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EXPERIMENTAL ACOUSTIC MODAL ANALYSIS

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INTRODUCTION

Three-dimensional acoustic cavities are found in many engineering, architectural, and musical applications. Noise and vibration problems in some machines and engineering systems are often related to the acoustic oscillations in these cavities, which could be excited either aerodynamically or structurally. Such objectionable oscillations can be reduced if suitable geometric changes are made such that acoustic resonances are avoided. Also, dissipative material liners or baffles can be placed in those regions where large acoustic amplitudes exist. However, the solution process requires that the cavity natural frequencies and modes be known. Since the cavities often encountered in practice are either geometrically irregular or possess complicated boundary conditions, computational or experimental methods must be adopted for the modal analysis. The focus of this paper is on the experimental aspects as an experimental acoustic modal analysis technique is proposed here. Such a method could be very useful in not only investigating the acoustic behavior of irregular shaped ducts, cavities and resonators but also could offer an alternative to the finite element method.

OBJECTIVES

This paper is a logical extension of the earlier paper by Nieter and Singh[1] who used the coincident-quadrature response technique to extract acoustic global properties for one-dimensional ducts. We intend to concentrate here on the three-dimensional annular-like cavities whose modal properties including damping will be extracted using the vector diagram method. Our excitation method relies on the utilization of the convertible speaker method[2], as opposed to piston-shaker method employed by Nieter and Singh[1], as we feel that this method is more suitable to three-dimensional acoustic systems. Unlike Nieter and Singh[1], all pressure measurements are conducted at the boundary of the cavity. The scope of the problem is limited to a linear three-dimensional acoustic cavity with zero mean flow. Two example cases will be considered over the frequency range of about 0-2000 Hz for the air medium having room temperature (22.2°C) at an ambient pressure.

The conceptual considerations as described in detail in References 1 and 3 lead to an experimental technique for the extraction of modal parameters (eigenvalues and eigenvectors) from measured acoustic impedances. Essentially, a continuous acoustic cavity is approximated by a lumped-parameter model with finite number of measurement points; this model can now yield the acoustic behavior of the cavity.

MEASUREMENT CONSIDERATIONS

Measurement of Cavity Impedances Figure 1 shows schematically the arrangement of the experimental test setup and instrumentation. The acoustic cavity of interest is bounded by a closed cylindrical duct outside and by a circular cylinder inside. The cavity is excited by a convertible acoustic driver mounted either on the side of the cylindrical duct or on the top cover plate of the duct. The speaker location is chosen such that all modes of interest can be excited. The convertible driver technique, used earlier by Singh and Schary[2] for one-dimensional acoustic systems, has been extended here to three-dimensional cavities. A small enclosure of known volume is attached to the back of the driver for calibration purpose. Pressure responses are measured at 25 locations with a 3mm microphone across the outer boundary. There are 6, 8, and 12 measurement stations along r, θ, and z directions, respectively.

Identification of Modal Parameters A number of techniques are available for identifying modal parameters from the measured frequency response functions[4]. If natural frequencies are not closely spaced and modal damping ratios are very small, one can easily use co-quad (real-imaginary) plot technique as demonstrated by Nieter and Singh[1]. But if natural frequencies are close and the damping is moderate, then the vector diagram method, also known as Kenndey-Panku method, is more suitable.

RESULTS AND DISCUSSION

Two example cases are discussed here. Example Case I is a cylindrical cavity whose theoretical eigensolutions are well known. Example Case II is an annular-like cavity whose theoretical solutions are not known and therefore experimental results will be compared with those given by a finite element model of the cavity.

Example Case I: A Cylindrical Cavity For a cylindrical cavity, with both ends closed and of radius R and length L, theoretical solutions for natural frequencies f (i,m,n) and pressure mode shapes Ψ(i,m,n) are well known in cylindrical coordinates (r,θ,Z) where i,m,n are modal indices in longitudinal (z), circumferential (θ), and radial (r) directions respectively[5]. Since a boundary condition along θ does not exist, we get a set of orthogonal modes for index m. However in the experiment the location of the finite acoustic source fixes the circumferential mode, we can describe the mode shape only by sin $m\theta$ where the origin of θ is maintained the same for the complete database. The experimental model constructed for testing purposes has a radius R of 119.1 mm and a length L of 298.5 mm. The boundary walls are nearly hard since the extracted modal damping ratios are less than 0.013 for all modes of interest as shown in Table I. The natural frequencies extracted from the transfer function data for air medium are compared with the exact solution (without damping) in Table I. Excellent agreement between theory and experiment is evident since the deviations from theoretical values are generally less than one percent of the theoretical values. Pressure mode shapes extracted experimentally are compared with the theoretical distributions in Figs. 2, 3 and 4 for the third mode (1,1,0) and the ninth mode (0,0,1). These mode shapes are normalized such that both theoretical and experimental modes have the same maximum amplitude at the reference antinode. Again, excellent agreement between theory and experiment is evident.

Example Case II: An Annular-Like Cavity Now we will consider a three-dimensional cavity whose theoretical solution is not known; it is bounded outside by a closed cylinder of radius r_o = 119.1 mm and length ℓ_o = 298.5 mm and inside by a concentric cylindrical insert of radius r_i = 90.5 mm and length ℓ_i = 228.6 mm. All walls are acoustically hard. The experimental model has nearly hard walls as extracted modal damping ratios are found to be less that 0.02 for all modes of interest (see Table II). The finite element model has 64 elements (8 x 8) for the "annular part" and 8 elements (4 x 2) for the "cylindrical part" as shown in Fig. 5. The mesh size is chosen based such that the second mode shapes in θ and z directions and the first mode shape in the r direction can be described adequately[6]. Only half of the cavity was modeled because of the symmetry[6]. The boundary conditions are that the derivatives of the acoustic pressure with respect to the boundary normals are zeros.

Table II shows the comparison of natural frequencies obtained from the experiment and the finite element model. Excellent correlation (1-8% differences) is found here with deviations being larger for higher modes since the mesh size of the finite element model can no longer describe these modes adequately. Figure 6 shows typical acoustic pressure modes extracted experimentally and as displayed on a terminal capable of animations. The "wire frame" figure for each mode displays the variation of the acoustic pressure in θ direction (top view) and z direction (front view); the shaded planes indicate the nodal surfaces corresponding to each mode. Comparisons of the seventh mode (1,2,0) for experiment and finite-element model are shown in Figs. 7 and 8; the agreement again is also very good.

CONCLUDING REMARKS

The examples presented here have been limited to cavities with nearly hard walls; consequently, modes have been measured to be almost real. However, an acoustic cavity with nonproportional damping where complex modes exist, can also be modeled with ease since the formulation proposed here is based on complex numbers. Using multi-degree-of-freedom curve fitting techniques, heavily damped modes could also be extracted accurately. It should be pointed out that in our example cases the pressure responses have been measured only on the boundary, which enables us to define modes of interest completely. There should be no difficulty in measuring pressure responses within the cavity by inserting slender microphone probes.

Overall the experimental technique has been found to be valid and accurate. Perhaps the most "dramatic" part of our study has been the animated representation of the measured pressure modes on a computer graphic terminal. One can view these modes very easily and obtain a good understanding of the nodal (or antinodal) surfaces; this knowledge is extremely valuable as acoustic design decisions such as geometric changes, placement of baffles and addition of absorptive material could be arrived at quickly and efficiently.

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TABLE I. Natural frequencies and damping ratios for a cylindrical cavity (case I)

Mode number k	Modal indices (i,m,n)	Natural frequency $f(i,m,n) = \omega_k/2\pi$: Theory, Hz (a)	Experiment, Hz (b)	%Error $((b-a)/a) \times 100\%$	Experimental damping ratio ζ_k
1	(1,0,0)	557	575	−0.35	0.013
2	(0,1,0)	848	845	−0.35	0.009
3	(1,1,0)	1025	1020	−0.49	0.007
4	(2,0,0)	1154	1150	−0.35	0.006
5	(0,2,0)	1406	1395	−0.78	0.004
6	(2,1,0)	1432	1425	−0.49	0.004
7	(1,2,0)	1520	1510	−0.66	0.007
8	(3,0,0)	1731	1730	−0.06	0.006
9	(0,0,1)	1764	1765	−0.05	0.006
10	(2,2,0)	1819	1805	−0.77	0.005
11	(1,0,1)	1856	1845	−0.59	0.004
12	(3,1,0)	1927	1925	−0.10	0.010
13	(0,3,0)	1934	1930	−0.21	0.004

TABLE II. Natural frequencies and damping ratios for an annularlike cavity (case II).

Mode number k	Modal indices (i,m,n)	Natural frequency $f(i,m,n) = \omega_k/2\pi$: Experiment, Hz (a)	Finite element method, Hz (b)	% Difference $((b-a)/a) \times 100\%$	Experimental Damping ratio ζ_k
1	(1,0,0)	482	487	1	0.02
2	(0,1,0)	584	400	2.7	0.013
3	(1,1,0)	877	903	3	0.009
4	(2,0,0)	1067	1124	5	0.009
5	(0,2,0)	1130	1143	1.2	0.009
6	(2,1,0)	1258	1347	7	0.010
7	(1,2,0)	1326	1432	8	0.005

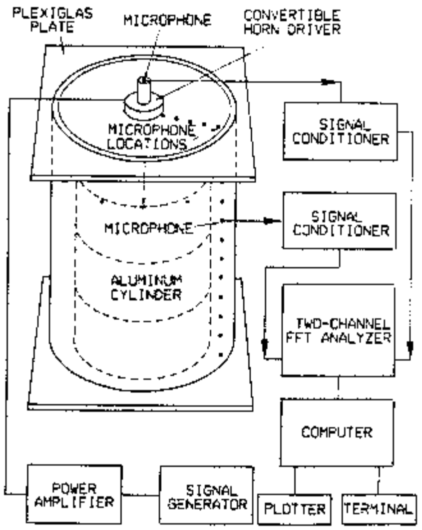

FIG. 1. Experimental setup and instrumentation.

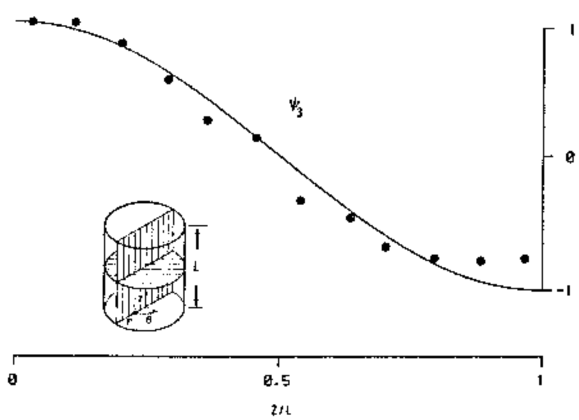

FIG. 2. Pressure mode ψ_3 along r=R, θ=90°, z=0-L; —theory ● experiment.

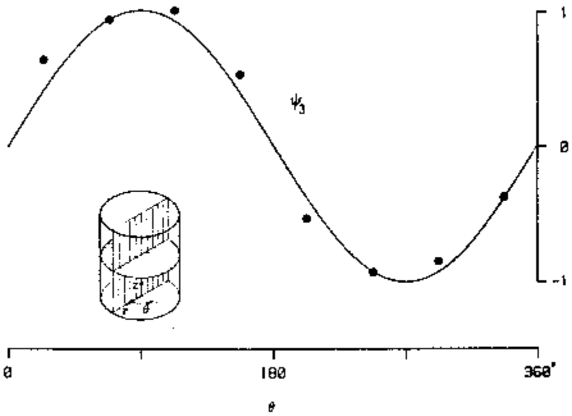

FIG. 3. Pressure mode ψ_3 along r=R, z/L=0.88, θ=0-360°; — theory, ● experiment.

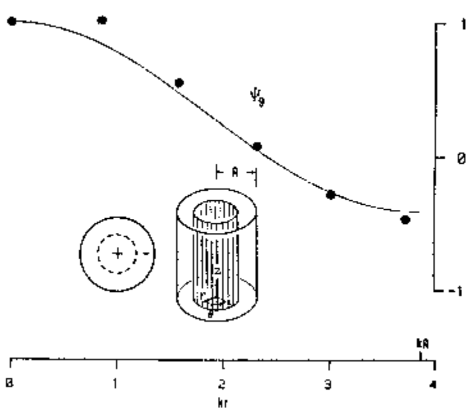

FIG. 4. Pressure mode ψ_9 along z=L, θ=0°, r=0-R; — theory, ● experiment.

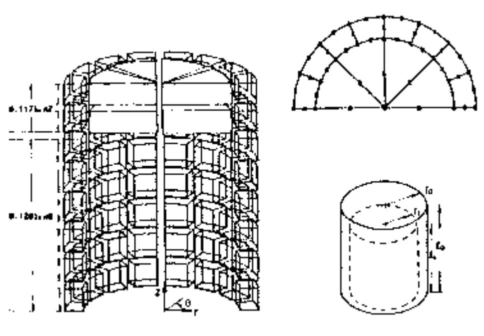

FIG. 5. Example case II: Annular-like Cavity and its Finite Element Model.

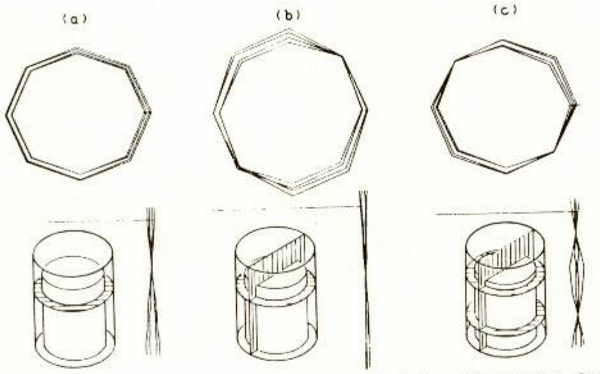

FIG. 6. Typical Pressure Modes Extracted from Experiment for Example Case II: (a) Mode (1,0,0). (b) Mode (1,1,0). (c) Mode (2,1,0).

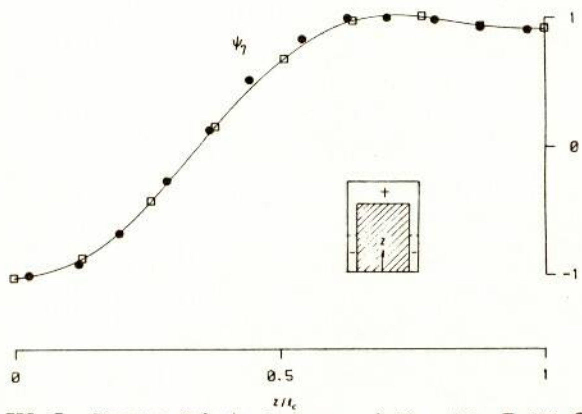

FIG. 7. Pressure Mode ψ_7 along $r=r_o$, $\theta=0°$, $z=0$-L; □ FEM, ● experiment.

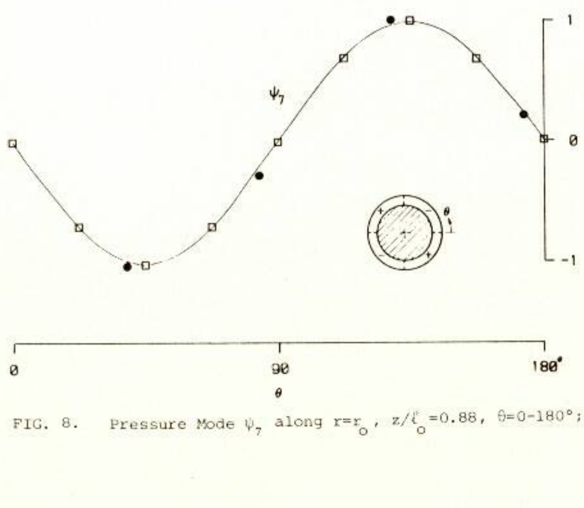

FIG. 8. Pressure Mode ψ_7 along $r=r_o$, $z/\ell_o=0.88$, $\theta=0$-180°; □ FEM,● experiment.