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AN ANALYTICAL STUDY OF AUTOMOTIVE NEUTRAL GEAR RATTLE

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ABSTRACT
A reduced order nonlinear model of an automotive manual transmission is presented and the neutral mode case is analyzed. For a single frequency excitation and primary resonance, approximate analytical solutions are developed using the method of harmonic balance. Based on these solutions, a simplified set of equations are developed which are suitable for the study of neutral gear rattle. The nonlinear frequency response characteristics are discussed and the results are compared to similar studies described in the literature. It is shown that this analysis provides an analytical basis for the nonlinear behavior reported in the literature and hence it is suitable for the development of design guidelines for reduced rattle.

LIST OF SYMBOLS

\( \Omega \) angular frequency of external excitation (rad/sec)
\( \Omega_{ij} \) stiffness coefficient for multi-degree-of-freedom case
\( \Omega_{iso} \) transition frequency between no impacts and single sided impacts
\( \Omega_{12} \) transition frequency between single sided impacts and two sided impacts

Subscripts

\( F_p \) excitation force
\( m \) mean
\( p \) alternating component
\( ss \) single sided
\( ts \) two sided

Superscripts

\( ' \) first derivative with respect to time
\( " \) second derivative with respect to time
\( - \) nondimensional variable

INTRODUCTION

Gear rattle is a major source of noise and vibration in automotive manual transmissions and is characterized by backlash induced vibro-impacts between meshing gears. Torsional vibration, caused by engine torque pulses, results in cyclic angular accelerations of the components in the geared system. At a gear mesh, when the inertia torque of the driven member exceeds the load torque, gear tooth separation and impacts occur. Repeated impacts, referred to as vibro-impact or rattle, may lead to excessive noise and large dynamic tooth loads. To relate vibro-impact to gear noise and gear vibration levels (gear rattle level) or dynamic stress, one must not only be able to predict the onset of vibro-impact but also the dynamic behavior of the system during vibro-impact.

A number of investigators have studied gear rattle; Nakamura [1967], Wang [1977] and Hayashi et al. [1979] studied vibro-impact between two gears; Azar and Crossley [1975, 1977], Hedges et al. [1979], Wang [1980], and Okada et al. [1981] considered systems with flexible shafts as well as gear pairs; and finally Sakai et al. [1981], Seaman et al. [1984], Ohuma et al. [1985], Fujimoto et al. [1987], Xie [1987], Reik [1986], and Fudala et al. [1987] considered gear rattle in automotive manual transmissions. Gear rattle has been found to be a steady state vibration problem which generally occurs at or near system resonant frequencies. It was also found that gear rattle levels (vibration levels) could be given in terms of the mean square value of the displacement or acceleration and that impact transients could be ignored. Experimental studies of actual
transmissions, as reported by Sakai et al. [1981] Seaman et al. [1984], Ohnuma et al. [1985], and Fujimoto et al. [1987], have shown that gear rattle is affected by the amount of backlash, the stiffness and damping in the clutch, the drag load on the gears (viscous drag from journal bearings), and the distribution of inertia in the transmission.

Although some of the general characteristics of gear rattle in automotive transmissions are known, no specific analytical models are available which can be used to understand the dynamic behavior of the transmission system. In addition, because previous studies have generally been based on simple linear models and/or the results from nonlinear digital and analog simulations, the global design concepts are understood but the effect of system nonlinearities on these concepts is not. Therefore, the objective of this analysis is to develop an approximate analytical solution for the nonlinear frequency response of the transmission system suitable for the study of neutral gear rattle.

**FORMULATION OF THE GOVERNING EQUATIONS**

A reduced order model of the automotive transmission is shown in Figure 1. It was shown by Comparini [1988] that the four degree of freedom lumped parameter model is sufficient for the study of neutral gear rattle; similar models have been used by other investigators (Sakai et al. [1981], Ohnuma et al. [1985], and Fujimoto et al. [1987]). The model consists of a flywheel-clutch (I-II) system coupled to a gear pair (III-IV) with a flexible shaft where the shaft stiffness, \( K_s \), is chosen to correspond to the lowest shaft torsional mode in the actual transmission system. The moment of inertia \( I_n \) includes the reflected inertia of the other elements of the transmission. There are three nonlinear stiffness in this model: the multi-stage stiffness of the clutch, \( K_s f_j(\delta) \), and the backlash in the clutch spline and gear pair, \( K_b f_s(\delta) \) and \( K_b f_s(\delta) \) respectively. A generic nonlinear element \( f(\delta) \), referred to as a clearance type nonlinearity, is shown in Figure 2. Here \( \alpha_j \) is a measure of the strength of the nonlinearity. When \( \alpha_j \) is close to one the system is weakly nonlinear and when \( \alpha_j \) is much greater or much less than one the system is strongly nonlinear. For the clutch nonlinearity \( \alpha_j \) is nonzero and in general \( \alpha_j < 1 \) and for backlash \( \alpha_j = 0 \). Therefore, the multi-stage clutch and backlash represents examples of strong nonlinearities. In the absence of damping, the equations of motion are given by:

\[
\begin{align*}
\dot{\delta}_1 + K_s f_1(\delta_1)R_1 &= T_1, \\
\dot{\delta}_2 - K_s f_1(\delta_1)R_{u1} + K_s f_2(\delta_2)R_{u2} &= - T_u, \\
\dot{\delta}_3 - K_s f_2(\delta_2)R_{u3} + K_s f_3(\delta_3)R_{u4} &= - T_w, \\
\dot{\delta}_4 - K_s f_3(\delta_3)R_{w4} &= - T_{w2}.
\end{align*}
\]

(1a) (1b) (1c) (1d)

where: \( \delta_i = \theta_i R_i - \theta_i R_{ui} \),

\[
\begin{align*}
\delta_1 &= \theta u R_{u1} - \theta w R_{w1}, \\
\delta_2 &= \theta u R_{u2} - \theta w R_{w2}, \\
\delta_3 &= \theta u R_{u3} - \theta w R_{w3}, \\
\delta_4 &= \theta u R_{u4} - \theta w R_{w4}.
\end{align*}
\]

The nonlinear functions \( f_j(\delta) \), \( j = 1, 2, 3 \) are defined in terms of a stiffness break point \( b_j \) and a relative stiffness between the stages \( \alpha_j \). The equations of motion are nondimensionalized as follows: length \( \delta_j = \delta_j / b_j \), \( \dot{\delta}_j \) \( b_j / b_j \), time \( \ddot{t} = \omega_{\theta} \ddot{t} \) where \( \omega_{\theta} = K_{\theta} / M_{\theta} \), force \( F_{\theta} = F_{\theta} / M_{\theta} b_j^2 \), \( \ddot{F}_{\theta} = F_{\theta} / M_{\theta} b_j^2 \) and frequency \( \Omega = \Omega / \omega_{\theta} \), \( \Omega_{\theta} = K_{\theta} / M_{\theta} \omega_{\theta} \) where \( K_{\theta}, i = 1, 2, 3 \), and \( j = 1, 2, 3 \) are the coefficients of \( f_j(\delta) \) in the ith equation respectively. The value \( b_j \) is any characteristic dimension of the system. The nondimensional equations of motion become:

\[
\begin{bmatrix}
\dddot{\delta}_1 \\
\dddot{\delta}_2 \\
\dddot{\delta}_3 \\
\dddot{\delta}_4
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & -\Omega_{\theta}^2 \\
0 & 0 & -\Omega_{\theta}^2 & 0 \\
0 & 0 & 0 & -\Omega_{\theta}^2 \\
0 & 0 & 0 & 0
\end{bmatrix}\begin{bmatrix}
f_1(\delta_1) \\
f_2(\delta_2) \\
f_3(\delta_3) \\
f_4(\delta_4)
\end{bmatrix} + \begin{bmatrix}
f_{\theta1} \\
f_{\theta2} \\
f_{\theta3} \\
f_{\theta4}
\end{bmatrix}
\]

(2a) (2b) (2c)

\[
\begin{bmatrix}
f_1(\delta_1) \\
f_2(\delta_2) \\
f_3(\delta_3) \\
f_4(\delta_4)
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \delta_1 - \delta_1 \delta_1 \delta_1 + b_1 \delta_1, & b_1 < \delta_1 \\
\alpha_1 \delta_1 + \delta_1 \delta_1 \delta_1 - b_1 \delta_1, & b_1 < \delta_1 \leq \delta_1 \\
\alpha_1 \delta_1 \delta_1 \delta_1 - b_1 \delta_1 + b_1 \delta_1, & \delta_1 < -b_1
\end{bmatrix}
\]

(3a) (3b)

In the equations of motion, the terms \( R_i \) refer to the radius on inertia element \( i \) associated with stiffness element \( j \). The multi-degree-of-freedom (MDOF) system is semi-definite and the equations of motion can be simplified by rewriting Equation (1) in terms of the relative displacement, \( \delta_j \), between the inertia elements. The simplified equations for the relative displacements in an analogous translational system are given as:

\[
\begin{align*}
M_1 \dddot{\delta}_1 + K_s f_1(\delta_1) &= F_{\theta1} + F_{\theta1}, \\
M_2 \dddot{\delta}_2 + K_s f_2(\delta_2) &= F_{\theta2} + F_{\theta2}, \\
M_3 \dddot{\delta}_3 + K_s f_3(\delta_3) &= F_{\theta3} + F_{\theta3}, \\
M_4 \dddot{\delta}_4 + K_s f_4(\delta_4) &= F_{\theta4} + F_{\theta4}.
\end{align*}
\]
STEADY STATE SOLUTION

The method of harmonic balance is used to obtain an approximate analytical solution of the MDOF system given by Equation (3). The method of harmonic balance is chosen for two reasons: an apriori assumption regarding the strength of the nonlinearity (magnitude of $\alpha$) is not required, and the method is well suited to the study of nonlinearities described by nonanalytic functions. The excitation is assumed to have the following form:

$$\ddot{F}_i = \ddot{F}_{m_i} + \dot{F}_p \cos(\Omega_p t + \phi_p), \quad \ddot{F}_2 = \ddot{F}_{m_2}, \quad \text{and} \quad \ddot{F}_3 = \ddot{F}_{m_3}$$  \hspace{1cm} (4)

Here $\ddot{F}_{m_i}$ represents the mean transmitted force associated with each relative displacement and $\Omega_p$ is the amplitude of the vibratory component acting on the first inertia at frequency $\Omega_p$ and relative phase angle $\phi_p$. The approximate solution is assumed to have the form:

$$\ddot{\phi}_j = \ddot{\phi}_{m_j} + \ddot{\phi}_p \cos(\Omega_p t + \phi_p), \quad j = 1, 2, 3$$  \hspace{1cm} (5)

The constant term $\ddot{\phi}_{m_j}$ is used to account for the steady state offset, or bias, introduced by the mean load component $\ddot{F}_{m_j}$. The term $\ddot{\phi}_p$ is the forced response due to the alternating force $\dot{F}_p$. The solution given by Equation (5) assumes that the forced response of Equation (3) is given by the first harmonic only and, therefore, neglects not only the higher harmonics and the possibility of superharmonics or subharmonics but also the possibility of combination resonances and internal resonances.

The nonlinearities $f(\ddot{\phi})$ are expanded in a Fourier series, retaining only the mean and first harmonic terms for each nonlinearity. The nonlinear functions given in terms of the describing functions $N_{m_j}$ and $N_{p_j}$ are:

$$f(\ddot{\phi}) = N_{m_j} \ddot{\phi}_{m_j} + N_{p_j} \ddot{\phi}_p \cos \varphi_{p_j}, \quad j = 1, 2, 3$$  \hspace{1cm} (6a)

$$N_{m_j}(\ddot{\phi}_{m_j}, \ddot{\phi}_p) = \int_0^1 f(\ddot{\phi}) \, d\varphi_{p_j} \quad \hspace{1cm} (6b)$$

$$N_{p_j}(\ddot{\phi}_{m_j}, \ddot{\phi}_p) = \int_0^1 f(\ddot{\phi}) \cos \varphi_{p_j} \, d\varphi_{p_j} \quad \hspace{1cm} (6c)$$

$$\varphi_{p_j} = \Omega_p t + \phi_{p_j} \quad \hspace{1cm} (6d)$$

Substituting Equations (4), (5), and (6) into Equation (3) and equating coefficients of like harmonics yields a set of coupled nonlinear algebraic equations. Solving these equations for $\ddot{\phi}_{m_j}$ and $\ddot{\phi}_p$ and redefining the mean components relative to the center of their respective nonlinearities yields the following set of nonlinear algebraic equations:

$$\ddot{\phi}_{m_j} = \frac{\dot{F}_{m_j}}{N_{m_j}}, \quad j = 1, 2, 3 \quad \text{and} \quad \ddot{F}_{m_1} = \frac{T_{m_1}}{R_1 b_k K_1}$$  \hspace{1cm} (7a,b)

$$\ddot{F}_{m_2} = \frac{T_{m_2} - T_{m_1}}{R_2 b_k K_2} \quad \ddot{F}_{m_3} = \frac{T_{m_3}}{R_3 b_k K_3}$$  \hspace{1cm} (7c,d)

$$\ddot{\phi}_p = \frac{\dot{F}_p}{N_{p_j}}$$  \hspace{1cm} (7e)

$$\ddot{\phi}_p = \frac{\dot{F}_p}{N_{p_j}}$$  \hspace{1cm} (7f,g)

$$\tan(\phi_p - \phi_{p_j}) = \frac{\dot{F}_p}{\dot{F}_{m_j}} \quad \hspace{1cm} (7h)$$

$$\Lambda = \left\{ \begin{array}{l} N_{p_j} - \frac{\ddot{\phi}_{m_j}}{\ddot{\phi}_p} \\ \frac{\ddot{\phi}_{m_j}}{\ddot{\phi}_p} \\ \frac{\ddot{\phi}_{m_j}}{\ddot{\phi}_p} \end{array} \right\} \Lambda$$  \hspace{1cm} (7i)

Here a positive sign is used when $\ddot{\phi}_p$ and $\ddot{F}_p$ are in phase and a negative sign when they are out of phase where the relative phase $\phi_p - \phi_{p_j}$, is defined by Equation (7i). The describing functions $N_{m_j}$ and $N_{p_j}$ are given by Equations (8a) and (8b):

$$N_{m_j} = 1 + \frac{\ddot{\phi}_p}{2N_{m_j}} \left\{ (1 - \alpha_j) \left[ G \left( \frac{\ddot{\phi}_p}{\ddot{\phi}_{m_j}} \right) - G \left( -\frac{\ddot{\phi}_p}{\ddot{\phi}_{m_j}} \right) \right] \right\}$$  \hspace{1cm} (8a)

$$N_{p_j} = 1 - \frac{(1 - \alpha_j)}{2} \left[ H\left( \frac{\ddot{\phi}_p}{\ddot{\phi}_p} \right) - H\left( -\frac{\ddot{\phi}_p}{\ddot{\phi}_p} \right) \right]$$  \hspace{1cm} (8b)

$$G(\mu) = \frac{\mu}{\pi} \left( \mu \sin^{-1} \mu + \sqrt{1 - \mu^2} \right), \quad |\mu| \leq 1$$

$$= \mu, \quad |\mu| > 1$$  \hspace{1cm} (8c)

$$H(\mu) = \frac{\mu}{\pi} \left( \sin^{-1} \mu + \mu \sqrt{1 - \mu^2} \right), \quad |\mu| \leq 1$$

$$= +1, \quad |\mu| > 1$$  \hspace{1cm} (8d)

$$\mu = \frac{\ddot{\phi}_p - \ddot{\phi}_{m_j}}{\ddot{\phi}_p}$$  \hspace{1cm} (8e)

To facilitate the development of the solutions in terms of the design variables, the functions $G$ and $H$ are replaced with truncated series expansions. The truncated series expansion is obtained by retaining only the first two terms in each series and adjusting the coefficient of the second term to yield the actual value of the series when the argument $\mu=1$. This modification is necessary because when the argument is close to 1, the contribution from the higher terms is significant. For small values of $\mu$, the contribution from the higher terms is small and the modification is not required as these terms have little or no effect on the functions $G$ and $H$. The truncated series expansion is obtained within $5\%$ of the original function $G$ and within $6\%$ of $H$ are found to be as follows for $|\mu| \leq 1$:

$$G(\mu) \approx \frac{\mu}{\pi} \left[ \left( \frac{\pi - 2}{2} \right) \mu \right], \quad H(\mu) \approx \frac{\mu}{\pi} \left[ \mu - \left( \frac{4 - \pi}{4} \right) \mu^2 \right]$$  \hspace{1cm} (9)
In all cases, when \( \alpha = 1 \) the describing functions \( N_{m1} \) and \( N_{m2} \), given by Equations (8a) and (8b), reduce to one, which is the linear case.

The MDOF system can be viewed as essentially consisting of three coupled systems called impact pairs (IP); the I-11 pair, the II-III pair, and the III-IV pair. To better understand the type of behavior which may be expected, consider the physical effect of the nonlinear stiffnesses on one of the impact pairs. The overall response can be separated into three different regimes: a no impact regime, a single sided impact regime, and a two sided impact regime. The different impact regimes are illustrated for backlash in Figure 3.

This type of nonlinearity can be viewed as an amplitude dependent stiffness with an average value given by the relative amount of time (over one period of vibration) that the IP is in one stage versus the other. As the amplitude of the alternating component \( \delta_p \) changes so does the stiffness. The IP is said to be hardening if the stiffness increases for increasing \( \delta_p \) and softening if the stiffness decreases for increasing \( \delta_p \). For the clearance type nonlinearity, the hardening or softening character will depend on the location of the mean component and on whether the IP is undergoing single or two sided impacts (the system is linear for the no impact case). For single sided impacts if the mean component (static deflection) is in the second stage, as \( \delta_p \) increases the average stiffness decreases because the amount of time the system spends in the first stage increases and the system is softening. For the two sided impact case the single sided case where the mean component is in the first stage the situation is reversed and the average stiffness will increase as \( \delta_p \) increases resulting in a hardening system. Each IP may undergo any or all of the different impact regimes, so the MDOF system may have a total of \( 3 \times 3 \times 3 = 27 \) cases. Further complicating the situation is the fact that the IPs are dynamically coupled; therefore, the behavior of a given IP will, in general, depend on the type of vibration experienced by the other two.

Since the MDOF nonlinear system can exhibit a wide range of dynamic behavior, general observations and conclusions usually cannot be made without considering special cases. Further a general nonlinear analysis can be very complicated and expensive so it is useful to know when a nonlinear analysis is actually required and when a linear analysis might be acceptable. Although the specific results from analyses of the special cases considered here can not be directly applied to a wider class of problems, the general methodology introduced should be applicable. To illustrate these points consider the frequency response for the neutral rattle case.

**NEUTRAL GEAR RATTLE - SIMPLIFIED SOLUTION**

In neutral the clutch is generally designed as a vibration isolator (Xie (1987)) and hence for low transmissibility the operating speed (idle speed) \( \Omega > \sqrt{2} \Omega_{n1} \) where \( \Omega_{n1} \) is the resonance associated with the stiffness of first stage. In general the idle speed \( \Omega \) is fixed so the stiffness of the first stage, \( \alpha \Omega_1 \), must be chosen to yield an appropriate resonant frequency. Space constraints limit the amount of wind-up allowed and place a lower limit on the spring stiffness and so the idle speed \( \Omega \) will, in general, only be slightly larger than the first stage resonance \( \Omega_{n1} \). The other resonances in the transmission are associated with the shaft and mesh stiffness and will be much higher. A typical idle speed would be 15Hz with the engine torque pulse excitation at \( \Omega_{n1} = 30 \)Hz (for a four cylinder engine). The first shaft resonance will usually be an order of magnitude larger, (say \( \Omega_{n2} = 10 \Omega_{n1} \)), and the gear mesh resonances are usually two orders of magnitude larger, (say \( \Omega_{n3} = 100 \Omega_{n2} \)). Assuming a linear shaft stiffness \( K_2 \), neglecting the clutch spline, it can be shown that \( \Omega_{n2} > \Omega_{n1} \), \( \Omega_{n3} > \Omega_{n2} \), and Equation (9) can be simplified to yield a single mode system as shown schematically in Figure 4. An approximate damped response is obtained by redefining the normalized excitation frequency and adding a hysteretic damping term \( \eta \) where it is assumed that the hysteresis can be represented by a complex stiffness in the frequency domain (see Thomson (1981)). The resulting equations are:

\[
\delta_p = \frac{\tilde{F}_p}{N_{m1}}, \quad \delta_p = \frac{\tilde{F}_p}{\sqrt{\left(\frac{N_{m1}}{\Omega_{n2}}\right)^2 + \eta^2}} (10a, b)
\]

\[
\delta_p = \frac{\tilde{F}_p}{\sqrt{N_{m1}^2 + \eta^2}} \quad \delta_p = \frac{\tilde{F}_p}{\sqrt{\left(\frac{N_{m2}}{\Omega_{n3}}\right)^2 + \eta^2}} (10c, d)
\]

where:

\[
\tilde{F}_p = \frac{\bar{F}_p}{\Omega_{n1}} (1 - \frac{1}{\Omega_{n1}^2}) (10e, f)
\]

\[
\tilde{F}_p = \frac{\bar{F}_p}{\Omega_{n2}^2 (1 - \frac{1}{\Omega_{n2}^2})} (10g, h)
\]

\[
\tilde{F}_p = \frac{\bar{F}_p}{\Omega_{n3}^2 (1 - \frac{1}{\Omega_{n3}^2})} (10i, j)
\]

From an examination of Equation (10) it is clear that for neutral rattle, the frequency response of the flywheel-clutch system \( \delta_p \) is given by the frequency response of a single impact pair and is independent of \( \delta_{p2} \) and \( \delta_{p3} \). As a result, the behavior of the clutch can be studied independently from the gear pair. Once the behavior of the flywheel-clutch system is known then the behavior of the gear pair can be found. Therefore, the analysis of the neutral gear rattle problem can be reduced to an analysis of a single impact pair.

**FREQUENCY RESPONSE OF AN IMPACT PAIR**

A detailed discussion of the frequency response of a simple impact pair is given by Companin and Singh (1988) and only a summary and application will be presented here. The overall solution is subdivided into three cases: no impacts, single sided impacts, and two sided impacts as shown in Figure 3. Substituting the appropriate form of the describing functions \( N_{m1} \) and \( N_{m2} \) into Equation (10) yields a set of equations which describe the dynamic behavior of the impact pairs. The complete equations are given by Companin (1988).

To understand the response of the flywheel-clutch system consider the response of a simple undamped impact pair:

\[
\tilde{\delta}_p + K(\tilde{\delta}_p) \delta_p = \tilde{F}_p (11)
\]

Here \( K(\tilde{\delta}_p) \) is a nondimensional amplitude dependent (nonlinear) stiffness given by:

\[
\text{No Impact (linear): } K(\tilde{\delta}_p) = \begin{cases} \alpha & \text{Type 1} \\ \eta & \text{Type 2} \end{cases}; \quad \tilde{\delta}_p < \tilde{\delta}_{p0} (12a)
\]
Single Sided Impact: \[ K(\delta_p) = \alpha_a \frac{F_p}{\delta_p} ; \delta_{p_{st1}} \leq \delta_p \leq \delta_{p_{st2}} \] (12b)

Two Sided Impact: \[ K(\delta_p) = 1 - \frac{F_p}{\delta_p} ; \delta_p > \delta_{p_{st2}} \] (12c)

where \( \delta_{p_{st1}} \) is the threshold displacement for two sided impacts and \( \delta_{p_{st2}} \) is the threshold displacement for single sided impacts. From Equation (12) it is clear that \( K(\delta_p) \) will be hardening when \( F_p > 0 \) and \( F_p > 0 \), softening when \( F_p < 0 \) and \( F_p < 0 \), and linear when \( F_p = 0 \) and \( F_p = 0 \). In addition the nonlinear stiffness saturates for large amplitudes of \( \delta_p \) as the stiffness approaches a limiting value: \( K(\delta_p) \rightarrow K_a \) (second stage stiffness) for two sided impacts and \( K(\delta_p) \rightarrow \alpha_a \) (average stiffness of the stages) for single sided impacts. For small amplitudes of \( \delta_p \), but still large enough for impacts to occur, the effect of the nonlinearity becomes more pronounced, but in all cases \( K(\delta_p) \geq 0 \).

The general shape of the nonlinear frequency response curve can be examined by considering a few limiting cases. Consider first the two sided impact case. When \( \delta_p = \delta_{p_{st2}} \) and \( \delta_p = 0 \) the term \( \delta_p = \alpha_a \delta_p \) and when \( \delta_p > \delta_{p_{st2}} \) the term \( \delta_p = \frac{4}{\pi} \alpha_a \delta_p \). Here \( \alpha_a = (1 - \alpha) \) is the deviation stiffness and the system will be hardening when \( \alpha_a > 0 \), softening when \( \alpha_a < 0 \), and linear when \( \alpha_a = 0 \).

From Comparin and Singh [1988], for zero damping, the response is given as:

\[ \delta_p = \frac{F_p}{\Omega_p^2} \] (13)

The positive sign is used when the force and displacement are in phase, \( \Omega_p^2 > 0 \), and the negative sign is used when the force and displacement are out of phase, \( \Omega_p^2 < 0 \). When \( |F_p| > |F_p| \) there will be two solutions below resonance and no solution above resonance for \( \alpha < 1 \) and two solutions above resonance and none below for \( \alpha = 1 \). Comparin and Singh [1988] have shown that when two solutions occur the larger amplitude solution is stable and the smaller amplitude is unstable. For \( |F_p| > |F_p| \) there will be a single solution both above and below resonance. It can be seen that when two sided impacts occur the general shape of the frequency response curve is essentially the same as the linear case except when \( |F_p| > |F_p| \), as solutions will only exist on one side of resonance. The transition from two solutions to no solutions results in a jump discontinuity in the frequency response function.

For single sided impacts, when \( \delta_p = \delta_{p_{st1}} \) the term \( \delta_p = \frac{1}{2} \alpha_a (b - \delta_m) \) when \( \delta_{p_{st1}} < \delta_p < \delta_{p_{st2}} \), and when \( \delta_p = \frac{2}{\pi} \alpha_a (b - \delta_m) \). The system will be hardening when \( \alpha_a (b - \delta_m) > 0 \) and softening when \( \alpha_a (b - \delta_m) < 0 \), but it is linear only when \( \alpha_a = 0 \). The response for single sided impacts is similar to the two sided case and, for zero damping, is given by:

\[ \delta_p = \frac{2F_p + \alpha_a (b - \delta_m)}{2\alpha_a} \]

Comparing Equation (14) with Equation (13) for the two sided impact case, it is clear when single sided impacts occur the general shape of the frequency response function remains similar to the linear case. But when \( |F_p| > |F_p| \), solutions will exist only on one side of resonance which is now at \( \Omega_p = \sqrt{\alpha_a} \). The single sided impact case, however, is more complicated than the two sided impact case because the sign of \( F_p \) depends on the term \( (b - \delta_m) \) as well as on \( \alpha_a \). For example, consider the case where \( \delta_p = \delta_{p_{st1}} \) and \( b > \delta_m \). As \( \delta_p \) increases \( \delta_p \) will decrease. If \( \delta_p \) becomes sufficiently large, \( \delta_p \) can move from the second stage \( b > \delta_m \) to the first stage \( b < \delta_m \) and in so doing the system will go from a softening type to a hardening type or vice versa depending on the sign of \( \alpha_a \). If \( \delta_p \) decreases the transition will reverse itself.

Representative frequency response functions are shown in Figures 4-7. Figure 5 represents a case where the system goes from softening to hardening as \( \Omega_p \) increases and therefore exhibits two jumps. Figure 6 is a strictly hardening system and Figure 7 is a strictly softening hence both have only one jump. In each of the Figures 4-7, the arrows indicate the paths for increasing and decreasing frequency \( \Omega_p \).

**APPLICATION TO NEUTRAL GEAR RATTLE**

The mathematical model presented earlier is ideally suited for the analytical study of the neutral rattle problem. For the neutral case, the clutch resonant frequency \( \Omega_{nn} \) is usually made as low as possible relative to the idle speed \( \Omega \) so that the clutch behaves as a vibration isolator (Xie [1987], Fujimoto et al. [1987], Reik [1986], etc.). The operating speed is essentially constant so the design objective is to select an appropriate set of system parameters such as the first stage stiffness \( \alpha_i K_i \), relative inertia \( M_i \), break point \( \delta_i \), and hysteretic damping \( T_i \), etc. to minimize or eliminate the gear rattle problem.

Consider the set of typical frequency response functions shown in Figures 8-10 for different impact conditions in the clutch: no impacts, single impacts, and two impacts respectively. The frequency response functions were obtained using a set of nondimensional values for an actual transmission system. The clutch behaves like a simple IP, as discussed above, and for the impact case the clutch represents a hardening system so a single jump occurs. The dynamic behavior of the gear pair follows that of the clutch but the amplitude ratio \( \delta_p / \delta_m \) varies due to impacting in the clutch as well as impacting in the gear pair.

It should be pointed out that the frequency response functions for the gear pair will appear somewhat distorted because the clearance space is very large with respect to the elastic deformation. This leads to very large slopes in the transition regions which appear as jumps in the frequency response functions. These "apparent" jumps are different from the jump transition which occurs in the clutch. The jump transition in the clutch was described earlier for the single impact pair and occurs at resonance because of the phase change between the applied force and the displacement. The apparent jumps in the response of the gear pair are not related to phase changes as they do not necessarily occur at resonance. Instead these are simply a function of the magnitude of \( \delta_p \).
DISCUSSION OF RESULTS

From a qualitative point of view, the neutral rattle case is strongly nonlinear because the overall response of the system is dominated by the nonlinear response of the clutch. Accordingly, a study of neutral rattle requires the use of a nonlinear analysis with an emphasis on the design of the clutch. This is consistent with similar studies reported in the literature by Xie [1987], Fujimoto et al. [1987], Reik [1986], Ohnuma et al. [1985], and Sakai et al. [1981], all of whom used nonlinear models as part of their studies of neutral rattle.

Because of the complexity of the nonlinear system, however, the design methodology is still based primarily on linear models. Specifically, if it is assumed that impacting does not occur, either in the clutch or at the gear pair, the nonlinear problem reduces to a linear one. Using a linear model it is possible to develop a set of equations for predicting the threshold at which rattle will begin (see Xie [1987] and Sakai et al. [1981]). These threshold equations then form the basis of the development of design guidelines.

When impacting in the clutch is unavoidable, the linear models are no longer valid and it is then necessary to return to the use of nonlinear models. Because of the lack of approximate analytical solutions, the design guidelines for the nonlinear case are based on nonlinear digital and/or analog simulations (Xie [1987], Fujimoto et al. [1987], Reik [1986], Ohnuma et al. [1985], and Sakai et al. [1981]), and/or experimental results from actual transmission systems (Ohnuma et al. [1985], Reik [1986], and Fujimoto et al. [1987]). As a result, the global design concepts are understood but the effect of the system nonlinearities on these concepts is not.

The major problem for the nonlinear case involves the region of multiple steady state solutions. The region exists just above the linear natural frequency associated with the first stage of the clutch and may be in the operating region for the neutral case. If the operating point falls in this region it is possible to get a low amplitude no rattle solution, or a large amplitude rattle solution depending on the initial conditions. Different initial conditions can be provided by gradual or abrupt clutching operations. Fujimoto et al. [1987], Reik [1986] and Ohnuma et al. [1985] observed a jump phenomenon associated with impacting in the clutches as well as the existence of multiple steady state solutions. While they do not provide analytical formulations for describing the region, their studies do acknowledge the importance of understanding this region for reducing rattle. Current design practice is to ensure the operating point is above and away from this region.

The analytical formulations presented in this study are suitable for the analysis of both the linear and nonlinear cases. For no impacting, Equation (10) reduces to a set of linear equations and can be used for the same threshold type analysis described by Xie [1987] and Sakai et al. [1981]. The analytical formulations are also valid for the nonlinear case and can be used to predict the range of multiple steady state solutions and the effect of the system parameters on this region. The nonlinear behavior discussed in the previous section is consistent, not only with the nonlinear digital and analog simulations described in the literature by Xie [1987], Fujimoto et al. [1987], Reik [1986], Ohnuma et al. [1985], and Sakai et al. [1981], but also with the behavior of actual automotive transmission systems as reported by Ohnuma et al. [1985], Reik [1986], and Fujimoto et al. [1987].

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Fig. 1 Generalized Four Degree-of-Freedom System. I: Flywheel, II: Clutch, III: Input Gear, and IV: Output Gear.

Fig. 2 Force Displacement Relationship of Nonlinear Stiffness $f(\delta)$.

Fig. 3 Illustration of Different Impact Regimes: (a) No Impacts (b) Single Sided Impacts (c) Two Sided Impacts.

Fig. 4 Neural Rattle Case (a) Physical Model (b) Typical Driving Point Frequency Response Function.

Fig. 5 Frequency Response $\delta_\alpha$ for $\alpha = 0$ (*Nonlinear) and $\alpha = 1$ (*Linear).

Fig. 6 Frequency Response $\delta_\alpha$ for $\alpha = 0.5$ (*Nonlinear) and $\alpha = 1$ (*Linear).
Fig. 7 Frequency Response $\delta_3$ for $\alpha = 1.5$ (Nonlinear) and $\alpha = 1$ (Linear).

Fig. 9 Frequency Response Functions for Neutral Rattle Case with Single Sided Impacting in the Clutch. (a) $\delta_{p1}$ and (b) $\delta_{p3}$

Fig. 8 Frequency Response Functions for Neutral Rattle Case with No Impacting in the Clutch. (a) $\delta_{p1}$ and (b) $\delta_{p3}$

Fig. 10 Frequency Response Functions for Neutral Rattle Case with Two Sided Impacting in the Clutch. (a) $\delta_{p1}$ and (b) $\delta_{p3}$