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FORCED VIBRATION AND ACOUSTIC POWER RADIATION RESPONSE OF
A PIPE TRANSPORTING OSCILLATING COMPRESSIBLE FLUIDS

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ABSTRACT
Pressure pulsations in pipelines often cause vibration, radiated noise, fatigue and reliability problems. This paper examines this problem through an example case where the pipeline is considered to be a thin elastic, simply supported cylindrical shell, placed within baffles. Green's function technique, along with the modal expansion, has been utilized to predict the axi-symmetric response of the shell due to a forcing function which is assumed to be a linear combination of the positive and negative direction travelling acoustic plane waves. The alternating stresses resulting from vibrations are also evaluated. Structural response and the acoustic wave equation in cylindrical coordinates are then used to predict the external acoustic radiation field. In our analysis, the back reaction of the vibrating shell on inner wave propagation, and the effect of external radiation on the vibrating structure are not included.

NOMENCLATURE

\begin{align*}
\alpha & \quad \text{radius of the shell} \\
\beta & \quad \text{speed of sound} \\
\epsilon & \quad \text{modulus of elasticity} \\
\gamma & \quad \text{Green's function} \\
\delta & \quad \text{shell thickness} \\
\iota & \quad \text{imaginary number} \\
\kappa & \quad \text{wave number} \\
\lambda & \quad \text{length of the shell} \\
p & \quad \text{pressure} \\
r & \quad \text{radial coordinate} \\
t & \quad \text{time} \\
\mu & \quad \text{particle velocity} \\
\nu & \quad \text{unit step function} \\
\psi & \quad \text{transverse or radial displacement} \\
\xi & \quad \text{longitudinal coordinate} \\
\rho & \quad \text{density} \\
\gamma & \quad \text{propagation constant} \\
\varphi & \quad \text{circumferential coordinate} \\
\sigma & \quad \text{stress components} \\
\mu & \quad \text{Poisson's ratio} \\
\omega & \quad \text{velocity potential} \\
\pi & \quad \text{acoustic power radiated} \\
\varepsilon & \quad \text{strain components} \\
\psi & \quad \text{structural mode preference angle} \\
\delta & \quad \text{damped natural frequency of structure} \\
\xi & \quad \text{modal damping factor of structure} \\
\alpha & \quad \text{wave number on the structure} \\
\beta & \quad \text{acoustic damping factor}
\end{align*}

Subscripts

\begin{align*}
i & \quad \text{inner} \\
L & \quad \text{left (-x)} \\
\mu & \quad \text{axial mode number} \\
\eta & \quad \text{circumferential mode number}
\end{align*}
waves coupling is of lower order, and hence their interaction is more effective. The interactions between the circumferentially varying motions are of higher order, which should take place generally at the higher frequencies.

The present paper considers damped acoustic damped pressure to be the forcing function which is given as follows; see the Nomenclature for the identification of symbols.

\[ p(x,t) = \rho_e \frac{\partial^2 \psi}{\partial x^2} + \rho L \psi \nonumber \]

\[ \dot{\psi} = \beta + \omega c = \beta + \omega c \nonumber \]

\[ \omega = \text{excitation frequency} \nonumber \]

With the above forcing function, the problem is now reduced to evaluation of the following.

1. Forced transverse vibration response of the damped shell.
2. Acoustic power radiation from the shell structure.
3. Stresses developed within the shell structure.

In our analysis, the back reaction of the shell on inner fluids is considered to be very small. Similarly, the shell vibrating surface should not be significantly influenced by the outer acoustic field.

**VIBRATION RESPONSE**

The piping can be modeled as a finite cylindrical shell of circular cross-section, as shown in Fig. 2, with the following assumptions: (i) Love's assumption of thin shells holds good, (ii) the cylindrical shell is simply supported, and (iii) Mubharli-Vlasov-Donnell approximation for the solution of Love's equations for natural frequencies and modes is valid. Natural frequency of a cylindrical shell, \( \omega_{mn} \) is given as\([6,7]\).

\[ \omega_{mn} = \sqrt{\frac{\rho A}{2}} \left[ \frac{m^2}{L^2} \right]^2 + \frac{m^2}{n^2} \left( \frac{\beta}{L^2} \right)^2 + \frac{m^2}{n^2} \left( \frac{\mu}{L^2} \right)^2 \left( \frac{\mu}{L^2} \right) \]  

The mode shapes in axial (\( U \)), circumferential (\( V \)), and transverse (\( W \)) directions are given as

\[ U_p(x,\theta) = A_{mn} \cos \frac{m\pi x}{L} \cos n(\theta-\psi) \]  

\[ V_p(x,\theta) = B_{mn} \sin \frac{m\pi x}{L} \sin n(\theta-\psi) \]  

\[ W_p(x,\theta) = C_{mn} \sin \frac{m\pi x}{L} \cos n(\theta-\psi) \]

For the forced vibration response, we shall be using the dynamic Green's function approach as formulated by Wilken and Soedel\([8]\). The dynamic Green's function is also known as dynamic influence

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Figure 1. Formulation of the Problem

Figure 2. Cylindrical Shell
function which describes the response at a point, in the structure due to a unit impulse load applied at some other point. It is a powerful analytical tool, and once it is known for a particular structure, then the response for any given loading condition can be evaluated fairly easily. For a cylindrical shell, the dynamic Green's function matrix has nine components. Since we have loading in only one direction, only three Green's functions are needed. Further, in-plane displacements are very small, and hence are of no practical interest. Therefore, we shall consider only one Green's function (G) which gives the response in transverse direction for the loading in the same (transverse) direction. Now, using the modal expansion technique, for damped shell case, G is

$$G(x, \theta, t; \bar{x}, \bar{\theta}, \bar{t}) = \frac{1}{2\pi} \sum_{p=0}^{\infty} \frac{1}{N_p \delta_p} W_p(x, \theta) W_p(\bar{x}, \bar{\theta})$$

$$\cdot e^{-\delta_p \omega_p(t-\bar{t})} \sin \delta_p(t-\bar{t}) U(t-\bar{t})$$

(8)

where a bar above the coordinates indicates the loading location and the coordinates without bar represent the response location. And \( \delta_p \) and \( N_p \) are

$$\delta_p = \frac{\omega_p \sqrt{1-\alpha_p^2}}{\alpha_p}$$

(9)

$$N_p = \int_0^L \int_0^{2\pi} W_p(x, \theta) a d\theta dx$$

(10-a)

$$= \frac{\pi a L}{\sin \theta_n}$$

(10-b)

where

$$\theta_n = \begin{cases} 1 & , \ n=0 \\ 2 & , \ n=0 \end{cases}$$

The normal modes orient themselves such that they offer the least resistance to the impulse i.e. \( v = \theta \). Modal index parameter range is \( p=0 \) to \( \infty \), which means \( m \) from 1 to \( \infty \) and \( n \) from 0 to \( \infty \). Thus the Green's function (8) is given as follows, with the help of expressions (7 and 10).

$$G(x, \theta, t; \bar{x}, \bar{\theta}, \bar{t}) = \frac{1}{2\pi} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{\sin m \pi x}{\sin \theta_n} \sin \frac{m \pi x}{L} \cos n(\theta - \bar{\theta}) S(t-\bar{t})$$

(11)

where

$$S(t-\bar{t}) = \frac{1}{\delta_m} e^{-\delta_m \omega_m(t-\bar{t})} U(t-\bar{t})$$

(12)

Transverse response \( W(x, \theta, t) \) is given as follows, for the excitation function expressed by (1).

$$W(x, \theta, t) = \int_0^L \int_0^{2\pi} G(x, \theta, t; \bar{x}, \bar{\theta}, \bar{t}) b(t-\bar{t}) d\bar{t}$$

(13)

Equations (11-13) and (1) give,

$$W(x, \theta, t) = \frac{1}{2\pi a} \int_0^L \int_0^{2\pi} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{e_n \sin m \pi x}{\sin \theta_n} \sin \frac{m \pi x}{L}$$

$$\cdot \sin \frac{m \pi x}{L} \cos n(\theta - \bar{\theta}) \frac{1}{\delta_m} e^{-\delta_m \omega_m(t-\bar{t})} U(t-\bar{t})$$

$$\cdot e^{-\delta_m \omega_m(t-\bar{t})} U(t-\bar{t}) e^{i\omega_m \theta} \right)$$

(14)

Since the forcing function is axisymmetric, only breathing mode in \( \theta \) direction (\( m=0 \)) will exist as,

$$\int_0^{2\pi} \cos n(\theta - \bar{\theta}) d\theta = 2\pi \quad , \quad n=0$$

$$= 0 \quad , \quad n \neq 0$$

(15)

And (14) is reduced to,

$$W(x, \theta, t) = \frac{2}{2\pi a} \int_0^L \sum_{m=1}^{\infty} \sin \frac{m \pi x}{L} \sin \frac{m \pi x}{L}$$

$$\cdot e^{-\omega_m \theta} \sin \delta_m(t-\bar{t}) U(t-\bar{t}) e^{i\omega_m \theta}$$

$$\cdot \left[ P_R e^{-\beta \omega_m \theta} + P_L e^{\beta \omega_m \theta} \right]$$

where \( \omega_m \) is given, from (1), as

$$\omega_m = \sqrt{E \frac{1}{\rho a^2} \left[ 1 + \frac{\pi^2}{12 a^2 (1-\mu^2)} \right]}$$

(17)

The solution of (16) consists of a steady state term and a transient term. Since the transient part would die out because of the damping, the system response would be the steady state response at the excitation frequency. It is this part which is of practical interest. The steady state response is

$$W(x, t) = \frac{2}{2\pi a} \sum_{m=1}^{\infty} B_m \sin \alpha x e^{i(\omega_m \theta - \gamma_m)}$$

(16-a)

where

$$\alpha = \frac{m \pi}{L}$$

(18-b)
\[ B_m = \frac{1}{\alpha^2 \omega_{m}^2} \left[ \frac{1 + (\alpha^2)^2}{\frac{k}{\alpha} - \frac{(k)}{\alpha} + \frac{2k}{\alpha^2}} \right]^{\frac{1}{2}} \left[ \left( \frac{L}{\omega_{m}} \right)^2 + \left( \frac{2L}{\omega_{m}} \right)^2 \right]^{\frac{1}{2}} \left[ \frac{P_R + P_L}{\alpha^2} \right] \]

\[ \psi_m = \tan^{-1} \frac{2k \beta}{\alpha^2 + \beta^2 - k^2} \]

From Equations (18-a) the following conclusions can be drawn:

1. For axisymmetric plane wave excitation, structural transverse response corresponds only to breathing circumferential mode \((m=0)\). However, the response exists for all axial modes \((m=1, 2, \ldots)\).

2. Resonance conditions exist, as expected, for the coincidence of plane wave frequency \((\omega)\) with the natural frequency of the structure \((\omega_m)\). The amplitude at this resonant condition will be governed by fluid \((\beta)\) and structural \((\sigma_{m})\) dampings.

3. Expression (18-c) can also be used to study coincidence of the wave numbers where \(K\) is the fluid pulsation wave number and \(\alpha = \frac{m}{L}\) is the wave number on shell surface which signifies wave propagation in the axial direction. Steady state response \(W(x,t)\) will grow without bounds when \(K\) and \(\alpha\) are equal for undamped wave propagation. Physically, it can be visualized as the case when the fluid standing waves couple strongly with a certain modal pattern on the shell.

DYNAMIC STRESSES

The vibration of the shell structure induces alternating stresses which in association with the material properties determine the fatigue behavior of the shell. The strain components due to the vibration are as follows:

(a) Membrane strains

1. Normal components

\[ \varepsilon_x = 0 \]

\[ \varepsilon_\theta = \left( \frac{1}{\alpha^2} \right) W(x,t) \]

\[ = \frac{2}{\alpha^2} \sum_{m=1}^\infty B_m \sin \alpha x e^{i(\omega t - \psi_m - \Phi_m)} \]

2. Shear component

\[ \gamma_{x\theta} = 0 \]

(b) Bending strains

1. Normal components

\[ \sigma_x = \frac{E}{1-\mu^2} \left[ \varepsilon_x + \frac{\mu}{2} \frac{\partial^2 W(x,t)}{\partial x^2} \right] \]

\[ \sigma_\theta = \frac{E}{1-\mu^2} \left[ \varepsilon_\theta + \frac{\mu}{2} \frac{\partial^2 W(x,t)}{\partial x^2} \right] \]

\[ \sigma_z = \frac{E}{1-\mu^2} \left[ \varepsilon_z + \frac{\mu}{2} \frac{\partial^2 W(x,t)}{\partial x^2} \right] \]

Substituting Eqs. (19-25) in Eqs. (26-28) we get

\[ \sigma_x = \frac{E}{1-\mu^2} \left[ \frac{\mu}{2} W(x,t) - \frac{1}{2} \mu \frac{\partial^2 W(x,t)}{\partial x^2} \right] \]

\[ = \frac{2E}{\mu L} \sum_{m=1}^\infty \left( \frac{\mu}{2} + \frac{\alpha^2 \mu}{2} \right) B_m \sin \alpha x e^{i(\omega t - \psi_m - \Phi_m)} \]

\[ \sigma_\theta = \frac{E}{1-\mu^2} \left[ \frac{\omega}{2} W(x,t) - \frac{1}{2} \mu \frac{\partial^2 W(x,t)}{\partial x^2} \right] \]

\[ = \frac{2E}{\mu L} \sum_{m=1}^\infty \left( \frac{1}{2} + \frac{\alpha^2 \mu}{2} \right) B_m \sin \alpha x e^{i(\omega t - \psi_m - \Phi_m)} \]

\[ \tau_{x\theta} = 0 \]

Thus the vibration of the shell structure, excited by inner fluid oscillations, result in principal stresses only.
ACOUSTIC RESPONSE

The axisymmetric motion of the shell also induces an axisymmetric acoustic field radiation outside the pipe line as shown in Fig. 6. The acoustic field is governed by the Helmholtz wave equation[10].

$$\nabla^2 \phi + k^2 \phi = 0$$  \hspace{1cm} (32)

$$\phi(r, x) = R(r) \times (x)$$  \hspace{1cm} (33)

The solution of Eqns. (32) and (33) in cylindrical coordinates is in terms of the Bessel functions,

$$R(r) = C_1 J_0(k_r r) + C_2 Y_0(k_r r)$$  \hspace{1cm} (34)

In order to investigate the radiation process, it is more convenient to write this solution in terms of the Hankel functions of I and II kind which are qualitatively similar to $e^{ik_r r}$ and $e^{-ik_r r}$; thus $H^{(1)}_0$ represents converging waves and $H^{(2)}_0$ represents diverging waves.

$$R(r) = C_3 H^{(1)}_0(k_r r) + C_4 H^{(2)}_0(k_r r)$$  \hspace{1cm} (35)

where

$$H^{(1)}_0(k_r r) = J_0(k_r r) + i Y_0(k_r r)$$  \hspace{1cm} (36-a)

$$H^{(2)}_0(k_r r) = J_0(k_r r) - i Y_0(k_r r)$$  \hspace{1cm} (36-b)

The boundary conditions are:

1. At the shell surface

$$u_r \bigg|_{r=a} = -\frac{\partial \phi}{\partial r} \bigg|_{r=a} = \frac{\partial W(x,t)}{\partial t}$$  \hspace{1cm} (37)

2. The vibrating surface generates only diverging waves. Thus $C_3 = 0$.

3. The cylinder is placed in the baffles, as shown in Figure 4.

$$u_x = -\frac{\partial \phi}{\partial x} = 0 \text{ at } x = 0, L$$  \hspace{1cm} (38)

Using boundary conditions (2) and (3), we get

$$\phi(r, x, t) = \sum_{q=0}^{\infty} A_q H^{(2)}_0(k_q r) e^{i \frac{\pi q x}{L}}$$  \hspace{1cm} (39)

where

$$k_q^2 = k^2 - \left(\pi q/L\right)^2$$  \hspace{1cm} (40)

Acoustic pressure and radial particle velocity are given as

$$p(r, x, t) = \int_0^1 \frac{\partial \phi(r, x, t)}{\partial t}$$  \hspace{1cm} (41)

$$u_r(r, x, t) = -\frac{\partial \phi(r, x, t)}{\partial r}$$  \hspace{1cm} (42)

Using Eqns. (39), (37) and (18), we get

$$A_q = \frac{-j(4\pi)}{\pi^2 \rho L} \frac{1}{H^{(2)}_0(k_q a)} \sum_{m=1}^{\infty} \frac{B_m}{m^2 + \pi^2}$$  \hspace{1cm} (43)

$$\left(\frac{m^2 - \pi^2}{m^2 + \pi^2}\right) \left[1 - \left(\frac{\cos m\pi \cos \pi q \pi + \frac{\pi}{m}}{\sin m\pi \sin \pi q \pi}\right)\right] e^{-j(\pi m \phi_m)}$$  \hspace{1cm} (44)

where

$$H^{(2)}_0(k_q r) = \frac{d}{dr} H^{(1)}_0(k_q r) = \frac{1}{2} \left[H^{(2)}_r(k_q r) - H^{(1)}_r(k_q r)\right]$$  \hspace{1cm} (45)

Acoustic pressure and radial particle velocity are

$$p(r, x, t) = \sum_{q=0}^{\infty} j \int_0^1 A_q \cos \frac{\pi q x}{L}$$  \hspace{1cm} (45)

$$H^{(2)}_r(k_q r) e^{i \alpha t}$$

$$u_r(r, x, t) = \sum_{q=0}^{\infty} A_q \cos \frac{\pi q x}{L} H^{(2)}_r(k_q r) e^{i \alpha t}$$  \hspace{1cm} (46)

Acoustic power radiated from a longitudinal differential element is[10].

$$d\Pi = \frac{1}{2} \text{Re} \left[p \cdot u_r^*\right] > 2\pi r dx$$  \hspace{1cm} (47)

Hence, the total acoustic power radiated from the surface is

$$\Pi = \frac{1}{2} \int_0^L \left[\text{Re} \left[p \cdot u_r^*\right]\right]_{r=a} dx$$  \hspace{1cm} (48)

Using Eqns. (43), (45), and (46), we get
\[ \pi = \frac{3}{\pi} \frac{\omega^3 a}{\epsilon^2_2 \epsilon^2_1} \sum_{q=0}^{\infty} \sum_{m=1}^{\infty} \frac{2}{m \pi q} \left( \frac{m^2 - q^2}{m^2 - q^2} \right)^2 \left[ 1 - \left( \cos \frac{m \pi}{\epsilon q} \cos \frac{q \pi}{\epsilon q} + \frac{\sin m \pi}{\sin q \pi} \right)^2 \right] \] (49)

An examination of the above equation reveals that the acoustic power \( \pi \) will grow boundless when

\[ H^{(2)}_\omega(kq \epsilon) \rightarrow 0 \] (50)

This case would correspond to a strong coupling between the shell structure and the outer acoustic medium. Of course, high acoustic power response could also be obtained when the displacement amplitude is large, as discussed earlier.

CONCLUDING REMARKS

This paper has examined the vibratory, dynamic stress and acoustic radiation response of a pipeline with fluid pulsations as the excitation. The back reactions of the outer acoustic medium on shell, and of shell vibration on the inner plane wave propagation have not been included. Perhaps the problem solved in this paper is somewhat ideal in nature; but it serves two purposes; firstly, it establishes a basis for an exact analytical solution, and secondly it clearly demonstrates the basic nature of the problem including strong couplings. In this general area there is obviously a vast potential for further analytical and experimental research.

REFERENCES

8. Wilken, I. D. and Soedel, W., "The Three Directional Dynamic Green's Function for Thin

Figure 3. Shell Stress Components

Figure 4. Vibrating Shell Surface and Outer Acoustic Field