MODELING STRATEGIES FOR MEDIUM AND HIGH FREQUENCY PLATE VIBRATIONS: RAY TRACING VERSUS FINITE ELEMENTS

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INTRODUCTION

Free and forced vibration responses of plate structures at low frequencies are commonly computed by using the Finite Element Method (FEM). However, at higher frequencies, much smaller element size is required in order to extract the required modal information. An eigenvalue problem of a very large dimension must be solved. Numerical errors introduced in this process are significant and only a limited number of modes, say first few, can be obtained accurately. As a consequence, there is much interest in exploring alternate computational methods which may be suitable over medium and high frequency regimes. One such method is the ray tracing technique that has been employed in acoustics, optics and electro-magnetics [1, 2]. But it has not been extended adequately to mechanical vibrations. The only exception seems to be some earlier work by Crandall et al. [3] who calculated regions of enhanced responses on strings and thin plates on a broad frequency band basis when excited by random point loads. We are developing a new ray tracing technique that is suitable to vibration problems since it accurately predicts the narrow band harmonic response of thin rectangular plates [4]. A comparative study of the relative merits (solution accuracy and computational efficiency) of the two modeling strategies discussed above, over a wide range of frequencies is presented in the current paper.

RAY TRACING METHOD

The ray tracing technique is based on a fundamental solution for an infinite plate and analogies from geometric optics. The biharmonic differential equation governs the flexural vibrations of a thin plate and its fundamental solution represents the transverse displacement $u(\mathbf{r}, t)$ at the observation point $\mathbf{r}_o$ due to a concentrated harmonic load of unit amplitude at the excitation point $\mathbf{r}_s$ in an infinite plate. Image sources are placed outside the plate to simulate the waves reflected from the plate edges. The strength of the image source determines the type of boundary condition satisfied on the plate edges. It is illustrated by the example of a semi-infinite plate.

Semi-infinite Plate: Consider Figure 1 where the source strength is $q$ and image strength is $q'$. The deflection, normal slope, shear force and bending moment at the point $P(x, 0)$ on the plate edge have contributions from both the source and the image. If $q' = -q$, the deflection and bending moment along the edge become zero simultaneously. This is the simply supported boundary condition. If $q' = q$, the plate will have the zero normal slope and the zero shear force along the edge. This is called the roller boundary condition. The free-free and clamped boundary conditions along the plate edge cannot be achieved by using point images. However at higher frequencies, the free-free boundary condition may be approximated by the roller boundary condition and similarly the clamped boundary condition may be replaced by simple supports [5].
Rectangular Plate: In Figure 2, the shaded region represents the plate and the lines E1, E2, E3 and E4 denote the plate edges. A '+' symbol indicates a positive image and a 'o' represents a negative image. The strength of each image source is equal to the magnitude of the applied harmonic force. This source-image configuration satisfies the simply supported boundary conditions on all four edges of the plate. For the sake of illustration, consider the edge E2. The row of images R5 is the mirror image of R6. This is analogous to the case of the semi-infinite plate which has equal and opposite forces acting, at the same distance, on either side of the edge to achieve the simply supported boundary condition. Therefore, their contributions to the deflection and bending moment along E2 get cancelled. Same is the case with rows R4 and R7. The only row of images which contributes to the deflection along E2 is R1. As more reflections are taken into account, the row of images contributing to displacement and bending moment moves farther from the plate edge. In the limit, edge E2 is simply supported. Similar arguments hold for edges E2, E3 and E4. By choosing proper amplitude for the image source strengths, different boundary conditions may be obtained along any edge.

Solution Accuracy: Two error norms $\epsilon_1$ and $\epsilon_2$ are defined as follows.

\[
\epsilon_1 = \sqrt{\frac{\sum g (u_i - u_m)(u_i - u_m)}{\sum g(u_m)(u_m)}}
\]
$e_2 = \sqrt{\frac{\sum_{bg}^{} u_i u_i^*}{\sum_{bg}^{} u_{i0} u_{i0}^*}}$ 

where $g$ denotes an interior grid point, $bg$ denotes a grid point on the plate boundary, $u_m$ is the exact solution obtained from analytical modal analysis, $u_i$ is the approximate solution from FEM or from ray tracing after $n$ reflections and $u_{i0}$ is the deflection due to the source alone. Note that $e_1$ measures the deviation of the approximate solution from either ray tracing or FEM from the exact solution in the interior of the plate. Conversely, $e_2$ is an estimate of convergence of the boundary conditions and it applies only to the ray tracing technique, since boundary conditions are explicitly specified in FEM. As $e_1$ and $e_2$ diminish, approximate solution approaches the exact solution.

RESULTS

Example Case: A rectangular steel plate with simple supports on all four edges is considered since the exact modal solution is known for this geometry. The dimensions of this plate are: length $a = 0.5842$m along $x$, width $b = 0.7366$m along $y$ and thickness $h = 0.762$mm. The loss factor for structural damping $\eta$ is taken to be 0.03. The undamped natural frequency distribution for the plate is given in Table 1.

<table>
<thead>
<tr>
<th>Frequency range (Hz)</th>
<th>0-100</th>
<th>100-1000</th>
<th>1000-5000</th>
<th>5000-10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal Count</td>
<td>13</td>
<td>151</td>
<td>744</td>
<td>1595</td>
</tr>
</tbody>
</table>

Low Frequency Regime: The cross point dynamic compliance spectrum $C(\omega)$, given $r_x$ at $(0.65a, 0.51b)$ and $r_y$ at $(0.91a, 0.96b)$ is plotted in Figure 3. The results from exact modal analysis and ray tracing method with ($k = 40$) show excellent agreement with each other. Similar results have been found for narrow plates (beams) and experimental measurements confirm the prediction capability of the proposed ray tracing method. See reference [4] for details.

![Figure 3: Dynamic compliance spectra at low frequencies. Key: ×, Ray tracing technique; ——, Exact solution.](image)
Behavior at Medium Frequencies: Using the ray tracing technique, we have observed that fewer reflections are needed to get accurate solutions as the excitation frequency is increased. In other words, for the same number of reflections, the accuracy improves with frequency. This behavior is not observed in conventional methods such as the FEM whose accuracy deteriorates at higher frequencies.

For the sake of illustration let us again consider the same rectangular plate example with simply supported boundary conditions, under a point harmonic load $F$ at $r_0(0.30a, 0.386)$. The response point is fixed at $r_o(0.74a, 0.666)$. In the ray tracing technique, the deflection is calculated using only $k = 5$ reflections i.e. deflection is obtained by simply adding the the contributions due to $(2 \times 5 + 1)^2 = 121$ image sources. Both predictions are compared with the exact modal analysis solutions over almost 4 decades of frequency from 1–10000 Hz. Two arbitrary frequency ranges, first at the low end from 5 to 2000 Hz and the second at the high end from 9000 to 9100 Hz are selected to compare the results. Given the wide range of frequencies, it is more appropriate to choose the acceleration $A(w) = \frac{-w^2}{k}$ as the sinusoidal transfer function in place of $C(w)$ where $-w^2$ is the transverse acceleration of the plate. Figure 4 shows the typical frequency response curves from 5–2000 Hz where only the ray tracing technique is compared to the exact solution. Both are almost indistinguishable.

![Graph](image)

Figure 4: Accelerance spectra at low and medium frequencies. Key: $\times$, Ray tracing technique $(k = 5)$; ---, Exact solution.

Behavior at High Frequencies: A finite element model is constructed by using 400 elastic shell elements [9]. Figure 5 shows the typical comparison at very high frequencies where 400 spectral points are distributed over 9000 to 9100 Hz. The ray tracing method is again in excellent agreement with the exact modal solution but FEM fails to predict the response. Further, no clear cut resonances are seen. This is because high modal damping values are observed given frequency-invariant loss factor of 0.03. It suggests that the asymptotic modal analysis or SEA type methods [6, 7] might be more suitable to predict this type of response. However, no apriori assumptions are needed in our method to find the narrow band frequency response.
Figure 5: Accelerance spectra at very high frequencies. Key: $\times$, Ray tracing technique ($k = 5$); $-$, Exact solution; $\Delta$, FEM.

To better understand these results, the solution error norm $e_1$ as given by equation (1) is plotted in Figure 6 for both methods with the exact modal solution $u_m$ as the benchmark. Even though either method could be made to yield improved behavior if more reflections are chosen for the ray tracing method or by decreasing the element size in FEM, certain trends are obvious. First, the FEM predictions are based on the eigensolutions which are prone to errors at the higher frequencies. Secondly, the ray tracing method seems to improve its prediction capability as the frequency is increased. It is believed to be related to the image placement distance which depends on dimensions of the plate in relation to the elastic wavelength of interest which becomes much smaller at higher frequencies. Finally, both methods show opposite trends over the spectral scale.

Figure 6: Solution error norm $e_1$ vs. frequency. Key: $-$, Ray tracing method with five reflections; $\ldots\Delta\ldots$, FEM with 400 elements.
CONCLUSION

A new ray tracing technique has been developed for the harmonic analysis of plate vibrations. Unlike the finite element method the solution accuracy of this method increases with an increase in the frequency and damping. Also, one can choose to analyze the response at any desired location unlike FEM where the complete solution has to be simultaneously obtained at all of the nodes. Our procedure also differs from the modal methods where all the modes must be known apriori before the harmonic response can be calculated. The ray tracing method seems very promising as a potential analysis tool for structure-borne noise problems because of its superior computational efficiency and accuracy over medium and high frequency regimes. Currently complicated plate shapes are being examined by using this method[8].

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References


