamics, which is the difference between the original filter output and the output of the feedback loop.

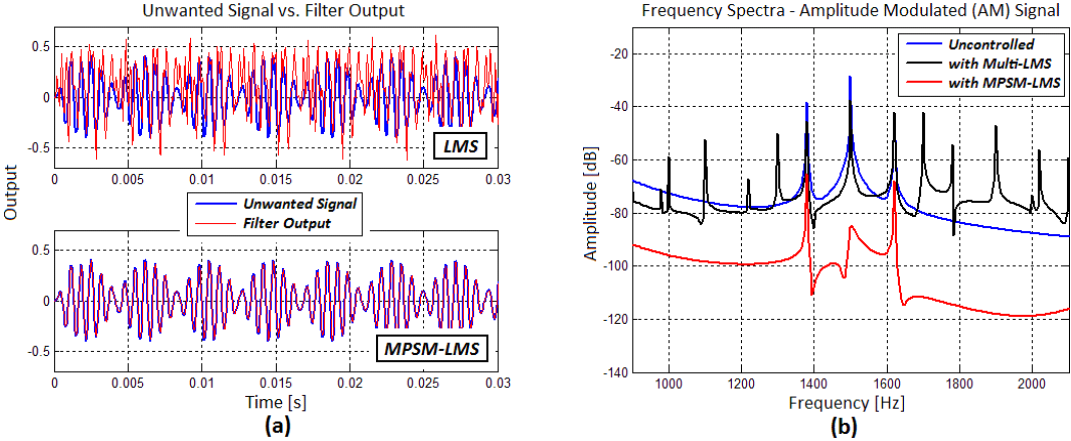


Figure 11 LMS vs. MPSM-LMS on (a) time domain tracking and (b) frequency spectra with AM signal

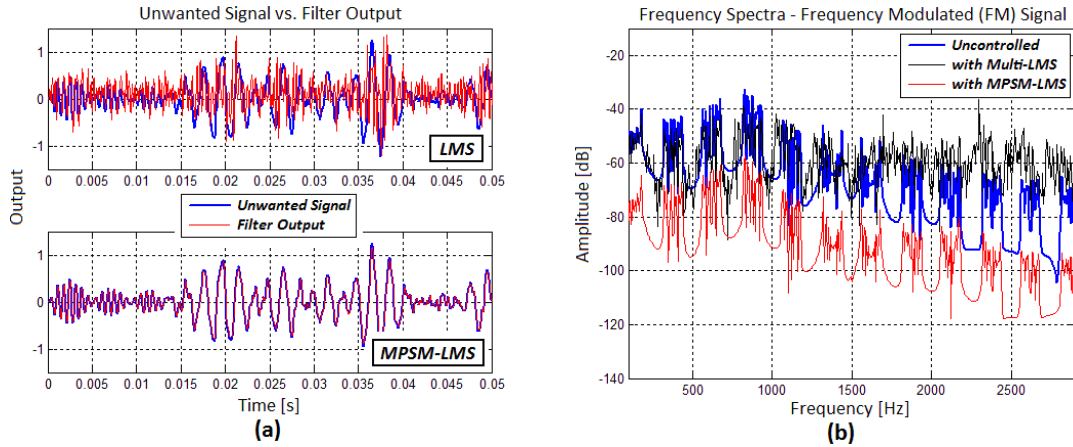


Figure 12 LMS vs. MPSM-LMS on (a) time domain tracking and (b) frequency spectra with FM signal

To investigate the ability of this algorithm, several types of signals with complex spectra are employed, including amplitude modulated and frequency modulated signals. These are especially selected since they contain side bands as the noise and vibration signals from the gearbox. As described in figure 11 and 12, although multi-channel LMS shows some reduction at the fundamental peaks, serious spillovers occur in other frequency range. With MPSM-LMS, broadband attenuation is possible and the estimation error is also significantly decreased.

In addition, this technique shows comparable features with the SM-LMS 2. First, it can act like FX-LMS without using filtered reference signal when the secondary path dynamics cannot be ignored. Secondly, it shows better tracking performance if the reference signal is mis-tuned or if the plant model for designing MPSMC is mis-tuned. Therefore, MPSM-LMS shows faster and more optimal convergence than conventional algorithms, stability with secondary path dynamics, and adaptiveness.

5. EXPERIMENTAL STUDIES

For experimental verification of proposed algorithms, a single active strut with a piezoelectric actuator is prepared and it is installed vertically to a shaker to make a condition with external excitations. An aluminum strut is employed with a piezoelectric actuator attached in the middle and the acceleration is measured at the tip of the strut, as shown in figure 13.

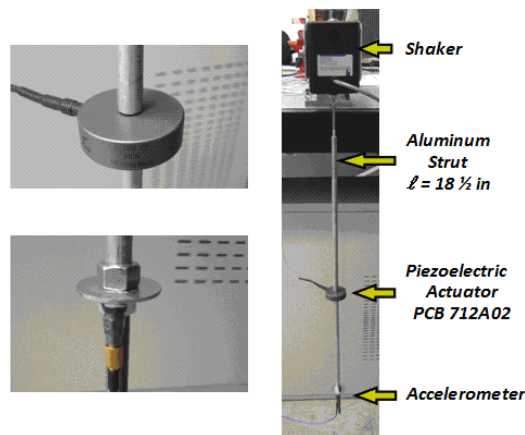


Figure 13 Single strut experiment

In the case I, the LMS algorithm is tested with a signal of three frequency components. Only one frequency at 50 Hz is targeted and 12 dB reduction at 50 Hz is achieved. In the case II, the SM-LMS 1 is tested under the same condition. Although only one frequency at 50 Hz is targeted in this case, reductions are observed at both 50 Hz and 70 Hz with 13 dB and 5 dB respectively.

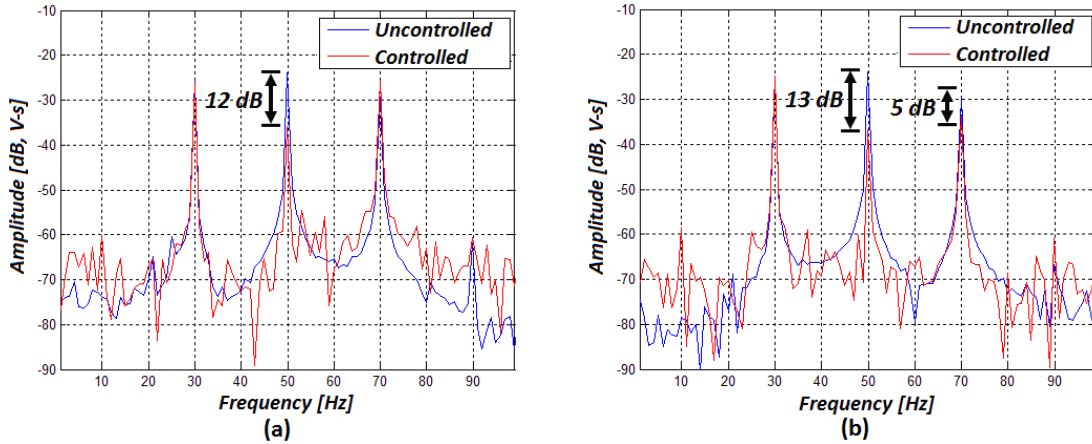


Figure 14 Frequency spectra of (a) case I with LMS and (b) case II with SM-LMS 1

In the case III, vibration reduction with the SM-LMS 2 is much better than the previous cases. Targeted peak at 50 Hz shows 22 dB reduction and other peaks at 30 Hz and 70 Hz show 20 dB and 3 dB reductions respectively. Therefore, two sliding mode methods of the modified LMS algorithms can accomplish reductions of multiple peaks with just one targeted peak at a certain frequency.

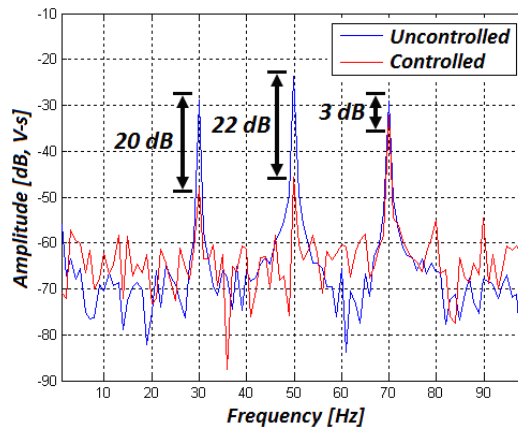


Figure 15 Frequency spectra of case III with SM-LMS 2

6. CONCLUSION

In this research, three novel enhanced LMS algorithms are proposed. The sliding mode control and the model predictive sliding mode control are implemented to obtain benefits from both nonlinear and model-based controllers. A modified structure of the adaptive filtering system with a feedback loop improves stability, adaptiveness, and capability of dealing with multi-spectral signals. Proposed algorithms are numerically verified in several cases with single sinusoidal, multi-spectral, and modulated signals. Above all techniques, the model predictive sliding mode LMS is likely to be the most appropriate for signals with complicated spectral contents. In addition, experiments are performed with a single strut and

the results show that the sliding mode LMS method 1 and 2 could reduce peaks that are not even targeted, which means that the tracking performance of these two are better than the conventional ones.

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