Vibro-Acoustic Model of a Disk Drive

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A new mathematical model of the vibro-acoustic characteristics of a computer hard-disk drive is presented in this paper. In particular, a mobility transfer function is defined that links sound radiated by a stationary or rotating disk to electromagnetic torque pulsations and structural dynamics. A simplified disk-drive system consisting of a brushless d.c. motor driving a single disk-spindle assembly, which is mounted on a flexible casing, is considered as the example case. Parametric studies illustrate the roles of bearing stiffness and disk geometry on the vibration and radiated sound.

INTRODUCTION

Typical acoustic measurements on certain computer hard-disk (3.5-inches) drives reveal many objectionable pure tones with side bands over the frequency range from 1.3–6.1 kHz. Though several experimental studies [Luttrell and Dunens (1987); Williamson et al. (1987); Woldermar and Kumano (1990)] have been carried out, the source mechanisms of such high frequency pure tones are poorly understood. Also, disk drive acoustic behavior is yet to be characterized analytically. To remedy this situation, a comprehensive theoretical investigation has been undertaken by the authors [Lee and Singh (1992, 1994); Lee et al. (1995)]. Through sound and vibration measurements, it has been determined that the motor torque pulsation of the brushless d.c. motor (BDCM) is the primary source. To facilitate the development of mathematical models, a simplified version of the disk drive is considered. As shown in Fig. 1, the model consists of a single disk driven by a motor (BDCM). Prior articles by the same authors have focused on the electromagnetic noise source [Lee and Singh (1992); Lee et al. (1995)] and sound radiation [Lee and Singh (1994)] characteristics. This paper extends the prior work by completing the missing link on structural path characteristics that couple source and radiator models.

Chief objective of the present article is to develop a new mobility transfer function that relates motor torque pulsations to the sound radiated by an annular disk. The overall mathematical model is then used to conduct several conceptual design studies. For example, the effects of bearing stiffness and disk geometry on mobility transfer functions and modal radiation characteristics are evaluated.

The organization of this article is as follows. Mathematical model of the path including bearings and casing is presented first in some detail. A list of symbols is included at the end to assist the reader even though all symbols are defined in the text or via sketches. Brief summaries of prior source and radiator models are included here for the sake of completeness. Finally, some parametric studies are presented.

MOBILITY TRANSFER FUNCTION

The mobility function by definition is the complex quotient of the structural velocity to excitation force. The authors have formulated the sound radiation of the radiator in terms of the disk natural
and rigid-body modes [Lee and Singh (1994)] and quantified the excitation as the harmonic pulsating torque of the BDCM [Lee and Singh (1992); Lee et al. (1995)]. Therefore, the mobility transfer function as developed in this study is defined as the complex quotient of velocity participation factors of disk vibration modes to a harmonic electromagnetic torque excitation. The mobility transfer functions serve as links between the sound radiation from the disk and the harmonic electromagnetic torque excitations.

**Problem Formulation**

The schematic and coordinate system of the disk-motor assembly is shown in Fig. 1. Several assumptions are made in developing the path model. First, the spindle (to which the disk is fixed on) is assumed to be rigid. The same assumption is made to the stator of the in-hub BDCM. Second, the casing, which is not included in disk sound radiation, is assumed to be flexible in this study and is modeled by two equivalent dynamic stiffness matrices. Third, the bearing dynamics is assumed to be modeled by a stiffness matrix in order to couple the rotor and the stator. Finally, the coupling between the electromagnetic torque and the disk flexural motion is assumed to exist via two paths:

1. Equivalent stiffness matrices of the bearing and the casing and
2. The clearance within the bearing. The first path induces indirectly moment and axial force excitation on the rotor. The inertial forces associated with the rigid translating and rocking modes then excite the disk natural modes. In the second path, the bearing clearance is assumed to cause an additional direct moment excitation on the disk.
Equivalent Stiffness Matrices of the Casing and the Bearing

The equivalent stiffness matrix of dimension 6 for the casing cover, as shown in Fig. 1, is defined by

$$
K_c = \begin{bmatrix}
K_{c,F_x u_x} & K_{c,F_x u_y} & K_{c,F_x \theta_x} & K_{c,F_x \theta_y} & K_{c,F_y \theta_x} & K_{c,F_y \theta_y} \\
K_{c,F_y u_x} & K_{c,F_y u_y} & K_{c,F_y \theta_x} & K_{c,F_y \theta_y} & K_{c,M_x \theta_x} & K_{c,M_x \theta_y} \\
K_{c,M_x u_x} & K_{c,M_x u_y} & K_{c,M_x \theta_x} & K_{c,M_x \theta_y} & K_{c,M_y \theta_x} & K_{c,M_y \theta_y} \\
K_{c,M_y u_x} & K_{c,M_y u_y} & K_{c,M_y \theta_x} & K_{c,M_y \theta_y} & K_{c,M_z \theta_x} & K_{c,M_z \theta_y} \\
K_{c,M_z u_x} & K_{c,M_z u_y} & K_{c,M_z \theta_x} & K_{c,M_z \theta_y} & K_{c,M_z \theta_x} & K_{c,M_z \theta_y}
\end{bmatrix}
$$
(1)

where \((F_x, F_y, F_z, M_x, M_y, M_z)\) and \((u_x, u_y, u_z, \theta_x, \theta_y, \theta_z)\) are excitation and displacement vectors, respectively. Consider a harmonic bending moment excitation of unit amplitude at location \((x_0, y_0)\),

$$
M_x = e^{j\omega t} \delta(x - x_0) \delta(y - y_0).
$$
(2)

The flexural motion of the casing cover under this excitation is then represented in terms of its normal modes as follows [Soedel (1981)]

$$
\begin{align*}
\psi(x, y) &= e^{j\omega t} \sum_k \frac{F_k(x_0 - y_0)}{(\omega_k^2 - \omega^2)} \psi_{c,k}(x, y)
\end{align*}
$$
(3a)

$$
F_k(x_0, y_0) = -\left. \frac{\partial \psi_{c,k}(x, y)}{\partial y} \right|_{x=x_0} \biggr|_{y=y_0}
$$
(3b)

where \(\psi_{c,k}\) is the natural mode of the casing. The dynamic influence coefficient at the driving point

$$
K_{c,M_x u_x} = K_{c,F_x \theta_y} = \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_x^2)} \frac{\partial \psi_{c,k}(x_0, y_0)}{\partial x} \psi_{c,k}(x_0, y_0)\right)}.
$$
(4a)

By using the same procedure, other components of the stiffness matrix are determined as follows:

$$
\begin{align*}
K_{c,M_y u_x} &= K_{c,F_x \theta_y} = \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_y^2)} \frac{\partial \psi_{c,k}(x_0, y_0)}{\partial y} \psi_{c,k}(x_0, y_0)\right)}
\end{align*}
$$
(4b)

$$
\begin{align*}
K_{c,M_z \theta_x} &= K_{c,M_y \theta_y} = \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_z^2)} \frac{\partial \psi_{c,k}(x_0, y_0)}{\partial x} \psi_{c,k}(x_0, y_0)\right)}
\end{align*}
$$
(4c)

$$
\begin{align*}
K_{c,M_z \theta_y} &= \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_z^2)} \left(\frac{\partial \psi_{c,k}(x_0, y_0)}{\partial y}\right)^2\right)}
\end{align*}
$$
(4d)

$$
\begin{align*}
K_{c,M_z \theta_x} &= \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_z^2)} \left(\frac{\partial \psi_{c,k}(x_0, y_0)}{\partial x}\right)^2\right)}
\end{align*}
$$
(4e)

$$
K_{c,M_z \theta_y} = \frac{1}{\sum_k \left(\frac{1}{(\omega_k^2 - \omega_z^2)} \psi_{c,k}(x_0, y_0)^2\right)}.
$$
(4f)

Values of other diagonal components of the stiffness matrix are assumed to be very large, and the off-diagonal components are assumed to be zero. Since the geometry of casing is very complicated,
the numerical values of modal functions $\psi_{c,k}(x_0, y_0)$, $\frac{\partial \psi_{c,k}}{\partial x}(x_0, y_0)$, and $\frac{\partial \psi_{c,k}}{\partial y}(x_0, y_0)$ are obtained from the finite-element analysis [Desalvo and Swanson (1983)].

Assuming $\theta_c$ as the angle between the spindle and the normal vector on the surface of the casing cover, the equivalent stiffness with respective to the spindle coordinate becomes

$$
\mathbf{K}_c = \mathbf{B}^{-1}\mathbf{K}_c
$$

(5a)

$$
\mathbf{B} =
\begin{bmatrix}
1 & 0 & \sin \theta_c & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\sin \theta_c & 0 & \cos \theta_c & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & \sin \theta_c \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \sin \theta_c & 0 & \cos \theta_c
\end{bmatrix}
$$

(5b)

A bearing stiffness matrix [Lim and Singh (1992)] of dimension 6, which has zero values for the 6th row and column, is used in the model to couple the flexural motion with the excitation. The parameters of the ball bearings used in the sample disk drive are listed in Table I, and the corresponding stiffnesses are given in Table II.

**Table I. Bearing parameters for example disk drive.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of balls</td>
<td>7</td>
</tr>
<tr>
<td>Contact angle</td>
<td>8°, 47°, 53°</td>
</tr>
<tr>
<td>Preload (kg)</td>
<td>21</td>
</tr>
<tr>
<td>Ball diameter (m)</td>
<td>2.778E-3</td>
</tr>
<tr>
<td>Path diameter (m)</td>
<td>10.5E-3</td>
</tr>
<tr>
<td>Radius of inner groove (m)</td>
<td>1.51E-3</td>
</tr>
<tr>
<td>Radius of outer groove (m)</td>
<td>1.57E-3</td>
</tr>
<tr>
<td>Radial clearance (m)</td>
<td>0.007E-3</td>
</tr>
<tr>
<td>Load-deflection constant (N/m^3/2)</td>
<td>8.5E7</td>
</tr>
</tbody>
</table>

**Table II. Bearing stiffnesses.**

<table>
<thead>
<tr>
<th>Stiffness term(s)</th>
<th>Values (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{b,P_wu_e}$</td>
<td>0.20197E6</td>
</tr>
<tr>
<td>$K_{b,P_yu_y}$</td>
<td>0.61573E8</td>
</tr>
<tr>
<td>$K_{b,M_xu_e}$</td>
<td>0.84856E3</td>
</tr>
<tr>
<td>$K_{b,M_yu_y}$</td>
<td>-0.5797E4</td>
</tr>
<tr>
<td>$K_{b,M_zu_e}$</td>
<td>0.5797E4</td>
</tr>
<tr>
<td>$K_{b,M_zu_y}$</td>
<td></td>
</tr>
</tbody>
</table>

**Governing Equations**

The Lagrange’s equations are used to derive the governing dynamic equations of the system. The displacement vectors at locations $a$ and $b$, which are defined as the joints between the stator and the casing as shown in Fig. 1, are expressed by

$$
r_{r,a} = (I + A)r_r ; \quad r_{r,b} = (I - A)r_r
$$

(6a,b)

$$
r_{a,a} = (I + A)r_a ; \quad r_{a,b} = (I - A)r_a
$$

(6c,d)
where \( I \) is an identity matrix with the same dimension as \( A \); \( r_r \) and are \( r_s \) displacement vectors at the midpoints of the rotor and the stator and defined by

\[
\begin{align*}
    r_r &= [u_{r,x} \quad u_{r,y} \quad u_{r,z} \quad \theta_{r,x} \quad \theta_{r,y} \quad \theta_{r,z}]^T \\
    r_s &= [u_{s,x} \quad u_{s,y} \quad u_{s,z} \quad \theta_{s,x} \quad \theta_{s,y} \quad \theta_{s,z}]^T.
\end{align*}
\]

The Potential Energy \( (PE) \) of the system consists of strain energy associated with the casing elasticity and the bearing stiffness matrices as well as the potential energy associated with the disk flexure. The Total Potential Energy \( (PE) \) and the Kinetic Energy \( (KE) \) are expressed by

\[
\begin{align*}
    PE &= \frac{1}{2} \{[I + A](r_r - r_s)]^T K_{b,a} ([I + A](r_r - r_s)] + [(I - A)(r_r - r_s)]^T + K_{b,b}([(I - A)(r_r - r_s)] \\
    &\quad \times [(I + A) r_s]^T K_{c,a} [(I + A) r_s] + [(I - A) r_s]^T K_{c,b} [(I - A) r_s] + r_d^T K_d r_d\} \\
    KE &= \frac{1}{2} \{r_r^T M_r r_r + r_s^T M_s r_s + r_d^T M_d r_d + r_r^T M_d r_d + r_s^T M_d r_d + r_r^T M_d r_d\}
\end{align*}
\]

where \( M_s \) and \( M_r \) are mass matrices of stator and rotor, and \( M_d \) is the modal mass matrix of the disk; and \( K_{b,a} \) and \( K_{b,b} \) are equivalent stiffness matrices of bearings. The energy associated with the casing and bearings are expressed in terms of physical coordinates and the potential energy of the disk is represented in the modal domain by using truncated natural modes. It should be noted that the last two terms in Eq. (8) are associated with the coupling between the rigid-body modes and the elastic modes of the disk. Since the rigid rocking motions can exist in both \( x \) and \( y \) directions, both \( c \) and \( s \) modes of disk flexure are included and these are denoted by subscript \( c \) and \( s \), respectively. In an earlier publication [Lee and Singh (1994)], it has been shown that the elastic modes are excited by the inertial forces associated with the rigid-body motion, and thus only \((m,0)\) and \((m,1)\) modes would be excited, where the first modal index \((m)\) is the number of nodal circles of the disk and the second index \((0 \text{ and } 1)\) is the number of nodal diameters. Therefore, only these modes need to be included in the displacement participation vector of disk, so that

\[
r_d = \frac{1}{\omega} [\eta_{00} \quad \eta_{01,c} \quad \eta_{01,s} \quad \eta_{10} \cdots \eta_{m0} \quad \eta_{m1,c} \quad \eta_{m1,s}]^T
\]

where \( \omega \) is the angular frequency of the harmonic excitation and \( \eta_{mn} \) are displacement modal participation factors. The modal stiffness matrix \( K_d \) and the modal mass matrix \( M_d \) of the disk are then expressed by

\[
\begin{align*}
    K_d &= \rho h \text{DIAG} \{\omega_{00}^2 \quad \omega_{01}^2 \quad \omega_{10}^2 \cdots \omega_{m0}^2 \quad \omega_{m1}^2 \}\} \\
    M_d &= \rho h \text{DIAG} \{1 \quad 1 \quad \cdots \quad 1\} \times 3(m+1) \\
\end{align*}
\]

where \( \omega_{mn} \) are natural frequencies of disk flexural modes, \( \rho \) and \( h \) are the density and thickness of the disk, respectively, \( \text{DIAG} \) means diagonal matrix, and subscript \( 3(m+1) \times 3(m+1) \) denotes the dimension of the matrix. The coupling mass matrix \( M_{dr} \) of the size \( 3(m+1) \times 6 \) is associated with the coupling between the rigid-body motion and elastic modes of the disk. By employing normalized
modal functions for rigid-body modes, $M_{dr}$ is expressed by

\[
M_{dr} = \begin{bmatrix}
0 & 0 & \langle \psi_{00}, \psi_{-1,0} \rangle A_P & 0 & 0 & 0 \\
0 & 0 & 0 & \langle \psi_{01,c}, \psi_{-1,1,c} \rangle A_R & 0 & 0 \\
0 & 0 & 0 & 0 & \langle \psi_{01,c}, \psi_{-1,1,c} \rangle A_R & 0 \\
0 & 0 & \langle \psi_{10}, \psi_{-1,0} \rangle A_P & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \langle \psi_{m0}, \psi_{-1,0} \rangle A_P & 0 & 0 & 0 \\
0 & 0 & 0 & \langle \psi_{m1,c}, \psi_{-1,1,c} \rangle A_R & 0 & 0 \\
0 & 0 & 0 & 0 & \langle \psi_{m1,c}, \psi_{-1,1,c} \rangle A_R & 0
\end{bmatrix}
\]

where

\[
A_P = \sqrt{\pi(b^2 - a^2)}, \quad A_R = \sqrt{\frac{\pi}{4}(b^4 - a^4)}
\]

\[
\langle \psi_{m0}, \psi_{-1,0} \rangle = \int_0^{2\pi} \int_a^b \psi_{m0,\psi_{-1,0}} \rho hr dr d\varphi = \frac{2\pi \rho h}{A_P} \sum_{q=0}^N \bar{c}_{m0,q}(b^{q+2} - a^{q+2}) q + 2
\]

\[
\langle \psi_{m1,c}, \psi_{-1,1,c} \rangle = \frac{2\pi \rho h}{A_R} \sum_{q=0}^N \bar{c}_{m1,d}(b^{q+3} - a^{q+3}) q + 3
\]

$a$ and $b$ are the inner and outer radii of disk respectively; $\psi_{-1,0}$ and $\psi_{-1,1}$ are disk rigid translating and rocking modes respectively; and $\psi_{mn}$ are the polynomial approximations of normalized disk natural modes defined by [Lee and Singh (1994)]

\[
\psi_{mn,c} = \sum_{q=0}^N \bar{c}_{mn,q} r^q \sin(n\theta), \quad \psi_{mn,c} = \sum_{q=0}^N \bar{c}_{mn,q} r^q \cos(n\theta).
\]

Applying the Lagrange's equations of motion, governing equations are derived as

\[
M_s \ddot{r}_s + [(K_{b,a} + K_{b,b}) + 2A^T(K_{b,a} - K_{b,b}) + A^T(K_{b,a} + K_{b,b})]r_s = -F
\]

\[
- \left( [K_{c,a} + K_{c,b}] 2A^T + A^T(K_{c,a} - K_{c,b}) \right) r_s = 0
\]

\[
M_r \ddot{r}_r + M_{dr} \ddot{r}_d + [(K_{b,a} + K_{b,b}) + 2A^T(K_{b,a} - K_{b,b}) + A^T(K_{b,a} + K_{b,b})]r_r = -F
\]

\[
- \left( [K_{b,a} + K_{b,b}] + A^T(K_{b,a} - K_{b,b}) + (K_{b,a} - K_{b,b})A + A^T(K_{b,a} + K_{b,b}) \right) r_s = F
\]

\[
M_d \ddot{r}_d + M_{dr} \ddot{r}_r + K_d r_d = 0.
\]

If that the equivalent casing stiffness matrices at the top and the bottom are identical and that both bearings are the same, one can simplify Eqs. (13) and (14) as

\[
M_s \ddot{r}_s + 2[K_{b} + K_{c} + A^T(K_{b} + K_{c})]r_s - 2[K_{b} + A^T K_{b}]r_r = -F
\]

\[
M_r \ddot{r}_r + M_{dr} \ddot{r}_d + 2[K_{b} + 2A^T K_{b}]r_r - 2[K_{b} + 2A^T K_{b}]r_s = F.
\]
If the effect of the bearing clearance is not included, the generalized force vector is simply a torsional excitation expressed by

$$F = [0 \ 0 \ 0 \ 0 \ 1]^T e^{j\omega t}.$$

If the bearing clearance causes misalignment between the rotor and stator, it should induce an additional direct-moment excitation on the disk. Then $F$ is modified as

$$F = [0 \ 0 \ 0 \ \theta_b\cos\varphi_r \ 0 \ \theta_b\sin\varphi_r \ 1]^T e^{j\omega t},$$

where $\theta_b$ is the angle between the normal vector of the disk and the stator, $\varphi_r$ is the angle between the projection of the normal vector on the $X-Y$ plane and $X$ axis. The displacement of the disk under a harmonic excitation is thus obtained by solving Eqs. (15)-(19). Consequently the participation factors of the disk rigid body ($\eta_{-1,0}$ and $\eta_{-1,1}$) and elastic modes and mobility functions ($M_{-1,0}$, $M_{-1,1}$, and $M$) are obtained by

$$\eta_{-1,0} = M_{-1,0} = \omega A_P u_{r,z}; \quad \eta_{-1,1c} = M_{-1,1c} = \omega A_R \theta_{r,z}; \quad \eta_{-1,1s} = M_{-1,1s} = \omega A_R \theta_{r,y}$$

$$(20a-c)$$

$$\eta = M = \omega r_d.$$  

$$(20d)$$

For the sample case, assume that only $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ modes are excited within the frequency range of interest (6 kHz). Since the casing is very stiff, the resulting rigid body motion is very small. Mobility transfer functions for a pure torsional excitation as given by Eq. (18) are shown in Figs. 2 and 3. It is seen that the amplitudes of mobility functions of elastic modes are very small compared to those of the rigid-body modes, and that the frequencies of the peaks in these mobility spectra correspond to the natural frequencies of the casing and the disk. Since the casing geometry is complicated, there are a lot of modes within the frequency range of interest. Consequently, high modal density is observed in mobility transfer-function spectra. If Eq. (19) is employed as the excitation, the induced direct-moment excitation dominates over excitations via other paths. While the mobility functions of the axisymmetric modes are almost the same as the previous case, the resonant frequencies of mobility functions of the rigid rocking modes and the non-axisymmetric elastic modes are primary corresponding to the disk elastic modes as shown in Figs. 4 and 5.

**ELECTROMAGNETIC NOISE SOURCE MODEL**

**Problem Formulation**

This section describes the modeling issues associated with the electro-magnetic sources of noise radiated. Main focus is on the prediction of motor torque pulsations primarily in the frequency domain over the applicable range; other noise source mechanisms are beyond the scope of this study.

Based on the experimental investigation and initial calculations, the following assumptions are made to simplify the analytical formulation of the idling disk:

1. Input d.c. voltage source is ideal and ripple free.
2. Current inverters are ideal switches.
3. Motor phase variables are balanced, i.e. each is offset by an electrical angle equal to $2\pi/3$.
4. Rotor magneto motive force (m.m.f.) wave is trapezoidal.
5. Torsional dynamics of the rotor can be given in terms of moment of inertia and damping coefficient. A typical three-phase $(a,b,c$ with $Y$ connection) BDCM includes six pairs of current inverters as shown in Fig. 6. Each phase circuit consists of a winding resistance $R$, inductance $L$, and back e.m.f. $E$. The input d.c. voltage is converted to nonsinusoidal a.c. voltage via an inverter switching logic and its amplitude is controlled by using the pulse-width-modulation algorithm.
FIG. 2. Mobility transfer function of axisymmetric modes (without the effect of bearing clearance): (a) rigid-translating mode, (b) \((0, 0)\) mode, and (c) \((1, 0)\) mode.
FIG. 3. Mobility transfer functions of asymmetric modes (without the effect of bearing clearance): (a) rigid-rocking mode, (b) (0, 1) mode, and (c) (1, 1) mode.
FIG. 4. Mobility transfer functions of axisymmetric modes (with the effect of bearing clearance): (a) rigid-translating mode, (b) (0, 0) mode, and (c) (1, 0) mode.
FIG. 5. Mobility transfer functions of asymmetric modes (with the effect of bearing clearance): (a) rigid-rocking mode, (b) (0, 1) mode, and (c) (1, 1) mode.
Analytical Model

Two different mathematical models, the state space-model and the Fourier-series representation, are developed for BDCM. The state-space model retains the governing equations in the state-space form. Results of the state-space model are obtained by using the numerical integration method followed by an FFT scheme. In the Fourier-series representation, an analytical series expansion is employed to derive the Fourier coefficients of torque pulsations. The state-space model provides more accurate predictions, however considerable computation time is needed for numerical solutions. The Fourier series is more efficient since it calculated the output torque directly in frequency domain. Even though a number of simplifications have been made in our noise-source model, predicted frequency contents of torque spectra associated with inverter switching logic, eccentricity and magnetic saturation match well with the sound measurements on two sample disk drives as shown in Fig. 7. Most of the pure tones are predicted consistently by either of the proposed models. They belong to one of the following three harmonic groups:

1. Dynamic interactions between back e.m.f. (E) and phase current (I).
2. Open stator slots.
3. Inductance variation. Table III summarizes these harmonic groups, where $P$ is the number of poles, $n$ is the number of stator slots, $\Omega_r$ is the mean rotational speed and $\omega_c$ is the switching frequency of Pulse Width Modulation (PWM) control scheme.
FIG. 7. Comparison of measured and predicted pure tones; \( P \) is the number of poles and \( n \) is the number of stator slots.

Table III. Harmonic groups.

<table>
<thead>
<tr>
<th>Harmonic group</th>
<th>Frequency contents</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Pulse-Width Modulation (PWM)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>harmonics of ( 3P\Omega_r )</td>
<td>dynamic interactions between ( E ) and ( I )</td>
</tr>
<tr>
<td>(ii)</td>
<td>odd harmonics of ( 0.5nP\Omega_r )</td>
<td>open stator slot</td>
</tr>
<tr>
<td>(iii)</td>
<td>( sP\Omega_r \pm m\Omega_c ) ( s ) is an integer ( m ) ( q,2q ), ( P,q,q = 1,2,3,\ldots )</td>
<td>inductance variation</td>
</tr>
<tr>
<td>Width Pulse-Width Modulation (PWM)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>( 3qP\Omega_r \pm \omega_c ) ( q = 1,2,3,\ldots ) ( \alpha = 0,1,2,3,\ldots )</td>
<td>dynamic interactions between ( E ) and ( I ) with side bands generated by PWM</td>
</tr>
<tr>
<td>(ii)</td>
<td>odd harmonics of ( 0.5nP\Omega_r )</td>
<td>open stator slot</td>
</tr>
<tr>
<td>(iii)</td>
<td>( sP\Omega_r \pm m\Omega_c \pm \omega_c ) ( s ) is an integer ( a = 0,1,2,3,\ldots ) ( m = q,2q ), ( P,q,q = 1,2,3,\ldots )</td>
<td>inductance variation with side bands generated by PWM</td>
</tr>
<tr>
<td>(vi)</td>
<td>harmonics of ( \omega_c )</td>
<td>carrier frequency ( \omega_c )</td>
</tr>
</tbody>
</table>

Seminumerical Model Using the Galerkin’s Method

The Galerkin’s method, essentially a quasi-analytical multi-term harmonic balanced method, is further employed to the complex nonlinear, time-varying BDCM model by converting the dual domain problem (time and spatial domains) into a single-spatial-domain formulation. The mechanical system dynamics is also incorporated into the computational scheme by expressing the unknown angular
velocity in terms of a Fourier series. The outlines of the methodology for the computation of the coefficients of the Fourier series are as follows:

1. First, represent the state-space variables as well as the various parameter effects in terms of truncated trigonometric series expansions of the rotor angular position. The coefficients of the trigonometric series are implicit functions of the unknown coefficients of state-space variables.

2. Substitute these expressions into the governing equations leading to nonlinear algebraic equations in terms of the unknown trigonometric coefficients.

3. These equations are assumed to hold only at selected discrete rotor angular positions (collocation). This leads the formation of the trigonometric collocation or Discrete Fourier Transform (DFT) matrices.

4. The numerical computation is started by assuming values for the state-space variables’ coefficients. The parameter coefficients are obtained from these by means of an IDF (Inverse Discrete Fourier Transform) followed by a DFT. These are then used in the evaluation of the nonlinear equations.

5. Finally, the corrections for the unknown coefficients’ values are obtained from a Newton-Raphson iterative scheme, which requires the construction and evaluation of a Jacobian matrix. The entire...
procedures outlined in steps 3 and 4 are repeated until the coefficient values converge and the truncation error involved is sufficiently small.

Figures 8 and 9 show comparison between the results from the numerical integration and the Galerkin's method.

DISK SOUND RADIATOR MODEL

Problem Formulation

This section presents a brief summary of modal base disk acoustic model. A single disk is assumed to be mounted flush with an infinite rigid baffle, and sound is radiated from only one side into free field as shown in Fig. 10.

![Diagram of a computer hard disk](image)

FIG. 10. Schematic of a computer hard disk.

Structure Modal Characteristics

For a stationary disk, the compression exerted by the collars simulates the clamped boundary condition at the inner edge; the outer edge is essentially free. A polynomial function is employed to approximate the shape function along the radial direction of disk natural mode. The Young's modulus is estimated by comparing the predicted first natural frequency of a free-free disk with the experiment result. Predicted natural frequencies match very well with the results from impact modal testing on a 3.5-inches computer disk as shown in Table IV.
Table IV. Selected natural frequencies of the sample disk.

<table>
<thead>
<tr>
<th>Case</th>
<th>(m,n)</th>
<th>(\omega_{mn}/2\pi) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.2)</td>
</tr>
<tr>
<td>Free-free</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prediction</td>
</tr>
<tr>
<td>Clamped-free</td>
<td>(m,n)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Experiment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prediction</td>
</tr>
</tbody>
</table>

Modal Radiation Efficiency

Modal sound power, which is defined as radiated sound power when the disk is vibrating at its elastic mode, is calculated using the far-field and the radiation impedance approaches. The far-field approach employed the Green's function to derive the expression of the sound pressure in the far-field. The sound power is then calculated by integrating the far-field sound intensity over a semi-spherical surface which encloses the disk. For the radiation impedance approach, the disk is discretized into several concentric annuli. The self-radiation impedance of each annulus and mutual radiation impedance between any two annuli are derived to calculate the sound power radiated from each annulus. The total sound power is then obtained by summing the radiated sound associated with each annulus. Both approaches yield analytical formulations of radiated sound power in terms of convergent power series of the acoustic wave number, which are computationally efficient. The radiation efficiencies of selected vibration modes as predicted by both approaches are compared with numerical results yielded by a boundary element program (BEMAP) in Table IV. Predictions show that axisymmetric vibration modes are more efficient radiators compared to those asymmetric disk modes which have the same number of nodal circles.

Table V. Predicted modal radiation efficiencies of a stationary disk.

<table>
<thead>
<tr>
<th>Modal index (m,n)</th>
<th>Natural frequency (Hz)</th>
<th>Modal radiation efficiency (\sigma_{mn})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>604</td>
<td>0.842E-1</td>
</tr>
<tr>
<td>(0,1)</td>
<td>602</td>
<td>0.272E-2</td>
</tr>
<tr>
<td>(0,2)</td>
<td>713</td>
<td>0.749E-4</td>
</tr>
<tr>
<td>(1,0)</td>
<td>3890</td>
<td>0.330</td>
</tr>
<tr>
<td>(1,1)</td>
<td>4050</td>
<td>0.322</td>
</tr>
<tr>
<td>(1,2)</td>
<td>4640</td>
<td>0.111</td>
</tr>
</tbody>
</table>

Modal Coupling Effects on Radiated Sound

Formulations are also extended to include the modal coupling effect associated with the elastic and rigid-body modes. This leads to the prediction of sound radiation given an arbitrary velocity distribution. The effect of coupling between disk vibration modes on the radiated sound is found to be significant if multi-modes are excited. Figures 11 and 12 show selected self (single mode) and coupled (multiple mode) terms of the radiated sound power of the sample disk for sinusoidal excitation. From these results, it is observed that self-radiated sound power spectra associated with elastic modes which have the same number of nodal circles (index \(m\)) essential have the similar patterns, except that the humps shift to higher frequencies as the number of nodal diameters (index \(n\)) is increased. For the sound power associated with the modal coupling effect, similar phenomena is observed for selected natural modes, and all of them have positive contributions to the overall
FIG. 11. Selected self-radiation terms of sound power for multimodal excitation.

FIG. 12. Selected mutual radiation terms of sound power between elastic modes.

FIG. 13. Radiation efficiency of an annular disk under a harmonic moment and axial force excitation.
radiated sound power. However, if a higher number of \( n \) is included, the destructive interference, i.e. of sound cancellation in the modal domain, may also exist. Predicted radiation efficiency spectra of concentrated harmonic force excitation is presented to demonstrate multi-modal excitation cases as shown in Fig. 13.

**Disk Rotation Effects on Modal Radiation Efficiency**

Additionally, the effect of disk rotation is investigated by employing the Green’s function of a moving point-sound source. Figure 14 shows normalized modal radiation efficiency of a rotating disk. It is observed from the predictions that disk rotation has more significant effect on the modal radiation efficiency than disk elastic modes. However, based on the predictions on the sample disk, the increase in modal radiation efficiency can be taken into account simply by considering an increase in natural frequency and the change of mode shape without losing much accuracy. Accordingly, a simple empirical equation has been developed to predict the modal sound radiation efficiency of a rotating disk for selected vibration modes.

\[
\frac{\sigma(\omega)}{\sigma(0)} = \left[ \frac{\Omega}{\omega(\omega=0)} \right]^{**2}
\]

**FIG. 14.** Normalized modal radiation efficiency versus normalized rotational speed.

**OVERALL VIBRO-ACOUSTIC MODEL**

Now the overall vibro-acoustic model can be constructed by combining the source, radiator, and mobility function models. From the radiator model, the sound power is expressed by

\[
W_{\text{rad}}(\omega) = \omega^2 \eta^T(\omega) \Pi(\omega) \eta(\omega)
\]  
\[(21a)\]

where

\[
\eta(\omega) = [\eta_{-1,0} \eta_{-1,1} \eta_{-1,2} \eta_{00} \eta_{01} \eta_{m1} \cdots]^T
\]  
\[(21b)\]

\[
\Pi(\omega) = \begin{pmatrix}
\Pi^{-1,-1} & \Pi^{-1,0} & \cdots & \Pi^{-1,m} \\
\Pi^{0,-1} & \Pi^{0,0} & \cdots & \Pi^{0,m} \\
\vdots & \vdots & \ddots & \vdots \\
\Pi^{m,-1} & \Pi^{m,0} & \cdots & \Pi^{m,m}
\end{pmatrix}
\]  
\[(21c)\]

\[
\Pi^u = \begin{pmatrix}
\Pi_{00}^u & 0 & 0 \\
0 & \Pi_{11}^u & 0 \\
0 & 0 & \Pi_{11}^u
\end{pmatrix}
\]  
\[(21d)\]
where the diagonal submatrices of $\Pi$ are named as self-modal sound power matrices and off-diagonal submatrices are coupled-modal sound power matrices. The self-modal sound power is radiated sound associated with individual disk mode and coupled sound power is associated with interaction between two disk modes [Lim and Singh (1990)]. Incorporating the mobility function spectrum developed in the previous section, one can express participation factor vector in terms of the torque excitation $T_e$ as

$$\eta(\omega) = T_e(\omega)M(\omega)$$

$$(22a)$$

$$M(\omega) = [M_{-1,0} \ M_{-1,1,c} \ M_{-1,1,s} \ M_{0,0} \ M_{0,1,c} \ M_{0,1,s} \ \cdots \ M_{m_1,0}]^T.$$  

$$(22b)$$

Consequently, the sound power spectrum is represented by

$$W_{rad}(\omega) = T_e^2(\omega)[M(\omega)^T\Pi(\omega)M(\omega)].$$

$$(23)$$

PARAMETRIC STUDIES

The sound radiated from a disk drive is affected by various parameters of the overall vibro-acoustic model. For the sake of illustration, a few parametric studies are presented on the effect of disk geometry such as thickness $h$ and outer radius $b$. In addition to sound radiation characteristics of the disk, the mobility functions are also affected by the disk dimensions. Limited parametric studies are conducted by varying the thickness from $0.2h^*$ to $2h^*$ and the outer radius from $0.75b^*$ to $1.5b^*$, where $h^*$ and $b^*$ are the current values of disk thickness and outer radius respectively.

The modal radiation efficiencies of $(0, 0)$ and $(0, 1)$ modes, as shown in Fig. 15(a), are found to increase monotonically as $h$ increases. On the other hand, local maxima are seen in Fig. 15(b) for $(1, 0)$ and $(1, 1)$ modes over the selected range of the disk thickness. It is also observed that the modal radiation efficiency of $(0, 1)$ mode is most sensitive to $h$ since $\sigma_{01}$ increases about 13 times when $h^*$ is doubled. Similar phenomena are found for $\sigma_{mn}$ as a function of $b$, as shown in Fig. 16. Modal sound power of two rigid-body modes and four elastic modes for selected values of outer radius ($0.75b^*, 1.125b^*$, and $1.5b^*$) are shown in Figs. 17–20. Inspection of these curves shows that humps shift to lower frequencies as $b$ increases. It is also seen that the magnitudes of these humps increase as $b$ increases, except for the second humps associated with the rigid-body modes. Mobility transfer functions for selected values of $h$ and $b$ are shown in Figs. 21 and 22, note that the effect of

![Graph](image)
FIG. 15. (Continued).

FIG. 16 Effect of disk radius on normalized modal radiation efficiencies.
FIG. 17. Effect of disk radius on self-modal sound power for asymmetric modes: (a) rigid-translating mode, (b) (0, 0) mode, and (c) (1, 0) mode.
FIG. 18. Effect of disk radius on self-modal sound power for asymmetric modes: (a) rigid-rocking mode, (b) (0, 1) mode, and (c) (1, 1) mode.
FIG. 19. Effect of disk radius on coupled-modal sound power for axisymmetric modes (a) \((-1, 0)\) and \((0, 0)\) modes, (b) \((-1, 0)\) and \((1, 0)\) modes, and (c) \((0, 0)\) and \((1, 0)\) modes.
FIG. 20. Effect of disk radius on coupled-modal sound power for asymmetric modes: (a) (1, 1) and (0, 1) modes, (b) (−1, 1) and (1, 1) modes, and (c) (0, 1) and (1, 1) modes.
FIG. 21. Effect of disk thickness on mobility transfer functions: (a) rigid-rocking mode, (b) (0, 1) mode, and (c) (1, 1) mode.
FIG. 22. Effect of disk radius on mobility transfer functions; (a) rigid-rocking mode, (b) (0, 1) mode, and (c) (1, 1) mode.
FIG. 23. Effect of bearing stiffness on mobility transfer function of rigid- translating mode: (a) 0.1 $K_b$, (b) $K_b$, and (c) 10 $K_b$. 
FIG. 24. Effect of bearing stiffness on mobility functions (asymmetrical modes): (a) rigid-rocking mode, (b) $(0, 1)$ mode, and (c) $(1, 1)$ mode.
$r_d$ displacement participation vector of disk (m)
$r_{r, a}, r_{r, b}$ displacement vectors of rotor (m)
$r_{s, a}, r_{s, b}$ displacement vectors of stator (mc)
$T_c$ oscillating component of motor torque (Nm)
$\omega(x, y)$ flexural motion of casing cover (m)
$W_{rad}$ radiated sound power (w)
$\delta(x), \delta(y)$ delta functions
$\eta_{mn}$ velocity participation factor of disk elastic mode (m/s)
$\psi_{c, k}$ natural mode of casing
$\psi_{mn}$ natural mode of disk
$\omega$ angular frequency of harmonic excitation (rad/s)
$\omega_k$ angular natural frequency of casing elastic mode (rad/s)
$\omega_{mn}$ angular natural frequency of disk elastic mode (rad/s)
$\Pi_{i_1, n_2}$ coupled modal sound power associated with $(i, n_1)$ and $(j, n_2)$ modes of disk (w)
$\Pi$ modal sound power matrix (w)
$\Pi_{ij}$ modal sound power matrix for disk modes with $i$ and $j$ nodal circles (w)
$\theta_b$ angle between disk normal and stator (rad)
$\varphi_r$ angle between disk normal projection on the $X$-$Y$ plane and $X$ axis (rad)

REFERENCES