ON THE DIGITAL GENERATION OF AN ACOUSTIC EXCITATION IMPULSE

1. INTRODUCTION

Recently [1], we described an experimental technique for evaluation of mufflers in the presence of mean flow, wherein a repeated digitally synthesized acoustic impulse was used for excitation; and digital time domain averaging of the pressure signals was used to extract the resulting incident, reflected, and transmitted waves (deterministic) from the flow noise (non-deterministic). The success of this technique requires a high degree of repeatability for successive impulses and precise synchronization of data acquisition with system excitation; otherwise, the needed signal-to-noise ratio enhancement will not be obtained.

Ideally, a unit impulse would be used; but this, of course, is not physically realizable. When the problem of generating a practical acoustic impulse with the required repeatability and synchronization is faced, several conflicting requirements, as stated below, become apparent. (1) Maximization of energy density implies high amplitude and long duration; however, maintaining linearity severely limits amplitude, and relatively short duration is required to avoid overlap of the incident, transmitted, and reflected waves. (2) The spectral content of the acoustic impulse must be reasonably smooth over the frequency range of interest. (3) The physical limitations of the electro-acoustic transducer must not be exceeded. (4) Finally, the synthesized electrical impulse must take into account the non-ideal transfer function of the transducer. Space limitations prevented discussion of these problems in the original publication [1]; it is the intent of the present communication to provide some of this information.

2. DIGITAL IMPULSE GENERATION

A minicomputer based digital system with Fast Fourier Transform capability is used for providing excitation along with data acquisition and processing. Here, the excitation profile is generated mathematically, and a digital to analog (D/A) converter is used to change this to a voltage suitable for driving the power amplifier and hence the acoustic horn driver. The D/A is coupled with the analog to digital converter (A/D) through a timing generator. The A/D is used to acquire acoustic pressure signals.

The output of the driver (i.e., the acoustic pressure signal \( p(t) \)) should resemble the impulse function. It is related to the input mathematical function \( x(t) \) in the band limited frequency domain, denoted by \( (f, T) \), as follows:

\[
p(f, T) = KH(f, T) \times (f, T),
\]

where \( H(f, T) \) is the transfer function of the acoustic driver (with amplifier), \( f \) is the frequency, \( t \) is the time, \( T \) is the time window, and \( K \) is the voltage translation factor. Since only discrete time samples are generated and acquired, \( p(f, T) \), and similarly \( x(f, T) \), is given as [2]

\[
p(f_m, T) = p(mAf, T)
= \Delta t \sum_{n=0}^{N-1} p(n\Delta t) \exp[-j 2\pi (mAf) (n\Delta t)], \quad m = 0, 1, 2, \ldots, (N/2) - 1,
\]

where \( m \) is the discrete frequency index, \( Af \) is the frequency resolution (1/T), \( \Delta t \) is the time resolution \((T/N)\), \( n \) is the time sampling index and \( N \) is the total number of samples. The 0022-460X/78/0608-0459 $01.00/0 © 1978 Academic Press Inc. (London) Limited
maximum frequency of analysis, \( f_{\text{max}} \), is \( (N/2T) \). Since D/A and A/D are coupled, the sampling parameters are the same for both.

An examination of equation (1) suggests that \( x(t) \) should be selected such as to provide the desired pressure impulse \( p(t) \) to be given despite the imperfect response of the speaker. This leads to the inverse problem solution which defines the input \( x(t) \) as

\[
x(t) = \frac{1}{K} \int_{0}^{T} p(t') y(t-t') \, dt',
\]

(3)

where \( y(t) \) is the inverse Fourier transform of the reciprocal of the speaker transfer function \( H(f,T) \). During the investigation, \( H(f,T) \) was determined experimentally, \( p(f,T) \), and thus \( p(t) \), was selected as per the measurement requirements, and then the convolution integral, equation (3), was performed to compute \( x(t) \). This, however, did not provide a symbolic impulse function; rather, the computed \( x(t) \) was spread over the entire time window. This could have resulted from either the inability to describe the phase of the desired function \( p(f,T) \) or from theoretical considerations involved with band limited analysis.

Instead, \( x(t) \) was selected by a trial and error procedure in which excitation spectra, \( p(f,T) \), were compared for various profiles. Perhaps a feedback loop system could have been constructed, but it was not found necessary.

The following parameters need to be selected judiciously for a particular impulse profile: (i) amplitude, (ii) duration (\( \delta \)), and (iii) sampling parameters. Because of the conflicting requirements, an optimum set of values must be found. The duration \( \delta \) is given by

\[
\delta = q\Delta t = q/2f_{\text{max}} = q/N\Delta f,
\]

(4)

where \( q \) is the number of points required to describe the impulse. \( \Delta t \) should be as small as possible so that impulse details are well described. But expression (4) also implies that the frequency resolution \( \Delta f \) should be as small as possible; otherwise “peaks/valleys” in the response spectrum could be missed. Moreover, a smaller \( \Delta f \) is also important for a good measurement dynamic range [2].

3. AN ILLUSTRATIVE EXAMPLE

Figure 1 shows \( x(t) \) and various sampling parameters. Pressure impulses are obtained without any pre-triggering and with the origin of \( x(t) \) as the reference point \( (t = 0) \). A syn-

![Figure 1. A typical impulse profile, \( x(t) \). \( q = 56 \), \( T = 50 \text{ ms} \), \( \Delta t = 0.1 \text{ ms} \), \( N = 512 \), \( \Delta f = 20 \text{ Hz} \), and \( f_{\text{max}} = 5 \text{ kHz} \).](image-url)
chronous time domain averaging of 100 cycles is performed to improve the signal to noise ratio.

Figure 2 shows the resulting pressure impulses, with and without flow, over the entire time window. A sharp impulse is seen in Figure 2(a). But with the introduction of flow, the amplitude drops and the shape is somewhat rounded off, as seen in Figure 2(b). This pressure impulse propagates through the medium without any apparent attenuation over the long piping lengths.

![Figure 2](image)

Figure 2. Pressure impulses $\rho(t)$, in air. (a) Without flow; (b) with 0.1 Mach flow.

![Figure 3](image)

Figure 3. Pressure impulse spectra, $|p(f)|^2$. Noise floors are shown by the thin lines. Continuous lines are values without flow, and dashed lines those with flow.

The spectral contents of the pressure impulse signal are shown and compared with the noise floor in Figure 3. The signal curve is fairly smooth and has a spread of roughly 50 dB over the entire frequency range. The signal to noise ratio is excellent as it is 30 dB at the minimum and 50 dB over a significant range. When the flow is introduced, the noise floor becomes slightly higher and the signal energy is somewhat lower, but still the signal to noise ratio is very good as it is 16 dB at the minimum. In general, the signal to noise ratio deteriorates with flow velocity [3]. The signal spectrum, as presented here, is for a signal propagating in the direction of fluid flow. For propagation in the opposite direction, the signal spectrum has been found to be more or less the same (within 3 dB). At higher flow velocities it is expected to be different [3].
4. CONCLUDING REMARKS

A minicomputer based digital system scheme for generating acoustic impulses has been presented. It offers several advantages over conventional analog pulse generators such as versatility, repeatability and high flexibility. Furthermore, it can be easily integrated into a measurement set-up and completely automated [1]. Also, feedback control loops can be easily incorporated for resonance control of the excitation. The major drawback of this technique is the cost and sophistication of the digital system, but it is a multi-purpose instrumentation facility.

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