INTRODUCTION

Elastomeric and hydraulic bushings are widely used in vehicle suspension, body and engine sub-systems for improved handling and ride performance characteristics and to reduce vibration and noise [1,2,3,4,5,6]. Such devices are often expected to provide a high viscous damping coefficient \( c \) and stiffness \( k \) for large amplitude excitations at lower frequencies. Further, lower \( k \) and \( c \) values are needed for controlling structure-borne noise at moderate to higher frequencies [1,2,7]. Since elastomeric bushings cannot satisfy such conflicting requirements, many fluid-filled bushing designs have been developed and utilized in vehicles [1-2], as evident from some articles [7,8,9,10] and patents [11,12,13,14,15,16,17,18,19,20]. Even though many patents [11,12,13,14,15,16,17,18,19,20] claim certain performance features, they provide no analytical justifications or specific stiffness properties. Further, very few scientific articles have addressed the hydraulic bushing characterization and modeling issues [7,8,9,10]. Therefore, this article aims to fill this void and experimentally and analytically study the dynamic characteristics of a family of hydraulic bushing design.

A typical fluid-filled bushing consists of two almost identical elastomeric chambers between inner and outer metal sleeves. These chambers are usually filled with the anti-freeze mixture, and they communicate fluid via a long passage (inertia track), and/or a short passage (or a controlled leakage orifice [1]). The relative deflection between the inner and outer metal sleeves causes chamber pressures to vary, and thus fluid flows back and forth through inertia track type and orifice-like passages, which provides effective damping over the desired frequency range, depending on the fluid system configuration and flow conditions. An accurate estimation of the dynamic stiffness of bushings is important for the prediction of dynamic loads in a vehicle sub-frame and suspension system. Although fluid-filled bushings exhibit frequency-dependent and amplitude-sensitive properties, a linear time-invariant (LTI) model must be developed first to estimate dynamic properties and to diagnose and tune such devices over the lower frequency regime (say from 1 to 50 Hz). Several different configurations of the hydraulic bushing should be experimentally and analytically examined to gain a better understanding of the underlying physics. Only radial damping is considered because the relative displacement between inner and outer metal parts is assumed to be only in the radial direction. Accordingly, specific objectives for this article are as follows: (a) Design a laboratory device with controlled internal configurations and conduct experiments on a production bushing and the laboratory prototype under sinusoidal excitation; (b) Analyze measured dynamic...
stiffness results and compare the dynamic properties of different configurations and examine their spectrally-varying properties amplitude-dependence; (c) Develop linear time-invariant (LTI) models of a fluid-filled bushing with alternate configuration using the lumped fluid parameter approach; (d) Validate the proposed models by comparing predictions with measurements under sinusoidal excitation only.

**EXPERIMENTAL STUDIES**

The static and dynamic properties of a practical fluid-filled bushing (designated Y1) is evaluated first using the non-resonant elastomer test machine (MTS 831.50, [21]) under a mean load \( f_m \). A sinusoidal displacement \( x(t) = A \sin \omega t \) is applied to the inner metal part in the radial direction, from 1 to 50 Hz in 1 Hz increments under amplitude \( X \), where \( \omega \) is the excitation frequency (rad/s), and \( X = 2A \) is the peak to peak (pp) amplitude. The force transmitted to the rigid base \( f_T(t) \) is typically measured. The dynamic stiffness is defined as:

\[
K_d(\omega) = |K_d(\omega)| \cos \phi_K(\omega) + i|K_d(\omega)| \sin \phi_K(\omega),
\]

(1)

where \( i = \sqrt{-1} \), and \( |K_d(\omega)| \) and \( \phi_K(\omega) \) are the magnitude and loss angle of \( K_d(\omega) \), respectively. In this paper, results are presented in the dimensionless form, and thus \( |K_d| \) is normalized by its static stiffness. The frequency ratio is defined as the excitation frequency \( (f = \omega/2\pi) \) divided by the frequency \( f^*_\phi \), at which the loss angle of a laboratory bushing prototype is maximum. One typical set of the measurements under three excitation amplitudes \( (X = 0.1, 0.5 \) and \( 1.0 \) mm (pp)) is shown in Figure 1. Significant amplitude and frequency dependence is observed from the \( K_d(\omega) \) spectra. Both \( |K_d| \) and \( \phi_K \) decrease as \( X \) is increased. The loss angle achieves its peak at \( f^*_\phi \) and then gradually decreases with \( \omega \), while \( |K_d| \) continues to increase until \( f^*_\omega \), which is about two times \( f^*_\phi \), and then settles down.

Fluid passages in practical bushings usually have irregular geometry and may be constructed with elastomer or metal. Moreover, experimentation with many bushing samples from different manufacturers would pose a difficult task. Thus, a laboratory device which can provide insights into various aspects is designed and fabricated for dynamic characterization and scientific examination. This device is able to provide different combinations of long fluid passage (inertia track) and short passage with adjustable flow restriction. Initial experiments are conducted on five configurations (designated B1 to B5, as listed in Table 1 and Figure 2) of this prototype using the MTS elastomer test machine. Additionally, two limiting cases (B0 with all the fluid passages closed and BR as the drained bushing) are also evaluated in the experiment. Similar to the dynamic stiffness measurements of the production bushing, a sinusoidal displacement excitation \( x(t) = A \sin \omega t \) is applied to the prototype device under a mean load. The dynamic stiffness \( K_d(\omega) \) spectra, along with the steady state time histories of the transmitted force \( f_T(t) \) are measured from 1 to 60 Hz with 1 Hz increment at two amplitudes, \( X = 0.1 \) and \( 1 \) mm. In addition, the internal pressures of two hydraulic chambers, \( p_1(t) \) and \( p_2(t) \), are measured by the dynamic pressure transducers installed within the fluid chambers of the prototype device.

**Table 1. Configurations of the prototype hydraulic bushing**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Description (Also see Fig. 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>One long flow passage of diameter ( d_f )</td>
</tr>
<tr>
<td>B2</td>
<td>One short fluid passage with a restriction of diameter ( d_o )</td>
</tr>
<tr>
<td>B3</td>
<td>One short fluid passage with a restriction of diameter ( d_o/3 )</td>
</tr>
<tr>
<td>B4</td>
<td>One long passage and one short passage with a restriction of diameter ( d_o ) in parallel</td>
</tr>
<tr>
<td>B5</td>
<td>One long passage and one short passage with a restriction of diameter ( d_o/3 ) in parallel</td>
</tr>
<tr>
<td>B0</td>
<td>All fluid passages closed</td>
</tr>
<tr>
<td>BR</td>
<td>Drained bushing</td>
</tr>
</tbody>
</table>
Figure 2. Multiple configurations of the hydraulic bushing constructed for controlled laboratory studies. (a) Configuration B1 with long fluid passage of diameter \( d_i \); (b) Configuration B2 with short flow passage with restriction of diameter \( d_o \); and configuration B3 of diameter \( d_o/3 \); (c) Configuration B4 with parallel long and short flow passages with \( d_o \) and B5 with \( d_o/3 \).

ANALYTICAL MODELS

Fluid Model I with Two Parallel Flow Passages

A fluid system model of the hydraulic path of a bushing with two passages in parallel is proposed in Figure 3, with many simplifying assumptions, based on lumped parameter method. These include an internal long passage such as the inertia track (#i) though shown as external tubing for the sake of clarity in Figure 3, and/or a short passage (#s) with a flow restriction or controlled leakage element (such as the orifice). Assuming the linear system, the long and short flow passages are represented by the fluid inertances \( I_i \) and \( I_s \) and fluid resistance \( R_i \) and \( R_s \), respectively. The compliances of two fluid chambers (#1 and #2) are \( C_1 \) and \( C_2 \), respectively, with effective pumping areas \( A_1 \) and \( A_2 \). The rubber element (#r, parallel with the hydraulic path, not shown in Figure 3) is modeled by rubber stiffness \( k_r \) and viscous damping coefficient \( c_r \).

Figure 3. Fluid model I of the hydraulic path with a long flow passage (inertia track, #i) and a short flow passage with restriction (#s).

Static and dynamic displacement excitations are applied to the inner metal part while the outer metal sleeve is relatively fixed. The mean force \( \bar{f} \) transmitted to the outer sleeve under a static displacement \( \bar{x} \) is as follows, where \( \bar{p} \) is the fluid chamber pressure under static equilibrium:

\[
\bar{f} = k_r \bar{x} + (A_s - A_r) \bar{p}.
\]  

The dynamic displacement excitation \( x(t) \), from the static equilibrium, is applied to the bushing under a mean load \( \bar{f} \). By applying the continuity equation to two fluid control volume elements (1 and 2), the following equations are obtained, where \( q_i(t) \) and \( q_s(t) \) are the volumetric flow rates through the long track and short passage, and \( p_1(t) \) and \( p_2(t) \) are the dynamic pressures inside the two fluid chambers,

\[
-A_i \dot{x}(t) - q_i(t) - q_s(t) = C_1 \dot{p}_1(t),
\]  

\[
-A_s \dot{x}(t) + q_i(t) - q_s(t) = C_2 \dot{p}_2(t).
\]

The application of the momentum equation to two flow passages (i and s) yields the following:

\[
p_1(t) - p_2(t) = I_i \dot{q}_i(t) + R_i q(t),
\]  

\[
p_1(t) - p_2(t) = I_s \dot{q}_s(t) + R_s q_s(t).
\]
The dynamic force is transmitted to the outer sleeve by rubber and hydraulic paths. Thus, the total transmitted force \( f_T(t) \) is defined below where \( f_{Tr}(t) \) is the rubber path and \( f_{Th}(t) \) the hydraulic path:

\[
f_T(t) = f_{Tr}(t) + f_{Th}(t) = \left(k_x x(t) + c_x \dot{x}(t)\right) + \left(A_p p_z(t) - A_p p_l(t)\right).
\]  

By transforming Eqs. (3), (4), (5) to the Laplace \((s)\) domain and assuming zero initial conditions, the following dynamic stiffness \( K_d(s) \) is obtained, where \( K_{dh}(s) \) and \( K_{dr}(s) \) are the dynamic stiffness of the hydraulic and rubber path, respectively, and \( Z_i = I_s + R_i \) and \( Z_s = I_s + R_s \) are the fluid impedance \((Z)\) of the long (inertia track) and short (orifice-like) passages respectively.

\[
K_d(s) = k_c + c_s s + \frac{(A_c^2 C_1 + A_c^2 C_2) Z_s (A_1 - A_2)}{C_c C_s Z_s + (C_1 + C_2) (Z_1 + Z_2)}.
\]  

When the short fluid passage behaves like a sharp-edged orifice, \( I_s \approx 0 \) can be assumed at the lower frequencies. Thus, Eq. (6) can then be converted into a reduced order form with standard parameters as follows, where \( R_o \) is the fluid resistance of the orifice-like element, \( \gamma \) is the static stiffness of the hydraulic chambers, \( \omega_n \) and \( \omega_n \) \((\text{rad/s})\) are the natural frequencies, and \( \zeta_1 \) and \( \zeta_2 \) are the damping ratios:

\[
K_d(s) = k_c + c_s s + \frac{\gamma}{s^2 + 2\zeta_1 \omega_n s + \omega_n^2},
\]  

\[
\omega_n = \sqrt{(A_1 - A_2)^2 \left(R_1 + R_2\right) \left(A_c^2 C_1 + A_c^2 C_2\right) I_c R_c}, \quad \omega_n = \sqrt{(C_1 + C_2) \left(R_1 + R_2\right) I_c R_c},
\]  

\[
\zeta_1 = \frac{(A_1 - A_2)^2 \left(R_1 + R_2\right) \left(A_c^2 C_1 + A_c^2 C_2\right) R_c I_c}{2\sqrt{(A_1 - A_2)^2 \left(A_c^2 C_1 + A_c^2 C_2\right) (R_1 + R_2) I_c R_c}},
\]  

\[
\zeta_2 = \frac{C_c C_s R_c I_c + (C_1 + C_2) I_c}{2\sqrt{(C_1 + C_2) (R_1 + R_2) I_c R_c}}, \quad \gamma = \frac{A_c^2}{C_1} + \frac{A_c^2}{C_2}.
\]  

The frequency response \( K_d(\omega) \) is obtained by substituting \( s = i\omega \) into the Laplace transfer functions, where \( i = \sqrt{-1} \).

**Fluid Model II with Only Inertia Track**

Some practical fluid-filled bushings have only one long passage (inertia track) \([11,12,13,14,17,18,19,20]\). Thus, fluid model II is developed without the short passage by setting \( q_s(t) = 0 \) in Eqs. (3) and (4) and transforming them into the Laplace domain with zero initial conditions. The resulting \( K_d(s) \) expression is now given as follows:

\[
K_d(s) = k_c + c_s s + \frac{(A_c^2 C_1 + A_c^2 C_2) Z_s (A_1 - A_2)}{C_c C_s Z_s + (C_1 + C_2) (Z_1 + Z_2)}.
\]  

Eq. (8) is also converted into a standard form as follows, where \( \gamma \) is the same as that in Eq. (7):

\[
K_d(s) = k_c + c_s s + \frac{s^2 + 2\zeta_1 \omega_n s + \omega_n^2}{s^2 + 2\zeta_2 \omega_n s + \omega_n^2},
\]  

\[
\omega_n = \sqrt{(A_1 - A_2)^2 \left(\left(A_c^2 C_1 + A_c^2 C_2\right) I_c\right)},
\]  

\[
\omega_n = \sqrt{(C_1 + C_2) \left(\left(C_1 + C_2\right) I_c\right)},
\]  

\[
\zeta_1 = \frac{1}{2} \sqrt{(A_c^2 C_1 + A_c^2 C_2) R_c^2 \left(\left(A_1 - A_2\right) I_c\right)},
\]  

\[
\zeta_2 = \frac{1}{2} \sqrt{C_c C_s R_c I_c \left(\left(C_1 + C_2\right) I_c\right)}.
\]  

**ANALYSIS OF MEASUREMENTS AND FREQUENCY DOMAIN PROPERTIES**

One typical set of dynamic stiffness measurements under excitation amplitude \( X = 0.1 \text{ mm} (\text{p-p}) \) is shown in Figure 4. As mentioned above, \( |K_d(\omega)| \) and \( \phi_{K_d}(\omega) \) are defined as the stiffness magnitude (modulus) and phase (loss angle) of \( K_d(\omega) \). Significant frequency dependence is observed from the \( K_d(\omega) \) spectra of each configuration. Responses of the five configurations (of Table 1) can be categorized into three groups as seen in Figure 4. First, the dynamic stiffness spectra of configurations B1 and B5 have high peak stiffness and loss angles at lower frequency, which implies that these two configurations are mainly controlled by the long inertia track and can provide higher damping levels at lower frequencies in a narrow bandwidth. Second, the \( K_d(\omega) \) of B2 and B4 have lower peaks \( |K_d(\omega)| \) and \( \phi_{K_d}(\omega) \) at higher frequencies, which are dominated by the short, restricted fluid passage. These introduce a broader damping peak at higher frequencies. Third, the measurements of B3 yield relatively lower \( |K_d(\omega)| \) and \( \phi_{K_d}(\omega) \) values since the small diameter of the restricted short fluid passage only permits limited fluid pumping effect. In addition, B4 and B5 exhibit two different functions of a bushing with parallel long (inertia track) and short (leakage) paths; here the short path allows only limited flow (and significant resistance) when excitation amplitude is small (B5) and is forced to be wide open when the amplitude is large (B4).
Figure 4. Measured dynamic stiffness spectra of five prototype configurations at low excitation amplitude. Key: B1; , B2; , B3; , B4; , B5.

To examine the real (Re) and imaginary (Im) parts of $K_d(\omega)$, Nyquist diagrams of B1 to B5 are displayed in Figure 5. For all configurations, the imaginary part of $K_d(\omega)$ is close to zero near $\omega \approx 0$ and the real part is approximated as $\Re(K_d) = k_r + \gamma c_r/\omega^2$ from Equation 7. As the frequency increases, the Nyquist plot assumes the form of a circle, an ellipse or semi-circle depending on the fluid passage characteristics.

Figure 5. Measured Nyquist diagrams of five prototype configurations at low excitation amplitude. Key: , B1; , B2; , B3; , B4; , B5.

Asymptotically, $\Re(K_d) \to k_r + \gamma$ and $\Im(K_d) \to c_r \omega$ as $\omega \to \infty$. Comparison of the five configurations leads to the following observations. The Nyquist plot of B1 has a much larger radius, and its shape is closer to a circle. Configuration B5 has a similar shape to B1 with a slightly smaller radius, which implies that the inertia track plays a dominant role in this configuration. The Nyquist diagrams of B2 and B4 are similar and closer to an ellipse, which suggest that the short fluid passage dominates and thus yields broader damping properties. Only an arc is formed for the B3 configuration since the value of imaginary part is relatively low.

The measured dynamic stiffness of the two limiting cases, BR and B0 are shown in Figure 6 and compared with the B3 configuration. The loss angles of BR and B0 are trivial compared with other configurations which exhibit the fluid pumping effect. The $|K_d|$ of B0 can be viewed as the sum of rubber stiffness ($k_r$) and the static stiffness of two fluid chambers ($\gamma$). Observe that stiffness magnitudes of B0 and BR both slowly increase with frequency, but with different slopes. This shows that the stiffness of rubber and two fluid chambers are also frequency-dependent, though it is not as significant in B1 to B5 cases. Moreover, $|K_d|$ of B3 is close to that of BR at lower frequencies, but gradually converges to that of B0 as the frequency is increased. This implies that some air might be constrained in the flow passage and only a small amount of fluid commutes between two chambers at higher frequencies.

Figure 6. Measured dynamic stiffness spectra of two limiting cases compared with B3. Key: , B3; , BR; , B0.

Amplitude Dependence

Three configurations (B1, B3 and B4) are selected from the three categories mentioned above to examine the amplitude dependence of hydraulic bushings. Their dynamic stiffness spectra under two excitation amplitudes $X = 0.1$ and 1.0 mm are compared in Figure 7. Both $|K_d|$ and $\phi_K$ decrease as $X$ is increased, especially for the long inertia track configuration. The frequencies of peak magnitude and loss angle $f_{k_d}$ and $f_{\phi_K}$ both decrease as well. The stiffness
magnitude difference of B3 at high frequencies under $X = 0.1$ and 1 mm is higher than that of BR (the drained bushing), which implies that the two chamber compliances may increase when the amplitude is increased. The Nyquist plots of three configurations at low and high amplitudes are compared in Figure 8. When $X$ is increased, the values of both the real and imaginary parts reduce and the shape of Nyquist plots becomes less circular.

**Figure 7.** Measured dynamic stiffness spectra of three configurations of the prototype bushing. Key: , B1 with $X = 0.1$ mm; , B1 with $X = 1.0$ mm; , B3 with $X = 0.1$ mm; , B3 with $X = 1.0$ mm; , B4 with $X = 0.1$ mm; , B4 with $X = 1.0$ mm.

**Figure 8.** Nyquist diagrams of measured stiffness data. Key: , B1 with $X = 0.1$ mm; , B1 with $X = 1.0$ mm; , B3 with $X = 0.1$ mm; , B3 with $X = 1.0$ mm; , B4 with $X = 0.1$ mm; , B4 with $X = 1.0$ mm.

**EXPERIMENTAL VALIDATION OF ANALYTICAL MODELS**

Frequency domain measurements of B4 and B1 configurations are compared with the dynamic stiffness predictions of linear models I and II. Parameters are estimated by linear fluid system formulae and/or bench experiments. However, theoretical formulation of the fluid resistance [22] tends to underestimate the $R_i$ and $R_s$ values. This implies that the momentum losses at fitting, valves, and sharp entrances and exits are significant. Consequently, simple bench experiments are conducted (with the water medium at room temperature) to measure the pressure drop $\Delta p$ and mean flow rate $q$ under steady flow conditions for each fluid passage [22]. Empirical $R_i$ and $R_s$ are then obtained by relating $\Delta p$ and $q$. Figure 9 compares the predictions of model II (for configuration B1) based on analytical and empirical $R_i$. Stiffness and loss angle spectra predicted with analytical $R_i$ have much higher amplitudes compared with the measurements, but the dynamic responses with empirical $R_i$ (from the bench test) match well with experimental results. Small discrepancies in the stiffness magnitude rise beyond $2f_\phi$ due to frequency dependent properties of the rubber path. Similarly, the dynamic stiffness predictions of model I are compared with measurements of configuration B4. Good agreement between theory and experiment is also observed.

**Figure 9.** Validation of model II (with long fluid passage only). Key: , predicted by linear model II with measured $R_i$; , predicted with theoretical $R_i$; , measured dynamic stiffness of B1; , measured $f_\phi$ and $f[k]$; , predicted peak magnitude and loss angle frequencies.

The measured time histories of $f_q(t)$ and $\Delta p(t) = p_2(t) - p_1(t)$ at several excitation frequencies and amplitudes are examined using the predictions of the linear models for each configuration. For instance, the transmitted force and
differential pressure time histories estimated by model II (B1 configuration) are compared in Figure 10 with measurements under 0.1 mm (p-p) excitation at 10 Hz. Figure 11 compares the measurements and predictions of \( f_p(t) \) and \( \Delta p(t) \) for B2 configuration with \( X = 0.1 \) mm (p-p) at 30 Hz. Excellent agreements are observed for the transmitted force and pressure differential. The measured signals of \( f_p(t) \) and \( \Delta p(t) \) are transformed to the frequency domain by the Fast Fourier Transform (FFT). Only the fundamental frequency component is found to be significant, and the Fourier magnitudes at the second and third harmonics are at least 30 dB lower than the first one. This implies that the time history of transmitted force can be approximated by a single harmonic term and the sinusoidal responses of \( f_p(t) \) are well represented by the proposed linear models.

![Figure 10. Time domain sinusoidal responses of B1 with X = 0.1 mm at 10 Hz. Key: ---, measured response; ---, prediction of model II.](image)

![Figure 11. Time domain sinusoidal responses of B2 with X = 0.1 mm at 30 Hz. Key: ---, measured response; ---, prediction of model II.](image)

**CONCLUSION**

This paper focuses on the dynamic analysis of hydraulic bushings. The main contribution is the combined experimental and analytical study of a laboratory prototype with the capability of varying key design features in a controlled manner. Frequency- and amplitude-dependent characteristics of each configuration are analyzed based on the experimental measurements of the dynamic stiffness spectra and Nyquist diagrams. It is observed that the dynamic stiffness spectra dominated by the short fluid passages with restriction have lower peak stiffness and loss angle but exhibit a broader bandwidth compared with those mainly controlled by the long fluid passage (inertia track). A linear model for a fluid-filled bushing with both a long inertia track and a short passage with restriction is developed, and the model for a device with only an inertia track has been examined as well. Further, the proposed models are validated by comparing predictions of dynamic stiffness and time domain sinusoidal responses with the measurements of the prototype device. The linear models presented in this article (with many assumptions of course) are capable of analyzing most practical designs [1-2, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Additionally, such models can be utilized to examine transient responses as described in the companion paper [23]. Future work will focus on an improvement of the rubber path model and analysis of fluid path nonlinearities.

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LIST OF SYMBOLS

\( A_1, A_2 \) - effective pumping areas of fluid chambers #1 and #2
\( c_r \) - damping coefficient of rubber
\( C_1, C_2 \) - lumped compliances of fluid chambers #1 and #2
\( f \) - frequency, Hz
\( f_T \) - transmitted fore
\( f_{[\kappa]}^*, f_{[\phi]}^* \) - frequency of maximum stiffness magnitude
\( f_\phi \) - frequency of maximum loss angle
\( I_I \) - inertance of the long fluid passage (inertia track)
\( I_s \) - inertance of the short restricted fluid passage
\( K_d \) - dynamic stiffness
\( |K_d| \) - magnitude of dynamic stiffness
\( k_r \) - stiffness of rubber
\( p_1, p_2 \) - pressure in fluid chambers #1 and #2
\( q_{i} \) - volumetric flow rate of the inertia track
\( q_s \) - volumetric flow rate of the short fluid passage
\( R_I \) - fluid resistance of the inertia track
\( R_s \) - fluid resistance of the short restricted fluid passage
\( s \) - Laplace transformation variable
\( t \) - time
\( x \) - excitation displacement
\( X \) - peak to peak value of sinusoidal displacement
\( Z \) - impedance of the fluid passage
\( \gamma \) - stiffness of two fluid chambers
\( \Delta p \) - pressure differential between two fluid chambers
\( \phi_K \) - phase (loss angle) of dynamic stiffness
\( \omega, \omega_n \) - circular frequency, rad/s
\( \zeta \) - damping ratio