Effect of Local Stiffness Coupling on the Modes of a Subframe-Bushing System

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ABSTRACT
The elastomeric joints (bushings or mounts) in vehicle structural frames are usually described as uncoupled springs (only with diagonal terms) in large scale system models. The off-diagonal terms of an elastomeric joint have been previously ignored as they are often unknown since their properties cannot be measured in a uniaxial elastomer test system. This paper overcomes this deficiency via a scientific study of a laboratory frame that is designed to maintain a high fidelity with real-world vehicle body subframes in terms of natural modes under free boundaries. The steel beam construction of the laboratory frame, with four elastomeric mounts at the corners, permits the development of a highly accurate, yet simple, beam finite element model. This allows for a correlation study between the experiment and model that helps shed light upon the underlying physical phenomenon. In particular, the effect of local stiffness coupling of elastomeric bushings or mounts is demonstrated through computational modeling and experimental validation. It is seen that the joint stiffness matrices strongly influence the modal properties of a laboratory subframe-mount system. For instance, a strong correlation between the rigid body modes (r = 1 to 6) and the first three elastic modes (r = 7 to 9) is only possible when the coupling (non-diagonal) terms are included in the bushing model.


INTRODUCTION
Elastomeric components are used as engine mounts, suspension bushings, torque struts, shock and strut mounts, jounce bumpers, cradle mounts, body frame mounts, and exhaust hangers. Their geometries are shaped to provide favorable properties in certain directions based upon the diagonal terms [1]. Nonetheless, non-diagonal (coupling) terms are often unknown though they are intrinsic to the design of complex automotive assemblies [2, 3]. Elastomeric joints present many modeling challenges such as amplitude and frequency dependent properties. Their small amplitude dynamic stiffness and damping characteristics influence vehicle noise, vibration, and harshness performance; their large-amplitude static force-deflection characteristics are crucial for vehicle ride and handling performance; on the other hand, when a vehicle is driven on a rough road, elastomeric components experience large-amplitude dynamic loads [4].

To complicate the matter further, the dynamic properties of an elastomeric joint embedded within an assembly may differ from those of components due to preload and boundary conditions [2]. Real world automotive frames, engine cradles, and suspension subframes are complicated structures; consequently, there is a need for a laboratory frame design permitting combinations of bushings and frames for experimental and analytical characterization. In view of the above-mentioned challenges, this article attempts to fill this void by designing and constructing a simple, yet high fidelity laboratory frame and investigate the influence of non-diagonal elastomeric joint properties on the modal properties through computational modeling and experimental validation.

PROBLEM FORMULATION
The scope of this investigation is limited to the frequency range of the first nine modes which includes six rigid body and three elastic deformation modes (say up to 300 Hz). Only natural frequencies and mode shapes will be examined. Since only lower frequencies are examined, the elastomeric mounts will be modeled as massless with lumped linear stiffness. Thick beam (Timoshenko) theory will be used for beam flexure and the joints are selected with low damping loss factors (say less than 5% in the small amplitude regime), thus the assembled structure is assumed to have normal modes. The specific objectives of this article are as follows: 1. design and construct a laboratory ladder beam frame with corner mounted (four) elastomeric mounts in the context of
AUTOMOTIVE SUBFRAMES (ENGINE CRADLE OR REAR SUSPENSION), 2. CONDUCT MODAL MEASUREMENTS IN FREELY SUSPENDED AND CONSTRAINED BOUNDARY CONDITIONS, 3. DEVELOP A COMPUTATIONAL MODEL OF THE FRAME AND FRAME-BUSHING SYSTEM EXPERIMENT, 4. EXAMINE THE INFLUENCE OF OFF-DIAGONAL STIFFNESS TERMS ON THE EIGENSOLUTION.

**DESIGN OF A LABORATORY FRAME**

A ladder frame design was selected due to its simplicity in construction, modeling, and robustness in capturing the modal properties of real-world frames. The laboratory ladder frame (Figure 1) is comprised of two symmetrical rail beams that connect together via two symmetrical cross-member beams. The dimensions of the laboratory frame were selected using modal similitude with that of real-world frames for the first three elastic deformation modes that are briefly described in Table 1. A simple design equation is used to estimate the natural frequency of the first rail beam bending mode which can be approximated with the free-free condition beam flexure formula:

\[
f_{rail} = \frac{(\beta L)^2}{2\pi} \sqrt{\frac{E I}{\rho A L^4}}
\]

where \(\beta L \approx (2r+1)\pi/2\), \(E\) is Young's modulus, \(I\) is the area moment of inertia, \(A\) is the cross-sectional area and \(L\) is the length of the rail beam. For beam models with the same geometry, the natural frequency ratio can be scaled for different materials (from material 1 to 2) as:

\[
f_{r/2} = \frac{E_1 \rho_2}{E_2 \rho_1}
\]

where \(r = 7\) of the vehicle frame.

**COMPUTATIONAL MODEL OF FRAME**

The computational model is comprised of 20 beam finite elements shown in Figure 2. A two-node Timoshenko beam element is superposed with a longitudinal rod element to generate a two-node six degree of freedom element. This formulation assumes that elastic longitudinal and beam bending are uncoupled. The work of Friedman and Kosmatka [5] contains a complete derivation of the finite element stiffness and mass matrices for a Timoshenko beam. The finite element model of the laboratory frame is implemented into a self-written script. The bushings are modeled with lumped dynamic stiffness matrices at nodes 1, 7, 10, and 16. The axi-symmetric bushings are assumed to have a stiffness matrix of the form:

\[
k_{bushing} = \begin{bmatrix}
k_{11} & 0 & 0 & 0 & 0 & k_{61} \\
0 & k_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{11} & -k_{61} & 0 & 0 \\
0 & 0 & -k_{61} & k_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & k_{55} & 0 \\
k_{61} & 0 & 0 & 0 & 0 & k_{44}
\end{bmatrix}
\]

when one end is attached to ground. Note that the vertical and torsional stiffnesses, \(k_{22}\) and \(k_{33}\), are uncoupled (lacking terms in the same row or column) and that the \(x\) and \(z\) translational and rotational stiffness terms are assumed identical as are the coupling terms. The negative signs for \(k_{61}\) that couple the \(\delta z\) and \(\delta \theta_x\) is due to the sign convention of the kinematic coupling.

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**Table 1. Comparison of measured normalized natural frequencies (\(\bar{\omega}_j\)) of elastic deformation modes between vehicle and laboratory frames under freely suspended conditions. Natural frequencies normalized by \(r = 7\) of the vehicle frame.**

<table>
<thead>
<tr>
<th>Mode, (r)</th>
<th>Vehicle Frame, (\bar{\omega}_j)</th>
<th>Laboratory Frame, (\bar{\omega}_j)</th>
<th>Mode Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.0</td>
<td>0.9</td>
<td>Torsion</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>1.4</td>
<td>Rail bending</td>
</tr>
<tr>
<td>9</td>
<td>1.8</td>
<td>1.8</td>
<td>Cross-member bending (vehicle) Out-of-phase rail bending (lab)</td>
</tr>
</tbody>
</table>

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**Figure 1. Schematic of the laboratory frame. Bushing supports are at the four corners.**
SUBFRAME-BUSHING MODAL EXPERIMENT

Four identical elastomeric mounts are connected to the laboratory frame, one to each of the four corners as shown in Figure 1. Driving point measurements were taken in three directions at points corresponding to nodes 6 and 7. The accelerance plots are shown in Figure 3 where the first nine vibration modes, including six rigid body modes and the first three elastic deformation modes are observable. Note that modes 2 and 4 prominently appear as pairs for excitations in the y- and z-directions. This experimental results suggests that strong modal coupling exists between modes 2 and 4 (rigid body modes).

The modal parameters are estimated for this work using a polyreference least-squares complex frequency-domain method. This implementation estimates the natural frequencies, damping parameters, and mode shapes in a global sense [6]. The mapping of the modes is illustrated in Figure 4, where the natural frequencies are normalized by the first elastic deformation mode in the freely suspended condition. The mode shapes 2 and 4 visually exhibit the greatest coupling as shown in Figure 5. The coupled modes...
include motion in the z-direction coupled to rotation about the x-axis. When animated, mode shapes 2 and 4 have a gliding action where mode shape 2 has a pivot point below the frame and mode shape 4 has a pivot point above the frame.

IDENTIFICATION OF MULTI-AXIS BUSHING PROPERTIES

Noll, et al. [2] and the flow-chart replaces the mathematical details and conceptually states the procedure from an applications viewpoint.

MODELING OF SUBFRAME BUSHING EXPERIMENT

The in-plane properties of two bushings are identified and averaged for use in the subframe-bushing experiment. The remaining multidimensional joint properties are estimated based upon the axisymmetric geometry of the bushings. All four bushing properties are assumed to be identical, although component variations have been observed. The nominal bushing properties are determined to be:

\[
\begin{bmatrix}
600 \text{ N mm} & 0 & 0 & 0 & 0 & 50 \text{ KN rad} \\
0 & 320 \text{ N mm} & 0 & 0 & 0 & 0 \\
0 & 0 & 600 \text{ N mm} & -50 \text{ N rad} & 0 & 0 \\
0 & 0 & -50 \text{ N rad} & 9 \text{ N mm rad} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \text{ KN rad} & 0 \\
50 \text{ KN rad} & 0 & 0 & 0 & 0 & 9 \text{ N mm rad}
\end{bmatrix}
\]

Only the \(k_{55}\) term is unmeasured in the simplified resonant beam experiment. This parameter is identified through an iterative manner. The identified bushing properties are incorporated into the computational frame model as elastic boundary conditions. The comparison of the theory with the experiment can be accomplished through both the natural frequencies and mode shapes. Table 2 contains the measured and predicted natural frequencies of the first six rigid body modes and three elastic modes (total of nine modes). The predicted and measured natural frequencies are within \(\pm 3.0\%\). with the exception of mode 4 (a highly coupled mode) which deviates from the experiment by 21\%. The modal assurance criterion (MAC) is used to correlate the mode shapes between theory and experiment. A modal assurance criterion value of 1.00 indicates perfect correlation and it was found that the correlation contained only strong diagonal terms (high correlation shown in Figure 8) with each greater than 0.93 with the exception of mode 4 (highly coupled) which exhibited a MAC value of 0.88, still a strong correlation. The non-diagonal MAC values fall in the range of 0.00 to 0.10, with the largest values appearing between modes 2 and 4 as well as 3 and 5 (highly coupled) as depicted in Figure 9.
Table 2. Comparison of natural frequencies between laboratory frame and computational model when the frame is restrained by four bushings.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experiment (Hz)</th>
<th>Theory (with coupling) (Hz)</th>
<th>Theory (without coupling) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>36</td>
<td>47</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>46</td>
<td>49</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>58</td>
<td>51</td>
</tr>
<tr>
<td>5</td>
<td>64</td>
<td>65</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>74</td>
<td>73</td>
<td>76</td>
</tr>
<tr>
<td>7</td>
<td>119</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>8</td>
<td>168</td>
<td>167</td>
<td>167</td>
</tr>
<tr>
<td>9</td>
<td>210</td>
<td>209</td>
<td>209</td>
</tr>
</tbody>
</table>

A separate computational analysis is conducted to assess the need of the non-diagonal stiffness terms as given in Eqn. (4). Here, the non-diagonal terms are set to zero and the natural frequencies and mode shapes are recalculated. When done so, the natural frequencies of the coupled rigid body modes (modes 2, 3, 4 and 5) are most affected. The natural frequencies for the first rigid body mode and the elastic modes (modes 1, 7, 8, and 9) for this bushing-frame system are unaffected. This is due to the orthogonal placement of the bushings relative to the frame, in that the vertical bushing stiffness ($k_{22}$) is uncoupled with the other directions due to the component symmetry. The most drastic effect is noticed in the MAC values (Figure 8). Here, the MAC values for modes 2 and 4 drop from 0.88-0.93 to 0.29-0.38. Coupled mode sets 2-4 and 3-5 exhibit a similar type of motion. The difference between the two coupled mode sets is the motion of the mode set 2-4 occurs with rotation about the $x$-axis with influence of the width ($w$) of the frame, whereas, the coupled mode set 3-5 exhibits rotation about the $z$-axis with influence of the length ($L$) of the frame. Thus, the most sensitive coupled mode set is 2 and 4 since the width dimension is one-third the length. The deviation in predicted natural frequency of mode 4 is attributed to component variation within the four bushings, since only limited multidimensional properties of two bushing are identified and average properties attributed to each of the four bushings in the computational model.

Figure 8. Diagonal terms of the modal assurance criterion (MAC) between theory and experiment for the bushing-frame subsystem. Values of 1.0 indicate perfect correlation. Key: Theory with coupling terms; Theory without coupling terms.

Figure 9. All terms of the modal assurance criterion (MAC) between theory and experiment for the bushing-frame subsystem.

**CONCLUSION**

The off-diagonal terms of elastomeric joints have been previously ignored as they are often unknown. This paper overcomes this deficiency via a scientific study of laboratory frame. The frame is designed to maintain a high fidelity with real-world vehicle body subframes in terms of natural modes under free boundaries. A correlation study between the experiment and model helps shed light upon the underlying physical phenomenon. In particular, the effect of local stiffness coupling of elastomeric bushings or mounts is demonstrated and experimentally validated. It is seen that stiffness matrices strongly influence the modal properties of a laboratory subframe-mount system. A strong correlation between the rigid body modes ($r = 1$ to $6$) is only possible when the coupling (non-diagonal) terms are included in the bushing model. The methods in this paper were limited in scope to the linearized properties of the bushings in the frequency domain. Additionally, the relationship between
bushing damping properties and the bushing-frame system modal damping is not considered. Despite these limitations, the proposed approach builds a clear foundation allowing for a greater understanding of the dynamic interactions between frames and elastomeric bushings.

Although superior modeling results are achieved utilizing the non-diagonal terms, it comes at the expense of having an accurate knowledge of these properties. Conventional uniaxial elastomer test machines cannot directly measure these properties, and thus indirect methods, such as the recent technique proposed by Noll, et al. [7], must be used to identify the stiffness and damping properties.

REFERENCES


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DEFINITIONS

- $a, b$ - length parameters
- $A$ - cross-sectional area
- $C$ - viscous damping matrix
- $E$ - Young's modulus
- $F$ - frequency, Hz
- $I$ - area moment of inertia
- $K$ - stiffness
- $K$ - joint stiffness matrix
- $K$ - stiffness matrix
- $L$ - length
- $M$ - mass matrix
- $W$ - width
- $x, y, z$ - Cartesian coordinates
- $B$ - natural frequency parameter
- $Δx$ - displacement, $x$-direction
- $Δy$ - displacement, $y$-direction
- $Δz$ - displacement, $z$-direction
- $δθ_x$ - rotational displacement about the $x$-axis
- $δθ_y$ - rotational displacement about the $y$-axis
- $δθ_z$ - rotational displacement about the $z$-axis
- $Z$ - damping ratio
- $Θ$ - rotational coordinate
- $P$ - density
- $Ψ$ - mode shape vector
- $Ω$ - frequency, rad / sec
- Superscript
  - $^*$ - normalized value
- Subscript
  - $R$ - modal index