

# Modeling of Automotive Gear Rattle Phenomenon: State of the Art

Chandramouli Padmanabhan, Todd E. Rook, and Rajendra Singh  
The Ohio State Univ.

## ABSTRACT

The sources of gear rattle in automotive transmission and other components are clearance non-linearities, which include backlashes, multi-valued springs, hysteresis, etc. Periodic vibro-impacts are generated because of the single sided or double sided impacts. It is obviously desirable to develop an appropriate computer simulation model which can aid in the control of transient noise and vibration signatures. Such models can also be used to design experiments and to interpret measured data. Several approaches have been attempted in the past, but most common is the digital simulation of the governing differential equations describing the non-linear, torsional dynamics of the drivetrain. Some progress has been made in understanding the basic rattle phenomenon and in developing suitable mathematical models. This article intends to be a status report on the mathematical or computer models including pre- and post-processing considerations. Various topics that will be briefly discussed include dimensionality and reduced order models, numerical difficulties encountered by solution algorithms, design analysis and optimization, etc. Two specific case histories are analyzed in this paper. The first deals with a reverse-idler gear pair rattle problem, and the second case examines flywheel-clutch-hub spline subassembly.

## INTRODUCTION

Automotive gear rattle phenomenon belongs to a general class of problems where periodic vibro-impacts are observed in noise and vibration signals. The basic source is usually a component or sub-system with a clearance non-linearity which includes gaps, multi-valued stiffness characteristics, dry friction, hysteresis, and the like. Dynamic excitation is usually at lower frequencies, say engine torque pulsations and the mean load of the system is usually very small. Also in some cases the load reversals may also take place within a cycle thereby introducing conditions for single sided or double sided impacts. Drag or mean friction torque also plays an important role along with dynamic system parameters.

Much of the research has concentrated on the neutral rattle problem in manual transmissions even though drive and coast rattles are now being viewed with concern. The authors of this article have considerable prior experience in addressing

neutral rattle-type problems in several physical systems. An attempt will be made here to present a set of cohesive rules that we have developed to facilitate the art and science of computer modeling. Before proceeding further, the reader should however recognize the following: (i) the rattle problem is generally a dynamic system problem i.e. it is not a gear dynamic issue (such as whine); (ii) a suitable nonlinear torsional model of the system is needed to understand the basic characteristics, to find optimal design solutions, and to examine signal processing and sound perception issues; and (iii) rattle problems are best analyzed in time domain even though we can use other domains to obtain some fundamental properties of a vibro-impact oscillator. Given the space limitations, only an overview of the state of art is presented.

## REVIEW OF AVAILABLE LITERATURE

Previous investigators have studied clearance induced vibro-impacts through a variety of techniques, such as digital [1, 2] and analog [3, 4] simulations, experimental noise perception studies [5, 6], and the multi-body dynamics approach [7, 8]. Many of these studies have focused on the neutral rattle phenomena in automotive manual transmissions. Yet many fundamental issues remain to be addressed [9]. For instance, even though researchers have discussed the difficulties in various stages of the problem [10], yet no thorough framework covering the entire investigation process of such problems currently exists. For example, consider some of the difficulties in the formulation stage. In the cases of many practical rattle problems, many different elements and clearances are encountered (e.g. gears, bearings, splines, and clutches). Consequently the dimension can be quite large, which is inefficient from a computer simulation point of view. Therefore, in the formulation, reduced order models must be utilized [1, 4, 11]; however the appropriateness of such models is difficult to assess a priori. Furthermore, the non-analytic nature of vibro-impact problems [3, 9, 12, 13] presents particular difficulties at the simulation stage, including ill-conditioning and numerical "stiffness" [9, 14]. A variety of numerical algorithms, including direct time integration [1, 11] and harmonic balance and shooting techniques [12, 13], have been investigated for these types of problems. Also not to be overlooked are the problems encountered in the classification of sound quality or

perception. Such issues have received little attention in the literature [5, 6].

## MODELING AND COMPUTER SIMULATION STRATEGY

Most models used to study the automotive rattle problem involve a multi-degree-of-freedom lumped parameter torsional system incorporating non-linear machine elements such as a multi-stage clutch and/or backlash in one or more gear meshes. Based on our extensive experience [1,3-5,9,11-13,15], we have developed the following steps to successful modeling: (i) formulate a suitable model for simulation (pre-processing stage), (ii) select a suitable numerical method to obtain solutions (processing stage), and (iii) choose or develop performance indices to evaluate and optimize various design parameters in order to reduce noise and vibration levels (post-processing stage). Although the study of the automotive rattle problem has been divided into three distinct stages so that various issues can be highlighted, it will become clear that these stages are often inter-dependent. For instance, problems encountered in the solution stage due to numerical "stiffness" are addressed by trying to develop a suitable non-dimensionalization scheme in the pre-processing stage.

### Pre-processing stage

Figure 1 shows a flowchart outlining the various issues involved in the formulation of the automotive rattle problem before a solution can be obtained; this is based on our experience with simulations of rattle type problems. The first issue in this process is often the size of the model; this dimension is dictated by the need to reduce the size of the non-linear model for efficient simulation while retaining all of the essential dynamic characteristics. For instance, if the primary concern is the optimization of clutch design parameters, a two or three-degree-of-freedom model is often sufficient. If however, there is a need to examine the dynamic interactions between the gear backlash non-linearity and the clutch, a larger model should be constructed.

An insight into this process can be gained by examining the natural frequencies of the linearized transmission system in the frequency range of interest. The non-linear model can be linearized about an operating point and the resulting linear model can be utilized to reduce the dimension of the system by lumping various inertias appropriately. As an example, consider a eight-degree-of-freedom model of an automotive transmission, with a single mesh gear pair, a double mesh gear set, a flywheel and a clutch as shown in Figure 2a. All of the stiffnesses are assumed to be linear and the resulting natural frequencies of the system are: 0(0), 37(37), 1551, 2129 (1991), 2272, 2555 (2551), 6296, 8851 Hz; the numbers in brackets are those of the reduced five-degree-of-freedom shown in Figure 2b. And the first two mode shape vectors are  $[-0.108 \ 0.464 \ 0.466 \ 0.466 \ 0.466 \ 0.467 \ 0.467 \ 0.467]^T$ , and  $[0.885 \ 0.322 \ 0.505 \ 0.516 \ -0.083 \ -0.149 \ -0.181]^T$ . The excitation to this system is periodic in nature and is due to engine torque fluctuations. Excitation frequencies are typically in the range of 25-75 Hz. So the first mode in the linear system would dominate the response and hence one can lump the last gear pair and inertia into the inertia  $I_1$ , reducing the dimension to five.

To achieve computational benefits beyond those afforded by a reduction of dimension, one may perform an appropriate

form of non-dimensionalization and coupling. Such benefits include the elimination of numerical drift associated with rigid body motion, and the reduction of the ill-conditioning arising from the numerical stiffness of the system; earlier investigations of automotive rattle [9] have discussed such computational difficulties due to the numerically "stiff" nature [14] of this problem. More specifically, the ill-conditioning occurs due to the presence of components in the solution whose frequency ratio is large [9,14]. To further explain the issue of numerical stiffness, first consider that the equations of motion can be generally written in first-order vector form as  $dy/dt = p(y,t)$ . Linearizing around an operating point,  $y_0$ , as do most numerical codes, and ignoring higher order terms, we have the following expression  $p(y,t) \cong p(y_0,t) + J(y - y_0)$ ,

where  $J = \left. \frac{dp}{dy} \right|_{y_0}$  is the Jacobian matrix. The local

eigenvalues of this Jacobian define the numerical "stiffness". The vibro-impact problem is classified as "stubborn" by Aiken [14], due to system derivative (Jacobian) discontinuities, varying mechanical stiffness, and the occurrence of highly oscillatory components. Various stiff categories are defined [9, 14], based on the ratio,  $R_\lambda$ , of the largest eigenvalue to the smallest (non-zero) eigenvalue. In order to estimate this ratio,  $R_\lambda$ , a concept of effective non-dimensional stiffness,  $\Gamma$ , is utilized here. The idea behind this procedure is to replace the time-dependent Jacobian components (associated with the nonlinear elements) with their time-averaged values. Consider a gear pair as an example. If two gears are connected by a backlash element, the pair may become uncoupled; this loss of contact is associated with a zero stiffness regime ( $\Gamma=0$ ), while full contact is associated with a finite non-zero stiffness regime (i.e.  $\Gamma=1$  on a non-dimensional basis). Hence, for a gear pair where contact is lost during the cycle of motion, the effective stiffness will be between zero and one (i.e.  $0 < \Gamma < 1$ ). Therefore the effective stiffness may be based on the time that the pair is uncoupled (e.g. if the system is uncoupled 50% of the time, one would expect that  $\Gamma=0.50$ ) or estimated by some other type of weighting based on the nonlinear system solution [15]. Then the eigenvalues of the averaged Jacobian correspond to a type of average eigenvalues for the physical system itself. The other computational improvement besides non-dimensionalization, coupling, is performed by posing the equations of motion in terms of the relative displacements, velocities and accelerations; these quantities do not possess rigid body drift as do the absolute motions. The coupling also has the advantage of improving the system "stiffness" as shown in Figure 3.

### Processing stage

Once the problem is formulated, a specific numerical technique must be chosen. A comparison of some different techniques is made in Table 1. Generally, the numerical techniques belong to either of two categories: initial-value or boundary-value problems. A wide variety of direct time integration techniques (e.g. the many Runge-Kutta variants--Fehlberg and Verner, and others such as Gears and Adams [9]), belong to the former, while harmonic balance and shooting methods belong to the latter. Whichever method is selected, it will be important that it be capable of successfully producing accurate time-domain results, as will be shown in the next section. Refer to [9] for a detailed discussion of such considerations.

### Post-processing stage

Of main interest is the study of the effect of various component or system designs on the vibration/shock isolation and noise perception indices with an emphasis on time domain approaches; a flowchart is given in Figure 4. Prior investigators [1, 10] have developed various rattle indices, based mostly on *intuition*, in order to evaluate the transmission design. Due to vibro-impacts, impulsive transients occur in a periodic fashion over one cycle of the acceleration time history. An increase in noise level caused by this phenomenon is a strong function of the acceleration peak amplitude(s) and the decay rate(s). Hence a single measure of acceleration attenuation across the drive-train is not sufficient to characterize the system behavior. This deficiency is overcome in our proposed scheme where the excitation frequency,  $\omega$ , is first filtered from the predicted time histories of the acceleration signals. Figure 5 compares the typical unfiltered and the corresponding filtered time domain acceleration of a component in the system (e.g. the gear) and one can clearly observe the characteristics of the impact transients. For instance, the impulsive transients of Figure 5c, show a sharp pulse followed by an exponentially decaying sinusoid. Using the unfiltered signal, two new indices,  $E_1$  based on the ratio of mean-square values of the accelerations and  $E_2$  based on the ratio of the peak to peak values, are developed to judge the effectiveness of any clutch from the noise control perspective. Further, two more indices, based on the filtered signals are developed specifically to assess sound perception; one is based on the mean-square energy,  $E_3$  and the other based on the energy contained within the initial sharp pulse,  $E_4$ .

$$E_1 = \left\{ \frac{\ddot{\theta}_{gear,ms}}{\ddot{\theta}_{flywheel,ms}} \right\}; \quad \ddot{\theta}_{i,ms} = \left\{ \frac{1}{\tau} \int_0^{\tau} \ddot{\theta}_i^2 dt \right\}; \quad \tau = 2\pi/\omega \quad (1)$$

$$E_2 = \left\{ \frac{\ddot{\theta}_{gear,p}}{\ddot{\theta}_{flywheel,p}} \right\}^2; \quad \ddot{\theta}_{i,p} = [\max(\ddot{\theta}_i) - \min(\ddot{\theta}_i)] \quad (2)$$

$$E_3 = \int_0^{\tau} \ddot{\theta}_{iF}^2 dt; \quad E_4 = \ddot{\theta}_{iF,p}^2 \Delta t; \quad \Delta t \text{ pulse duration};$$

$$\ddot{\theta}_{iF} \Rightarrow \text{Filtered} \quad (3, 4)$$

where in the above equations  $i \equiv$  station index.

### CASE HISTORY I: REVERSE-IDLER STUDY

First, we study a reverse-idler geared system in which there are multiple clearances which may be the source of much rattle. We consider two different models to investigate this problem; a 3-DOF model shown in Figure 6a, and 4-DOF model shown in Figure 6b. Both these models have only torsional degrees of freedom and can be obtained by lumping the appropriate inertias about the reverse-idler pair in Figure . Their equations of motion can be written in general coupled [15] matrix form as

$$I\ddot{q}(\tau) + \Xi \dot{q}(\tau) + \Psi f(q(\tau), \dot{q}(\tau)) = Q(\tau) \quad (5)$$

where  $q$  is a vector of the relative displacements,  $\Xi$  is the coupled damping matrix,  $\Psi$  is the coupled stiffness matrix,

and  $Q$  is the coupled excitation. The system parameters are also labeled in the figures. The comparison of the two models with experimental results will help demonstrate the importance of proper dimension as discussed in Section 3.1. First, the basic 3-DOF model is used with the results shown in Figure 6a. When compared to the experimental results, it is observed that the impacts in the simulation results are out of phase. This deficiency in the model was due to the fact that significant torsional deflections of the shaft were found in the experiment. Consequently, the 4-DOF model which included shaft dynamics (Figure 6b) was utilized. The results of this model compare quite favorably to the experimental results; the impacts are in the proper phase, and of similar duration. Therefore, it is illustrated that oversimplifying a computer model (3 rather than 4-DOF) can mean the difference between excellent and poor correlation with the experiment.

### CASE HISTORY II: CLUTCH DESIGN STUDIES

Second, we highlight the role of various clutch design parameters in controlling noise and vibration generated by the rattle phenomenon. To achieve this objective the generic five degree-of-freedom model of Figure 1b is reduced to a three degree-of-freedom torsional model as shown in Figure 8a by lumping inertias  $I_4$  and  $I_5$  into

$I_3 = I_3 + (R_3^2/R_4^2)I_4 + (R_3^2/R_5^2)I_5$ . The model retains non-linear elements such as a multi-stage dry friction damping, multi-stage torsional clutch stiffness and hub-spline backlash. Governing equations of motion, which are strongly non-linear, are given by:

$$I_1 \ddot{\theta}_1 + C_c(\dot{\theta}_1 - \dot{\theta}_2) + T_c[\theta_1, \theta_2] = T_e \sin(\omega t) \quad (6)$$

$$I_2 \ddot{\theta}_2 - C_c(\dot{\theta}_1 - \dot{\theta}_2) + C_h(\dot{\theta}_2 - \dot{\theta}_3) - T_c[\theta_1, \theta_2] + T_h[\theta_2, \theta_3] = 0 \quad (7)$$

$$I_3 \ddot{\theta}_3 - C_h(\dot{\theta}_2 - \dot{\theta}_3) - T_h[\theta_2, \theta_3] = T_m - d_3(\omega/2 + \dot{\theta}_3) \quad (8)$$

where  $I_1$  and  $I_2$  represent the clutch disk and clutch hub inertias respectively, while  $I_3$  corresponds to the reflected gear inertias;  $\theta_i$ ,  $i=1..3$ , are the torsional displacements,  $\theta_c, \theta_h$  correspond to the clutch disk breakpoint and the hub-spline backlash values,  $C_c$  and  $C_h$  the viscous damping coefficients of the clutch disk and hub-spline respectively,  $d_3$  the drag torque damping coefficient and  $T_e$  is the engine torque. Typical non-linear clutch torque-displacement characteristics  $T_c$  and hub-spline backlash torque  $T_h$  are also shown graphically in Figures 8b and 8c respectively. The equations are solved by using a special Runge-Kutta 5th order (4th order error estimator) variable step-size numerical integration routine which performs fairly robustly even while encountering numerical *stiffness* problems caused by high frequency vibro-impacts [9].

The main objective of this study is to examine the influence of the first stage stiffness  $K_1$  and mean torque  $T_m$  on vibration and noise control during idling, as the second stage stiffness and hysteresis values are usually selected to achieve optimum vibration reduction during driving conditions. Figure 5 shows a sample comparison between simulation and experiment [16]. As can be seen from these results the agreement is excellent. A fairly reasonable match between simulation and experiment, was found for a wide range of parameters. Table 2 summarizes the rattle



indices (for both the single-stage and two-stage clutch designs) from both noise control and sound quality perspectives. This table corresponds to a low value of mean torque,  $T_m$ . While  $E_1$  shows a steady increase,  $E_2$  exhibits a decline for intermediate values of  $K_1$ . This effect can be attributed to the stronger interaction between the torque characteristics of the clutch and the hub-spline backlash. Further, the acceleration attenuation is better in this range with the indices being smaller for the dual-staged clutch case. So a dual-staged design with moderate  $K_1$  values would perform well from a noise control perspective. The  $E_4$  values decrease for moderate  $K_1$  values, similar to the reduction in  $E_2$  values. In this range the peaks are smaller and broader. The total energy  $E_3$ , increases steadily with  $K_1$  as does the frequencies associated with impact transients. This implies that the ringing effect of the impacts will become more objectionable.

Next, we consider the effect of increasing the mean/drag torque. Figure 9 shows conceptually the effect of increasing the mean or drag torque. The operating regime of the clutch can cover both stiffness stages instead of usually one stage for the low mean or drag torque case. The transition from the soft stage to the stiffer stage (or vice-versa) can cause significant torsional excitation behind the clutch, leading to an increase in vibration and noise due to vibro-impacts between the clutch hub and the splines on the input shaft. A parametric study indicates that it is beneficial to have a softer first stage stiffness  $K_1$  in order to reduce the overall acceleration levels due to vibro-impact. This design requirement contradicts the requirement of larger  $K_1$  values when the mean torque  $T_m$  is low. Hence, one has to make a judicious compromise in selecting a value for the first stage stiffness  $K_1$  for optimal noise and vibration performance under varying mean torque conditions.

## CONCLUSION

This article has presented a state of the art in the modeling of rattle. Specifically we have developed a step-by-step approach to address the automotive rattle problem. Although the overall problem solving procedure is broken up into three key steps, it is essential to remember that each of these is inter-dependent on the other two steps. This approach contributes towards overcoming any potential numerical difficulties through proper system dimension and algorithm selection. Further, the signal processing procedure, along with the newly developed performance indices  $E_3$  and  $E_4$ , can be used to understand noise control and perception issues related to the study of neutral rattle. Future research will be directed towards drive and coast rattle problems.

## ACKNOWLEDGMENT

We wish to acknowledge the Nissan Technical Center (Japan) and the Center for Automotive Research at the Ohio State University for supporting this research.

## REFERENCES

1. R. Singh, H. Xie and R.J. Comparin 1989 Journal of Sound and Vibration 131, 177-196. Analysis of automotive neutral gear rattle.
2. M. Kataoka, S. Ohno, and T. Sugimoto 1991 JSME International Journal Series III 34, p. 345-354. A two-degree-of-freedom system including a clearance with a two-step hardening spring.
3. R.J. Comparin and R. Singh 1989 Journal of Sound and Vibration 134, 259-290. Frequency response characteristic of an impact pair.
4. C. Padmanabhan and R. Singh 1992 Journal of Sound and Vibration 155, 209-230. Spectral coupling issues in a two-degree-of-freedom system with clearance nonlinearities.
5. C. Padmanabhan and R. Singh 1993 Proceedings of NOISE-CON 93, 607-612. Influence of clutch design on the reduction and perception of automotive transmission rattle noise.
6. T. Abe, I. Fukunaga and T. Kohgo 1992 Transactions of the JSME. Development of evaluation method of transmission gear noise and gear rattle using acoustic simulator (in Japanese).
7. F. Küçükay 1984 Proc. of IMechE Third International Conference on Vibrations in Rotating Machinery, 81-90. Dynamic behavior of high speed gears.
8. F. Pfeiffer 1991 Proc. of JSME International Conference on Motion and Power Transmissions, Japan, 43-48. Modeling problems of rattling in gearboxes.
9. C. Padmanabhan, R.C. Barlow, T.E. Rook and R. Singh 1995 ASME Journal of Mechanical Design. Computational issues associated with gear rattle, vol. 117.
10. A. Rust, F.K. Brandl and G.E. Thien 1990 Proc. of IMechE Conference on Gearbox Noise and Vibration, 113-120. Investigations into gear rattle phenomena- key parameters and their influence on gearbox noise.
11. A. Kahraman and R. Singh 1990 Journal of Sound and Vibration 144, 469-506. Nonlinear dynamics of a geared rotor-bearing system with multiple clearances.
12. C. Padmanabhan and R. Singh 1995 Journal of Sound and Vibration. Analysis of periodically excited non-linear systems by a parametric continuation technique (in press).
13. C. Padmanabhan and R. Singh 1995 Journal of Sound and Vibration. Dynamic analysis of a piecewise non-linear system subject to dual harmonic excitation using parametric continuation (in press).
14. R.C. Aiken (editor) 1985 Stiff Computation. Oxford University Press, Oxford.
15. T.E. Rook and R. Singh 1995 Journal of Sound and Vibration. Dynamic analysis of a reverse-idler gear pair with concurrent clearances (in press).
16. Personal Communications, Nissan Technical Center, Japan.

**Table 1: A summary of solution algorithms for rattle-type non-linear systems.**

Method	Ref.	Key Features
<i>Initial Value Methods</i>		time consuming transients
Numerical Integration	9,11,14	no apriori assumptions
<i>Boundary Value Methods</i>		assumes periodicity
Harmonic Balance		
• Multi-term	12,13,15	more intensive as spectra become broadband
• Describing function	3,4	calculates first harmonic only
Shooting	12, 13	more intensive as DOF increases

**Table 2: Rattle Indices with dry friction damping****A. Single-staged Clutch ( $\theta_c = \infty, C_c=0$ )**

$K_1$ (Nm/rad)	$E_1$	$E_2$	$E_3, \ddot{\theta}_3$	$E_4, \ddot{\theta}_3$	Comments
125	1.25	2.14	1299	141	Two transients per cycle
250	1.25	1.98	1417	128	Lowest impulse energy
500	1.27	1.82	1745	164	Freq. of oscillation rises; Sharper initial pulse
750	1.30	1.88	2198	241	
1000	1.34	1.88	2651	313	-----
1500	1.36	1.68	3034	354	Pulse broadens but peak amplitude rises
2000	1.39	1.63	4079	306	

**B. Two-staged Clutch ( $\theta_c \neq \infty, C_c=0$ ).**

$K_1$ (Nm/rad)	$E_1$	$E_2$	$E_3, \ddot{\theta}_3$	$E_4, \ddot{\theta}_3$	Comments
125	1.22	1.96	1212	136	Two transients per cycle; Lower impulse energy range bigger
250	1.23	1.81	1336	115	
500	1.23	1.42	1567	115	
750	1.27	1.47	2057	214	Freq. of oscillation rises
1000	1.31	1.57	2550	332	Peaks remain sharp $\Rightarrow$ ringing effects
1500	1.34	1.58	3391	362	

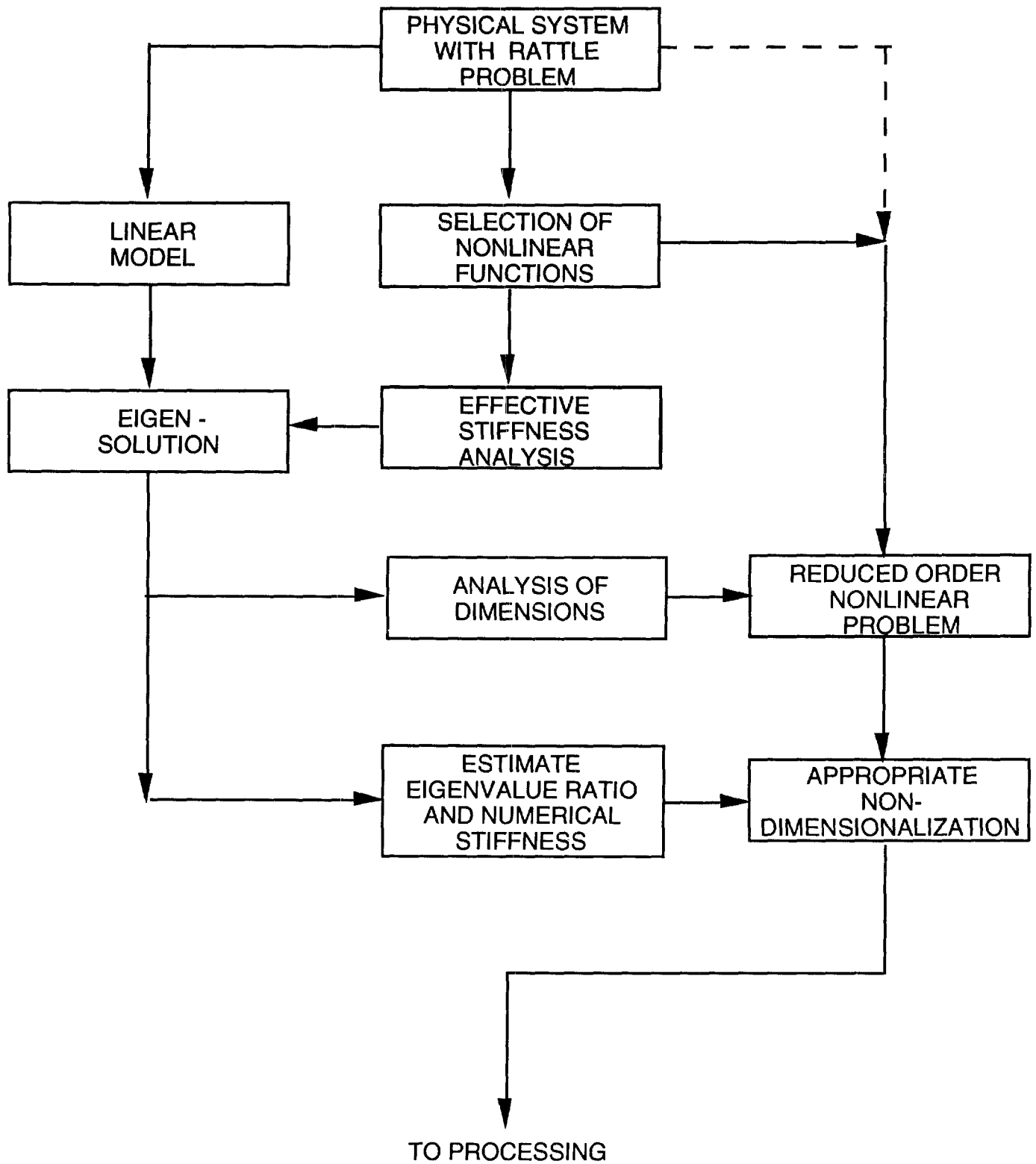
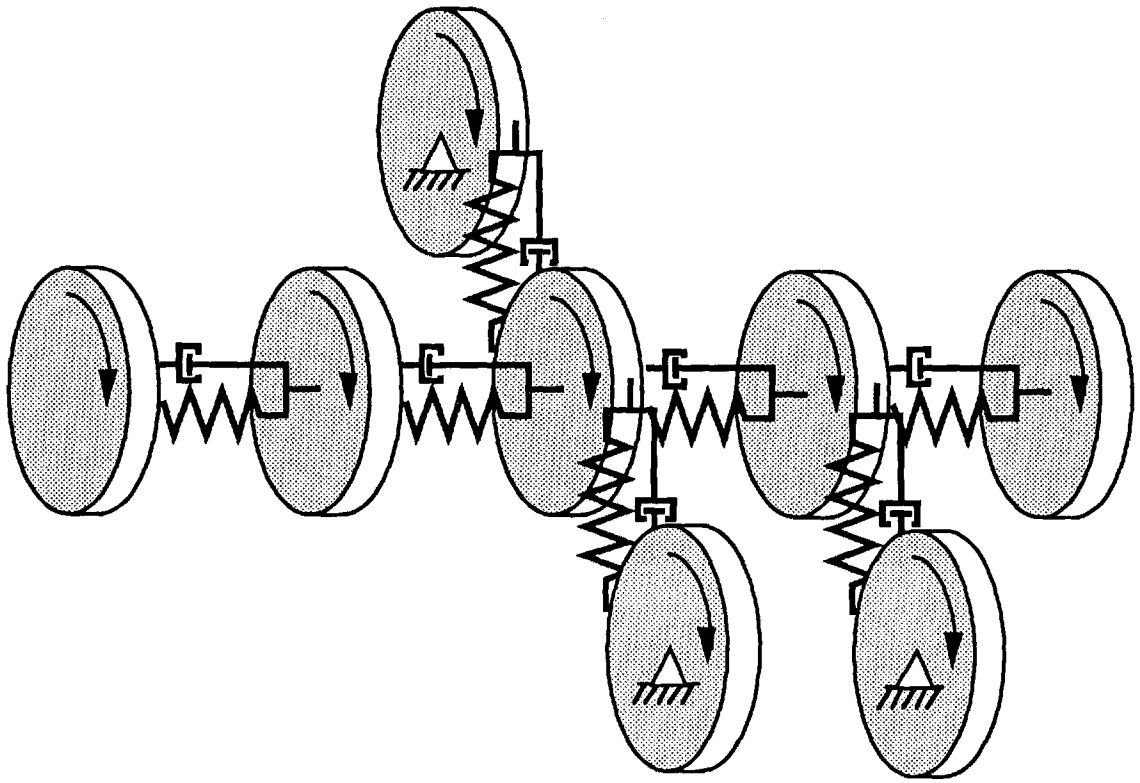


Figure 1: Flowchart of pre-processing stage.

a.



b.

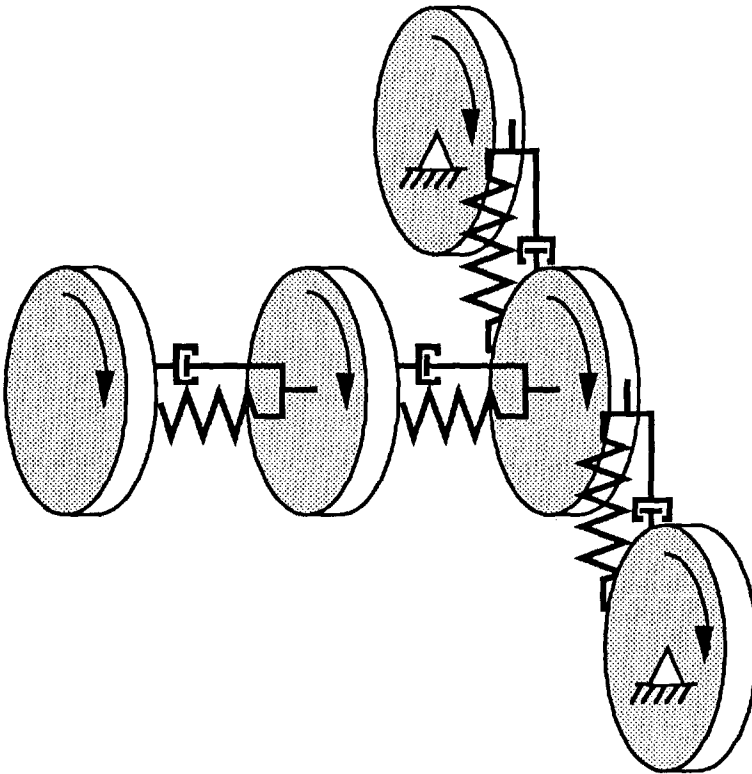
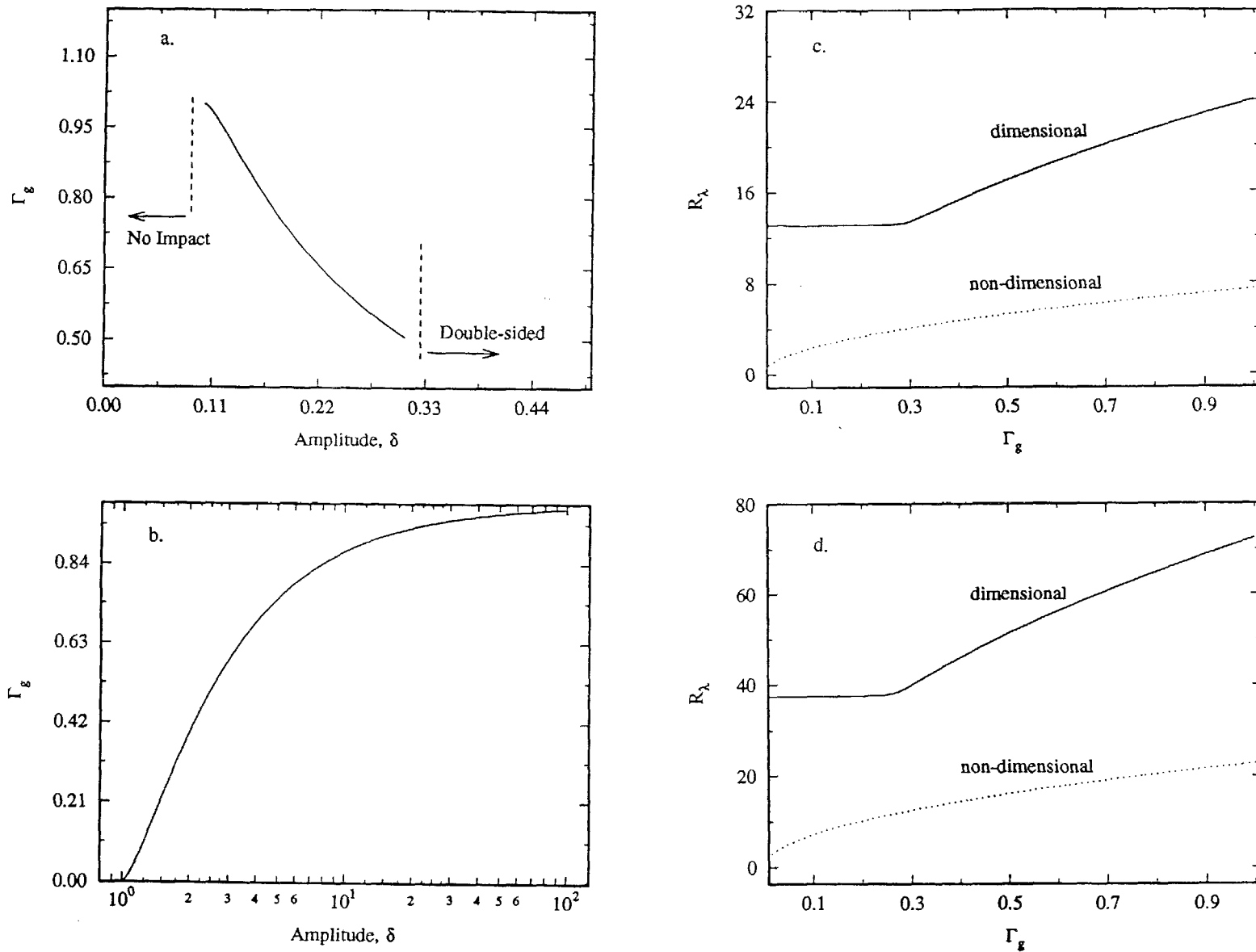


Figure 2: Schematic of geared system. a. original model, b. reduced order model.



**Figure 3: Effective stiffness and "stiffness" ratio analysis [9]. a. variation of effective stiffness with displacement for high mean load. b. variation of effective stiffness with displacement for low mean load. c. "stiffness" ratio for system with low clutch effective stiffness. d. "stiffness" ratio for system with high clutch effective stiffness.**



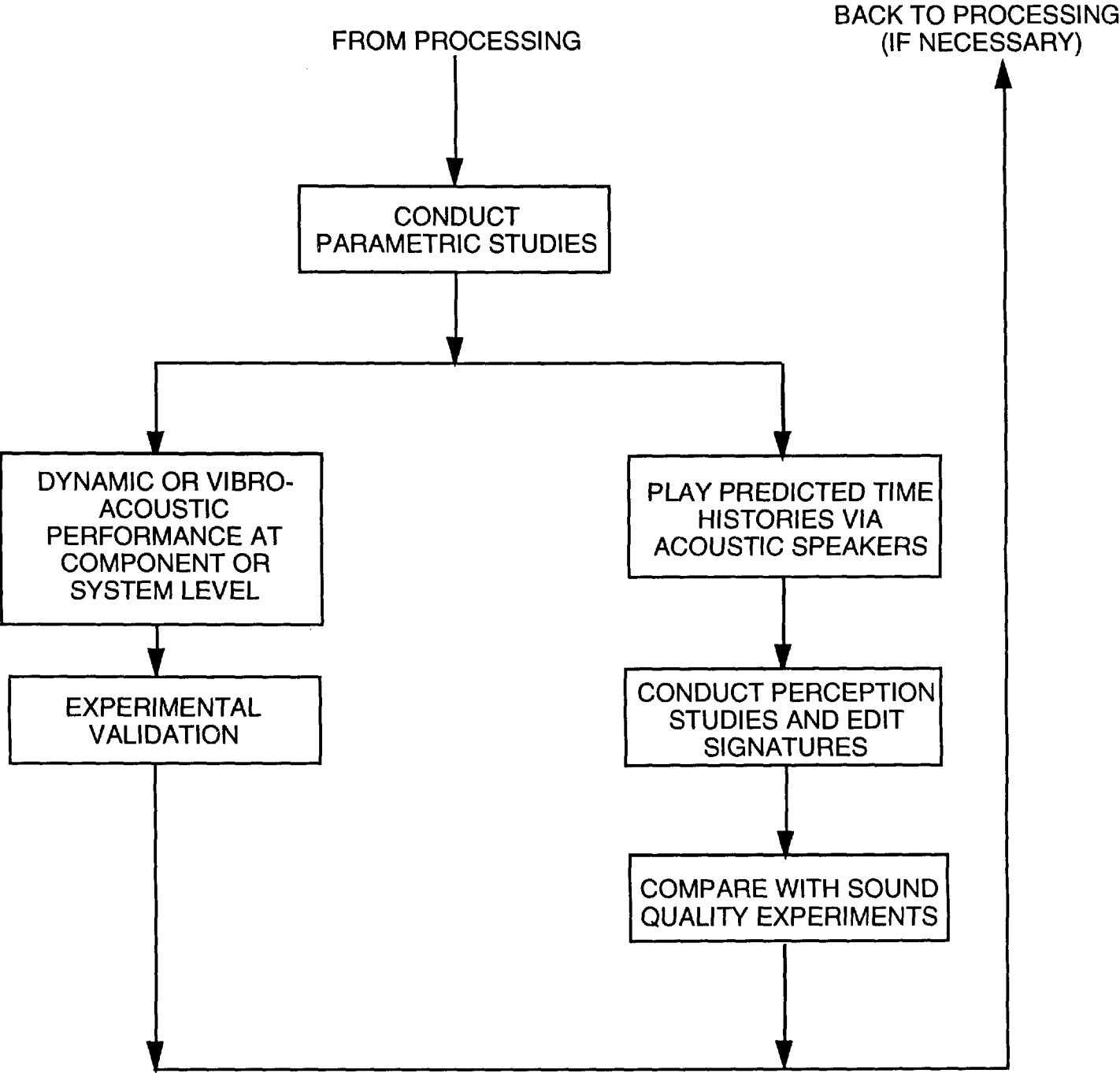


Figure 4: Flowchart of post-processing stage.

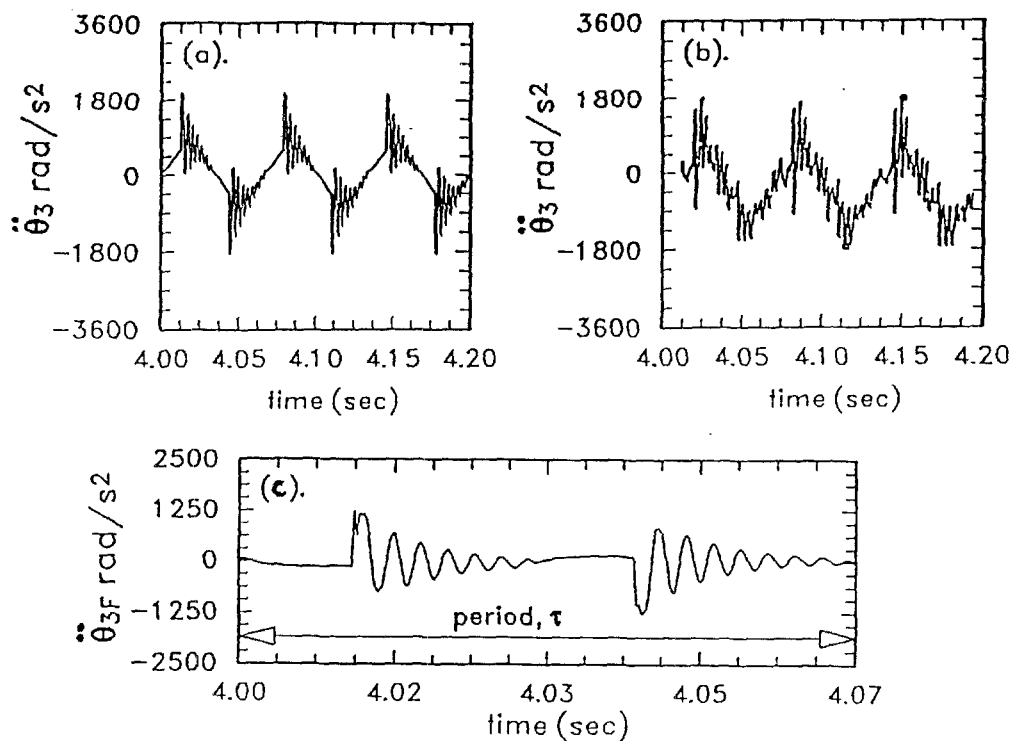


Figure 5: Acceleration time profiles obtained from a. computer simulation, b. experimental measurements and c. filtering signal shown in a.

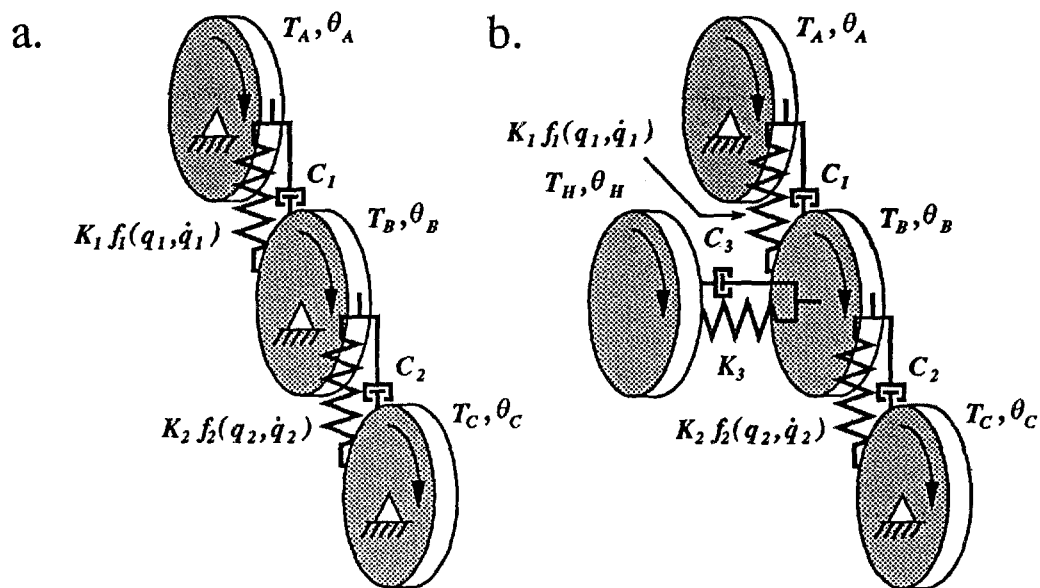
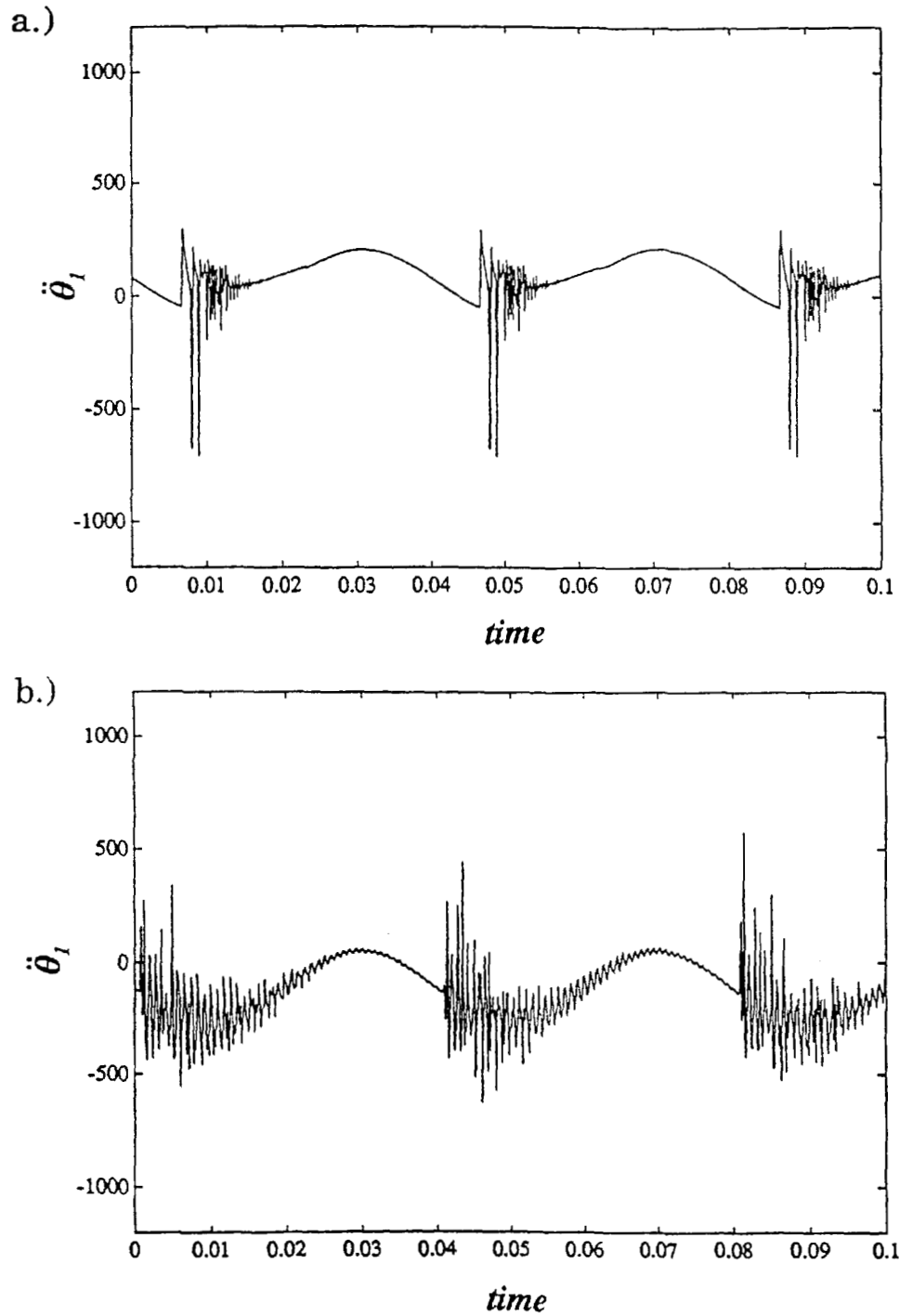


Figure 6: Case History I, the reverse-idler pair in a simplified automotive transmission. a. 3-DOF model, b. 4-DOF model.



**Figure 7: Simulation results for Case History I. a. from 3-DOF model (Fig. 6a). b. from 4-DOF model (Fig. 6b).**

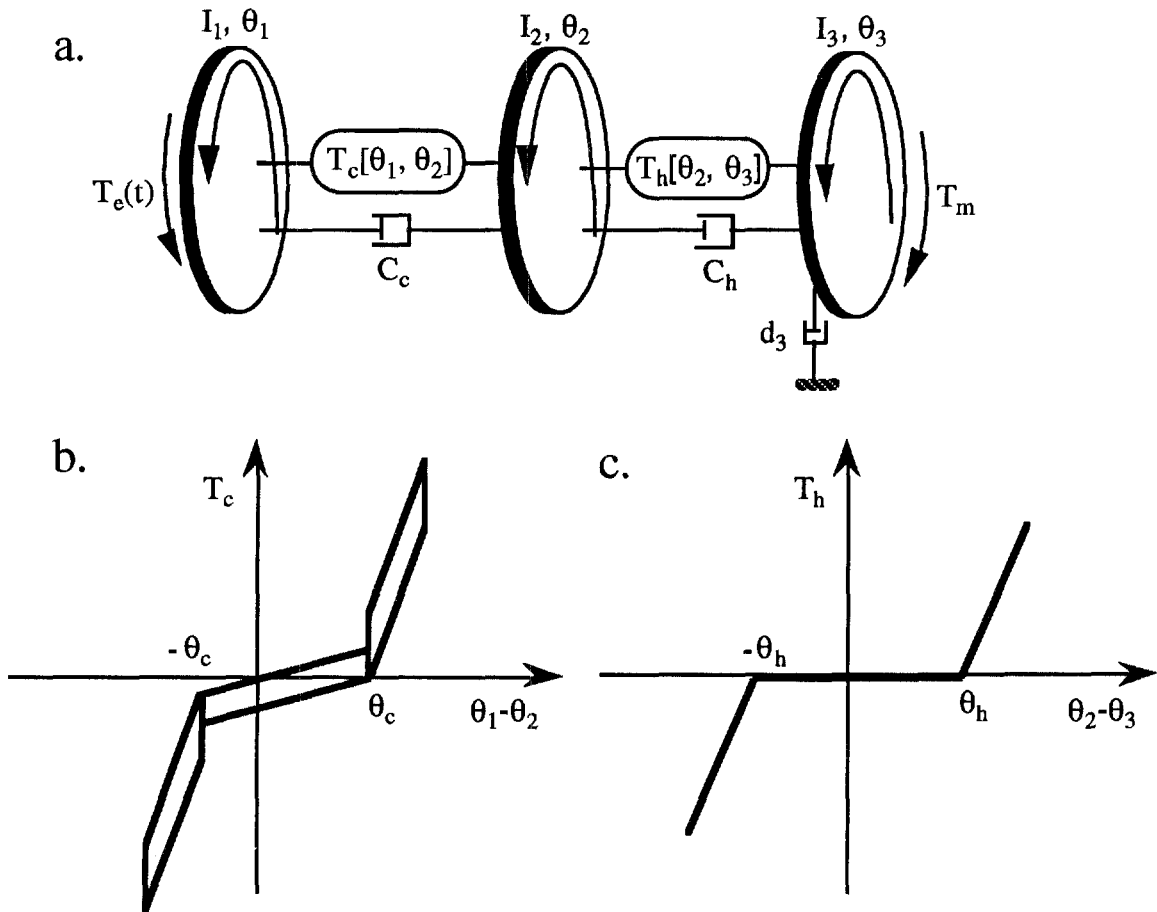


Figure 8: Case History II, a. schematic of clutch-gear system, b. nonlinear torque characteristic of clutch and c. nonlinear force characteristic of hub-spline.

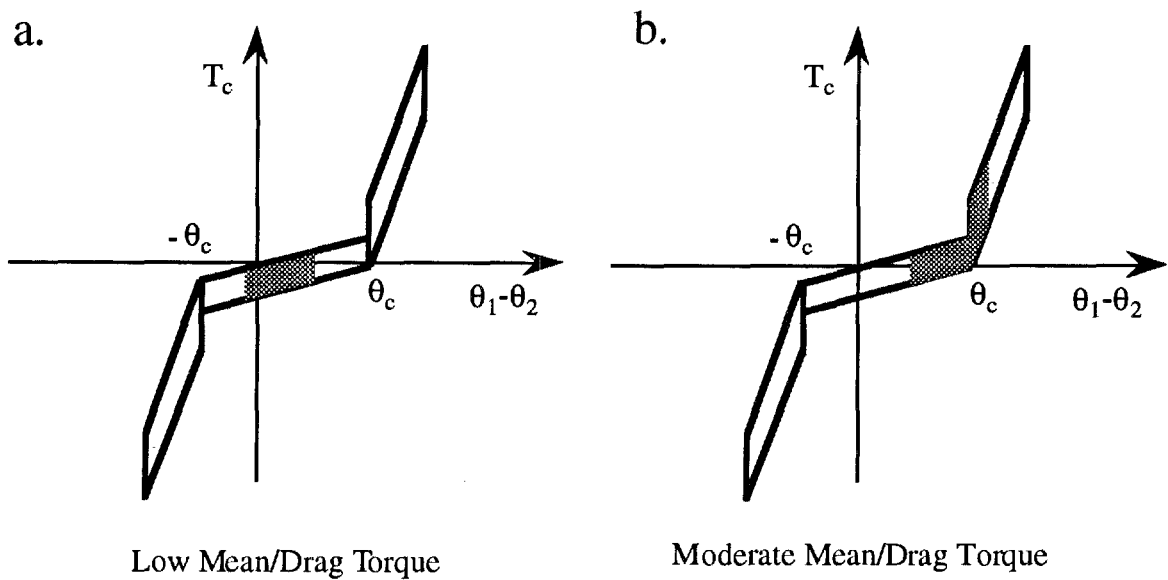


Figure 9: Effect of mean/drag torque on operating point, a. effect of low mean/drag torque, and b. effect of moderate mean/drag torque.