FINITE ELEMENT MODELING OF FLUID SYSTEMS USING
THE MOBILITY ANALOGY APPROACH

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Proceedings of the

1st International Modal Analysis Conference & Exhibit

November 8-10, 1982
Holiday Inn, International Drive
Orlando, Florida

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ABSTRACT

One-dimensional fluid systems can be analyzed for natural frequencies and modes using an available structural finite element program, with the aid of the mobility analogy. In this paper, the methodology, strength and limitation of the solution technique are discussed. This method is validated by considering several example cases and comparing results with theory, experiment or other numerical techniques.

INTRODUCTION

The physical variables of a dynamic system could be classified either according to the dynamic and kinematic variables (impedance approach), or according to the across and through variables (mobility approach) [1-3]. However, in order to develop an analogy between the mechanical and fluid systems, the mobility approach is often more attractive because these physical systems are generally excited by a through (flow/current) variable T(t), and the response is often expressed in terms of the across (effort/potential) variable A(t); see Table 1 for the classification of mechanical and fluid variables per mobility analogy.

The impedance/mobility analogy is used extensively to solve dynamic problems for the fluid systems [1-6]. This is because the analogous mechanical or electrical systems are easier to analyze; also, there is an abundance of solution techniques and literature available for these physical systems. Conversely, the literature dealing with the fluid transients, especially the eigenvalue solution of fluid systems, is very limited [4,7]. Cory and Hatfield [6] have demonstrated that the force-flow analogy can be used to determine natural frequencies f_n and modes Ψ_n of fluid oscillations. They used a structural finite element program, SUPERB [8], for a refinery piping example case, and obtained good agreement between SUPERB and WAVENET (a fluid transients computer program based on the method of characteristics [9]). However, in order to establish the mobility analogy solution technique, a sufficient number of example cases must be analyzed, and an adequate error analysis should be performed. This is the focus of this paper as we will consider several one-dimensional fluid systems and compare the computed results, using the mobility analogy-finite element analysis, with the solutions obtained by theory, experiment, or other numerical techniques.

SCOPE

We are interested in obtaining an eigenvalue solution (i.e. f_n and Ψ_n) of a fluid system over the plane wave regime. Accordingly we assume the following: (i) the fluid system is linear, homogeneous, undamped, and perfectly elastic, (ii) the fluid is at rest, (iii) the fluid is bounded by rigid walls and therefore solid-fluid interactions are not included, and (iv) one-dimensional plane wave propagation exists over the frequency range of interest.

METHODOLOGY

1. Convert a fluid system into a one-dimensional mechanical system using the mobility analogy:

   a. geometry: $l_m = l_{fe}$, where $l_e = l_g + \Delta$ [7,10]; $S_m = S_f$

   b. properties: $\rho_m = \rho_f$; $E_m = E_f$ (bulk modulus) ; poisson's ratio ≈ 0

   c. boundary conditions: (i) open fluid end ($p = 0$) $\rightarrow$ fixed mechanical termination ($\xi = 0$), (ii) closed fluid end ($q = 0$) $\rightarrow$ free mechanical termination ($F = 0$), or a very compliant spring, and

(iii) fluid branch ($q = q_1 + q_2$) → mechanical branch ($F = F_1 + F_2$).

2. Develop a finite element model of the analogous mechanical system such that only translation in the longitudinal direction (ξ_x) is allowed. For modeling a fluid branch, the element-overlapping method should be used to ensure this.

3. Choose parameters related to the eigenvalue solution. The number (N_M) and locations of master nodes [11] must be chosen judiciously in the x direction only, i.e. $\xi_y = \xi_z = \Theta_x = \Theta_y = \Theta_z = 0$.

4. Run the finite element model for eigenvalue solution.

5. Finally, the results of the analogous mechanical system can be interpreted for the fluid system as follows: $(f_n)_m = (f_n)_f$, and $(\Psi_n)_\xi = (\Psi_n)_p$.

RESULTS

We will now apply our method to some basic fluid components which are often encountered in the machines and piping networks. For the finite element analysis, we have used SUPERB with linear beam type element [8].

Example Case I: Closed-Open Tube

Table 2 and Figure 1 show comparison between the mobility analogy-finite element analysis and the closed-form solutions for f_n and Ψ_n. We note excellent correlation between theory and finite element analysis, especially for the lower modes. The end corrections or additional kinetic energy effects must be applied at the open end [10]. Since the finite element analysis ignored the end corrections, its predictions are closer to the theoretical solution based on the geometric length (i.e. $l_e = l_g$).

Similar comparisons between theory and finite element analysis have been found for the closed-closed and open-open tubes [12].

Example Case II: Helmholtz Resonator

We have examined a Helmholtz resonator whose geometry is described in Reference [13]. Table 3 and Figure 2 show our results for the first mode; these are compared with theory [7,10] and other three-dimensional finite element analyses [13,14]. We observe that our method predicts f_1 well only when we employ only one master mode and apply an end correction; this model, however, does not predict the same mode shape as given by other three-dimensional finite element analyses. Conversely, we predict mode shape well by employing a large number of master modes (say $N_M = 11$).

Example Case III: Composite System

Figure 3 shows a composite system consisting of three volumes and two orifices [15]. For this example case, we observe a substantial variation in f_1 and f_2 values with different finite element models. In Table 4 and Figure 4 the results of two typical models are given; these are also compared with an experiment and a two degrees of freedom lumped parameter analysis. We note that the finite element model requires a careful selection of the master nodes. Similar results have been found for other combinations of volumes and orifices [12].

CONCLUDING REMARKS

The space limitation here prevents us from a detailed discussion of the example cases presented above. Based on these and other example cases we have studied, we can conclude that the mobility analogy-finite element method predicts natural frequencies and modes of one-dimensional fluid systems reasonably well. However, we have to pay adequate attention to the following modeling aspects: (i) the number of elastic and inertia elements, (ii) the number and locations of master nodes, and (iii) the employment of end corrections. For some physical systems, a selection of large number of master nodes may not yield the "more correct" f_n value as demonstrated here. This aspect of the finite element analysis is generally not discussed well in the literature [6,8,11,13]; and therefore, it should be investigated further with reference to the fluid systems. Also, more fluid components and systems should be studied in order to establish some modeling guidelines.

The mobility analogy-finite element analysis method is an attractive solution technique for practical one-dimensional fluid systems as any available structural finite element code could be used readily. However, we should point out that sometimes it is difficult to construct an analogy between a mechanical and a fluid system. Therefore, some user judgement and discretion is advised.

LIST OF SYMBOLS

A	across variable
c	speed of wave propagation
E	modulus of elasticity
f	frequency
F	force
l	length
n	number of modes
N	number of nodes
p	pressure
q	volume flow rate
r	radius (hydraulic)
S	cross-sectional area
T	through variable
t	time
x	longitudinal coordinate
ρ	density
Ψ	mode (pressure)
Θ	angular displacement
ξ	displacement (longitudinal)
Δ	end correction (length)

Subscripts

e	effective
f	fluid

g geometric
i inside
m mechanical
M master
n modal index
O outside
T total/elastic

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Table 1. Mobility Analogy Between Mechanical and Fluid Systems

	Mechanical System	Fluid System
T(t)	F	q (or $\int q dt$, $\dot{q}$)
A(t)	$\dot{\xi}$ (or ξ, $\ddot{\xi}$)	p
Wave equation A(x,t)	$\frac{\partial^2 \xi}{\partial t^2} = c_m^2 \frac{\partial^2 \xi}{\partial x^2}$	$\frac{\partial^2 p}{\partial t^2} = c_f^2 \frac{\partial^2 p}{\partial x^2}$
c	$c_m = \sqrt{E_m/\rho_m}$	$c_f = \sqrt{E_f/\rho_f}$

Table 2. Natural Frequencies of Example Case I: Closed-Open Tube. Medium: Fresh Water at 20°C

Method	Natural Frequency (Hz)					
	f_1	f_2	f_3	f_4	f_5	f_6
Theory [10]						
(i) without end correction ($l_e=l_g$)	37.2	111.6	186.1	260.5	334.9	401.4
(ii) with end correction ($l_e=l_g+\Delta$)	35.7	107.0	178.4	249.8	321.1	392.5
Finite-element analysis using mobility analogy • N_T=22, N_M=19 • without end correction ($l_e=l_g$)	36.5	109.7	183.7	258.8	335.6	414.5

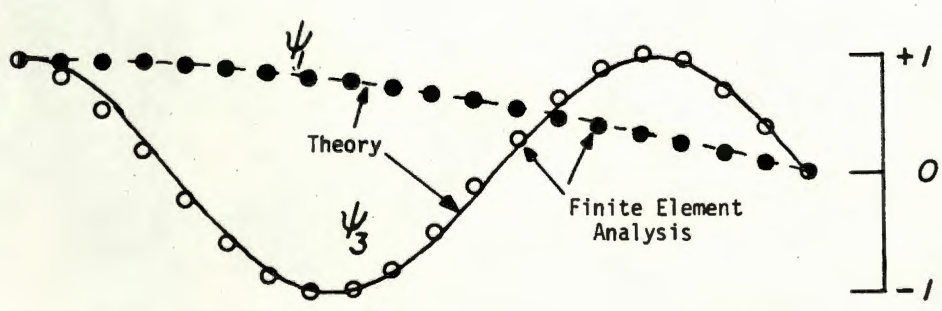

Figure 1. Pressure Modes of Example Case I: Closed-Open Tube. Note That the End Corrections are Not Applied Here for Both Analyses, i.e. $l_e=l_g$.

Table 3. First Natural Frequency of Example Case II: Helmholtz Resonator. Medium: Air at Room Temperature.

Method	Natural Frequency f_1 (Hz)	End Corrections	
		Δ_i	Δ_o
Theory - single degree of freedom analysis [7,10]			
(i) without any end correction ($l_e=l_g$)	45.3	0	0
(ii) with one end correction	44.0	0.85r	0
(iii) with both end corrections	43.0	0.85r	0.64r
One-dimensional finite element analysis using mobility analogy			
(i) $N_T=9$, $N_M=1$	43.5	0.85r	0
(ii) $N_T=13$, $N_M=11$	33.5	0.85r	0.64r
Three-dimensional finite element analysis using an acoustic element [13]			
• $N_M \simeq 600$	43.3	0	0
Three-dimensional finite element analysis using transient heat conduction analogy [14]			
• $N_T=92$, $N_M=9$	43.3	0	0

Table 4. Natural Frequencies of Example Case III: Composite System. Medium: Air at Room Temperature

Method	Natural Frequency f_n		End Corrections Included
	f_1	f_2	
Experimental [15]	206.2	385.6	-
Finite element analysis using mobility analogy			
(i) $N_T=8$, $N_M=4$	179.4	375.0	Yes
(ii) $N_T=26$, $N_M=24$	190.5	393.8	No
Lumped parameter analysis - two degrees of freedom [15]	218.8	396.5	Yes

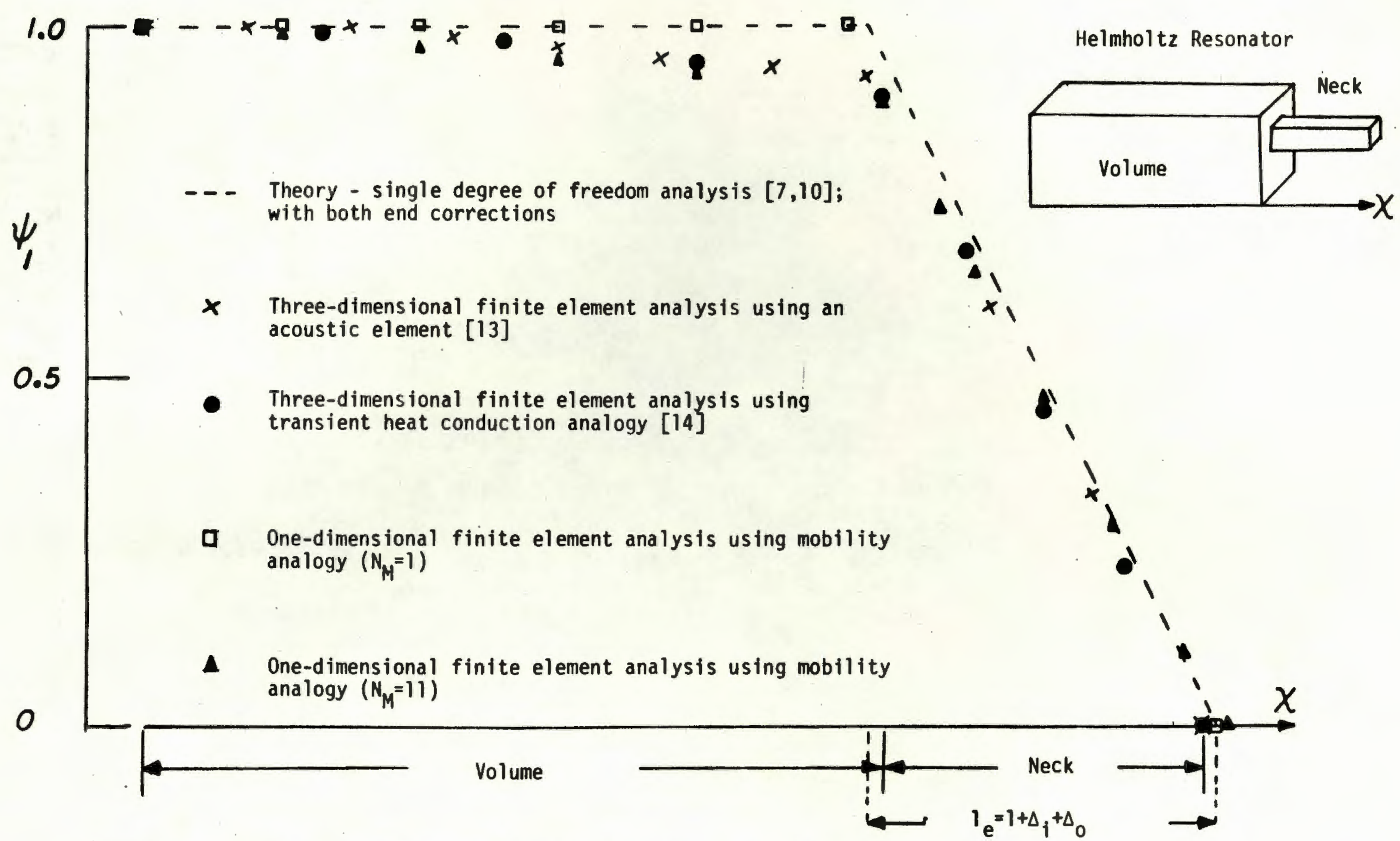

Figure 2. First Pressure Mode of Helmholtz Resonator. See Table 3 for f_1 Values.

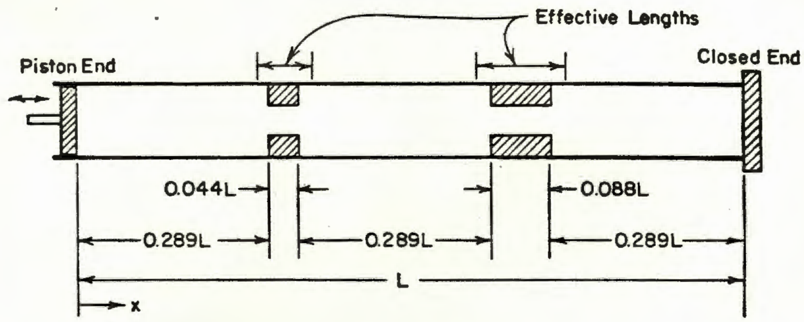

Figure 3. Schematic of the Example Case III: Composite System.

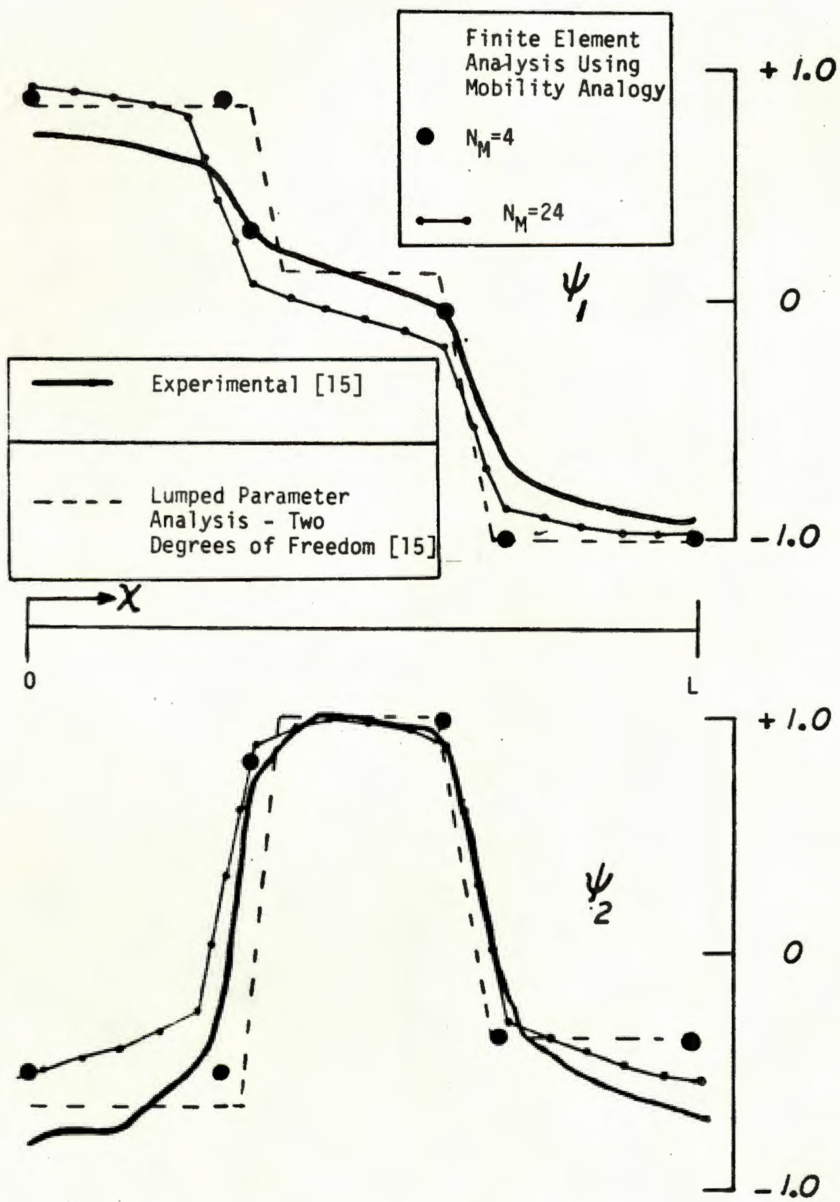

Figure 4. Pressure Modes of Example Case III: Composite System (Fig. 3). See Table 4 for Natural Frequencies.