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DEVELOPMENT OF AN IMPULSE TECHNIQUE FOR MEASUREMENT OF MUFFLER CHARACTERISTICS

R. SINGH AND T. KATRA

Acoustics/Dynamics Engineering, Carlyle Compressor Company, Division of Carrier Corporation, Syracuse, New York 13221, *U.S.A.*

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It is often desirable to evaluate mufflers experimentally, especially when either complex configurations are involved or flow effects are of interest. Available measurement techniques have certain inherent limitations and strengths. The present paper describes an acoustic impulse technique which offers a combination of advantages not found in any other single method known to the authors. An acoustic impulse of short duration is generated and applied to the muffler specimen. Isolated incident, reflected and transmitted pulses are captured by appropriately located pressure transducers. A synchronous time domain averaging operation is performed to eliminate random flow noise components from these pressure signals. Desired muffler characteristics are then computed, in the frequency domain, from the Fourier transforms of the isolated wave time histories. Both magnitude and phase are obtained as continuous functions of frequency (with resolution limited only by the available computing facility). The technique is reliable, efficient and versatile. Its attractiveness lies in the fact that the muffler response, over a broad frequency range, is obtained from a single impulse excitation, which is especially advantageous in the case of difficult and unstable flow conditions. The major drawbacks to this technique are a somewhat limited dynamic range, cost and complexity of the hardware and software required, and limited applicability for the study of non-linear phenomena. The paper describes the various conceptual, physical and measurement considerations involved in the development of an impulse technique. Instrumentation details include its adaptation to a minicomputer based digital data acquisition and processing system (conventional discrete instruments can also be utilized as described in Appendix A). Excellent agreement with theory and standing wave tube method measurements verify the validity of the technique.

1. INTRODUCTION

Muffler performance is often evaluated by various indices such as insertion loss, noise reduction, attenuation, transmission loss, etc. [1]. It should be noted that insertion loss, noise reduction and attenuation are not uniquely related to the physical properties of a muffler as they include source and load impedance effects, and are thus system properties. Thus, with a changing operating system, insertion loss and/or noise reduction are no longer indications of fundamental muffler characteristics. However, since the total sound pressure levels are easy to measure and also it is difficult to isolate a muffler acoustically, these indices have found usage as they describe overall application system properties. On the other hand, transmission loss is a characteristic of a muffler element alone and does not change from one acoustic environment to another. Although it is often stated in the literature [1–3] that it is imperative that only transmission loss (or sound power transmission coefficient) be considered an inherent muffler performance index, it has remained primarily an analytical concept because in general measurement of the transmission loss has been hampered by the lack of an acoustic wattmeter. Nonetheless, it has been derived from sound pressure measurements under certain limited conditions such as employment of an anechoic termination after the muffler, etc. Acoustical transfer impedance also describes muffler properties but is not convenient for performance evaluation purposes. Furthermore, acoustical input impedance and sound power reflection coefficient (measured with an anechoic termination) are also inherent characteristics but these do not deal directly with the sound transmission through a muffler.

Recent investigations [4–14] have studied the phenomenon of flow-sound interaction in acoustic mufflers and resonators, and have revealed certain inadequacies in "no-mean flow" type of linear acoustical formulations. Much more intensive work needs to be done in this area before comprehensive and general mathematical models are available for design prediction purposes. Singh [14] demonstrated an excellent agreement between theory and experiment for a compressor manifold containing air, but in an operating compressor, with refrigerant and lubricating oil as the media, theory could not predict realistic damping values. However, an empirical frictional coefficient, based on measurements in the operating system, showed excellent results. Thus a need exists to explore the physical properties of a muffler or manifold element when subjected to actual or simulated operating system fluid environments.

One need not argue about the desirability of obtaining a continuous frequency spectrum (over the range of interest, of course) of the muffler characteristics. This is particularly highlighted when a muffler is evaluated in actual operation with, say, a reciprocating machine; since the pressure spectrum will usually have significant energy only at fundamental running speed and its harmonics, the attenuation can be determined only at these frequencies. This would not provide a complete picture, and peaks and valleys in the attenuation spectrum could easily be missed. Also, this information may not be sufficient for assessment of muffler performance if the running speed may vary over a range. Furthermore, a continuous frequency spectrum could aid in the design process, as points of high transmission loss in the spectrum could be lined up with harmonics of operating speed.

From this discussion and the literature review to follow in the next section, it is obvious that a reliable and efficient experimental technique for measuring continuous frequency transmission loss characteristics is needed. It should also allow measurements with a steady fluid flow superimposed on propagating sound waves. The present paper aims to propose an impulse technique to satisfy these requirements. The development of the impulse technique, including conceptual considerations, procedures, instrumentation and results, constitutes the major thrust of this paper.

2. LITERATURE REVIEW

Perhaps the most commonly used technique is the standing wave tube or impedance tube method [15]. In this method a traversing microphone is used to determine the magnitude and location of successive maxima and minima of the standing wave pattern in a tube terminated by the muffler or the unknown acoustic system. From this information, the input impedance, reflection and absorption coefficients can be calculated. With the use of an anechoic termination after the muffler, the standing wave method can also be extended to transmission loss measurements [16, 17]. Gatley and Cohen [16] in 1969 reviewed previous experimental methods and tried several techniques to measure small muffler characteristics. They recommended the standing wave tube method and proposed a "unique design". It has been used extensively by a number of investigators [6, 8, 9, 11, 12, 14, 16–19] for resonators, mufflers and manifolds. The technique could be tedious and time consuming as the traverse is often manually operated and measurement can be conducted at only one frequency at a time. The microphone traversing requires continuous pressure measurement along the tube, which is especially cumbersome and inaccurate at low frequencies. Although it is a rather difficult process for measurements with moving fluids, especially in small tubes, some researchers [6, 8, 9, 11, 12, 17, 19] have allowed for superimposed fluid flows and measured acoustical characteristics. The lower frequency limitation [15] makes it unattractive in many applications (for example, the required microphone traverse is about 20 feet for measurements at 50 Hz in air). This poses measurement reliability problems because of the dissipation [15]. Also, errors can occur in the exact location of minima and maxima. Kathuriya and Munjal [20] suggest an improved calculation procedure which does not involve the location of the pressure maxima.

A two pressure or two microphone method has been used in a variety of ways. Melling [18] placed microphones at the front and the back of an absorbing material and measured impedance from magnitude and phase differences at the microphone locations. Sullivan [17] used a similar technique for perforated plates. Johnston and Schmidt [13] developed a procedure in which two microphones were used upstream of an unknown impedance termination and the standing wave was decomposed into incident and reflected waves by using correlation techniques. This procedure was used to measure reflection coefficients of piping obstructions. Seybert and Ross [21] refined this technique by using band limited white noise excitation. This has been extended to transmission loss by utilizing an anechoic termination downstream of the unknown impedance system and locating a third microphone there [13, 21]. Gatley and Cohen [16] also investigated a three pressure method and found it unsuitable.

Bardone-Sacerdote and Sacerdote [22] designed an impedance measuring device for porous materials. It consists of a tube closed at its ends by two speakers and two microphones located on each side of the sample, which is located in the tube at unequal distances from the ends. With one sound source operated at a time, four measured pressure amplitudes and phases provide the necessary information on characteristic impedance and propagational constant. Single frequency measurements were made over the desired frequency range. This could probably be improved by using random excitation, and we see no reason why it should not work for muffler impedance measurements. Miwa and Igarashi [23] developed a technique for the measurement of four-pole coefficients. It utilized microphones placed on either side of a muffler, and terminated each end into glass wool (or an anechoic termination). Salava [24] constructed a constant volume velocity acoustic source and used it for impedance measurements.

Singh and Soedel [25] demonstrated an efficient and direct measurement procedure for determining acoustical impedances of manifolds. It utilized a known volume velocity input and harmonic, random and transient excitations were attempted. However, it is relatively difficult to extend this to measurements with flow. Also, it cannot be readily used for determination of such muffler characteristics as transmission loss.

Gatley and Cohen [16] used a transient/pulse method which utilized several cycles of a sinusoidal signal as the wave incident upon an unknown termination. The incident and reflected wave were captured by the same microphone, and together they provided information sufficient to compute reflection coefficient magnitude and phase. This technique is really a quasi-steady method as the signal is long enough to approximate a steady state response but short enough to avoid a reflection problem. Thus it is inherently a single frequency measurement. Gatley and Cohen [16] found that the long tube length necessary for the separation of incident and reflected waves resulted in significant attenuation, and also questioned its accuracy and suitability for the direct measurement of reflection and transmission factors. Ingard and Singhal [8, 9] refined the pulse method by using two microphones. Harmonic wave trains were emitted from a sound source, and incident and reflected waves were measured by different microphones. This also made it possible to make appropriate corrections for the attenuation of the pulses. Louden [26], on the other hand, used a single pulse technique where a rectangular electric pulse was fed to the sound radiator. (A single input and output pulse ideally contains all required frequencies, and reflection and/or transmission characteristics of any acoustic system could thus be determined.) He [26] applied it to measure the end corrections of open pipes and the conductivity of a side hole in a pipe. He also extended it to the transmission loss of partitions by measuring the amplitude of the transmitted pulse with and without the presence of a partition. A single channel oscilloscope, triggered by the pulse generator, and a camera were used to record the single pulses. These were analyzed to yield their Fourier components, either with the help of a mechanical curve analyzer or a computer.

Because of the inherent limitations of these existing methods, a desirable objective is to develop a reliable technique which would (1) measure transmission loss directly, (2) provide continuous frequency measurements over the frequency range of interest, (3) incorporate mean fluid flow effects and (4) also be utilized for the measurement of complex acoustical characteristics such as input and transfer impedances of mufflers and manifolds. To achieve these objectives, an impulse technique has been developed and is described in the following sections.

3. IMPULSE TECHNIQUE: CONCEPTUAL CONSIDERATIONS

An ideal impulse or delta function may usefully be thought of as a function having zero length in time and infinite amplitude. Because of its unique time and frequency domain properties it has been extensively used for mathematical analyses [5], although, of course, an ideal impulse is not physically realizable. This is not necessarily a serious limitation, as impulse-like physically realizable signals can be generated. Such functions are adequate over typical frequency ranges of interest [27], and pose little or no theoretical compromises in measurement accuracy or completeness if proper techniques are utilized. Therefore, it should be possible to determine the response of a system at all frequencies, over a range of interest, with a single impulse excitation.

In structural dynamics, an impulse technique is now commonly used [28]. System characteristics are determined by striking the structure with a force gauge equipped hammer, and measuring the vibration response to the measured force input. Its direct analog is impractical in acoustical systems as in general measurement of the particle or volume velocity is not feasible. But the other analogous physical quantity, pressure, can be measured reliably over a broad frequency range. This is especially easy in the plane wave propagation region, a condition which is very often met in small mufflers over the typical frequency range of interest.

For plane wave propagation in the presence of a superimposed fluid flow, the transmission loss (TL), input impedance (Z_{11}) and transfer impedance (Z_{12}) are given as follows [3–5]:

$$TL(\omega) = 20\log_{10}|A_I/A_T|, \tag{1}$$

$$Z_{11}(\omega) = \left\{A_I \exp\left(-\mathrm{j}\frac{\omega}{c_+}x_1\right) + A_R \exp\left(\mathrm{j}\frac{\omega}{c_-}x_1\right)\right\} \Big/ \left\{\left(\frac{1}{\rho c_+} A_I \exp\left(-\mathrm{j}\frac{\omega}{c_+}x_1\right)\right) - \left(\frac{1}{\rho c_-} A_R \exp\left(\mathrm{j}\frac{\omega}{c_-}x_1\right)\right)\right\}, \tag{2}$$

$$Z_{12}(\omega) = \rho c_+ \left\{A_I \exp\left(-\mathrm{j}\frac{\omega}{c_+}x_1\right) + A_R \exp\left(\mathrm{j}\frac{\omega}{c_-}x_1\right)\right\} \Big/ A_T \exp\left(-\mathrm{j}\frac{\omega}{c_+}x_2\right), \tag{3}$$

where

$$c_+ = c(1+M), \qquad \text{and} \qquad c_- = c(1-M). \tag{4}$$

(A list of nomenclature is given in Appendix B.) Here points 1 and 2 are considered upstream and downstream, respectively, of the muffler. It should be noted that A is a complex quantity. Thus, Z_{11} and Z_{12} are complex, and TL represents only the magnitude levels.

In the above expressions, only the convective effect of the flow is considered and higher order terms have been ignored. Thus, expressions (1)–(3) are strictly applicable for the linear case, because it is difficult to deduce intensity or particle velocity information from a single pressure signal measurement for a non-linear case.

Expressions (1)–(3) are derived for an excitation frequency ω. With an impulse function it is possible to excite the system at all frequencies. Therefore, an acoustic pressure impulse can be used as an incident wave, with the resulting reflected and/or transmitted pressure signals being taken as the system response. Since these are traveling waves in an essentially non-dispersive system, they can be sorted out in time and analyzed separately. All the information necessary to determine muffler characteristics, as demonstrated by expressions (1)–(3), is contained in these two responses to the known input excitation. Hence, the technique can be used efficiently over the plane wave frequency range under a variety of conditions, including with flow.

4. PHYSICAL CONSIDERATIONS

Figure 1 illustrates the concept of the impulse technique. The reference for both the longitudinal co-ordinate (x) and the time has been chosen at the origin of the impulse: i.e., at the sound source $x = 0$ and $t = 0$. The leading edge of the impulse, of duration δ_I, approaches the upstream transducer (# 1) in t_I, then strikes the muffler at t_m. The reflected wave, of duration δ_R, starts appearing at transducer # 1 at t_R; and the transmitted wave, of duration δ_T, reaches

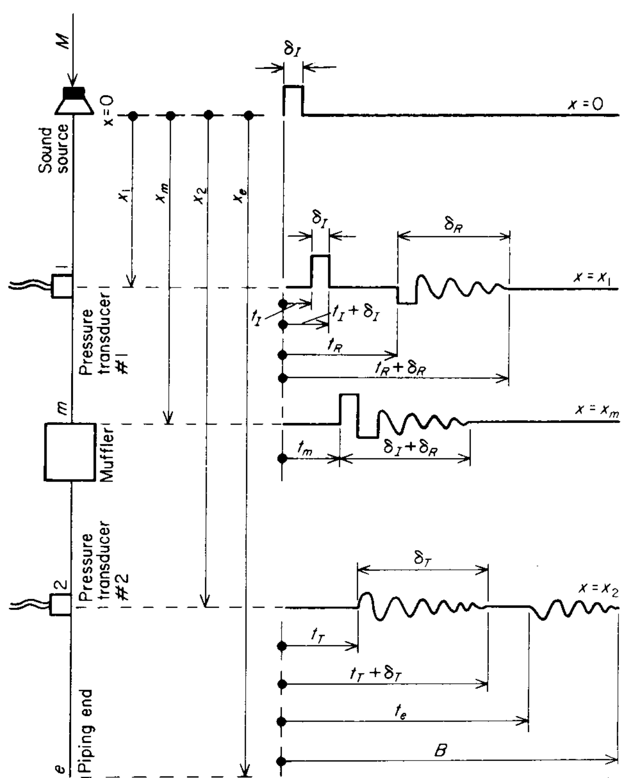

Figure 1. Illustration of the concept of the impulse technique.

the downstream transducer (#2) at t_T. The reflection of the transmitted wave from the open end, e, should start arriving at transducer #2 at t_e.

Since the requirement is to capture isolated incident, reflected and transmitted waves, the proper selection of the following parameters is very critical: transducer locations, piping lengths, impulse duration.

The criteria for capturing isolated waves requires the satisfaction of the following conditions.

1. Transducer #1 should capture the incident wave before the reflection from the muffler arrives: i.e.,

$$t_R > t_I + \delta_I, \tag{5a}$$

or, in terms of the piping lengths,

$$(x_m - x_1) > \tfrac{1}{2}c(1 - M^2)\,\delta_I. \tag{5b}$$

2. Transducer #2 should be located so that it picks up the transmitted wave completely before the reflection of the transmitted wave from the open end starts coming back: i.e.,

$$t_e > t_T + \delta_T, \tag{6a}$$

or

$$(x_e - x_2) > \tfrac{1}{2}c(1 - M^2)\,\delta_T. \tag{6b}$$

Note that for expression (6b) it is assumed that the flow velocity in the muffler element is the same upstream and downstream. This is the limiting case, belonging to the unity expansion ratio. Since the intention is to design the piping lengths, this case will provide a conservative estimate which should suffice for all conditions.

3. The time window or record length of data also influences the set-up design. It is critical that the time window, B, be long enough to capture the entire pressure wave signal during its first appearance at the transducer location, and short enough so that no undesirable reflections are acquired. Otherwise, they will have to be eliminated as a part of the data processing. The time window should be large enough to capture the wave reflected from the muffler entirely: i.e.,

$$B \geqslant t_R + \delta_R, \tag{7a}$$

or

$$B \geqslant [\{2x_m - x_1(1 + M)\}/c(1 - M^2)] + \delta_R. \tag{7b}$$

4. Similarly, the time window must also be large enough to capture the transmitted wave: i.e.,

$$B \geqslant t_T + \delta_T, \tag{8a}$$

or

$$B \geqslant [x_2/c(1 + M)] + \delta_T. \tag{8b}$$

Note that the selection of the time window, B, cannot be arbitrary. It is further complicated by the mathematical relationships between it and other data acquisition parameters (discussed in the next section). Thus, depending upon the evaluation criteria and available instrumentation, a compromise between various parameters must be selected.

The above mentioned criteria have been applied in the design of the measurement rig used by the authors. It is shown schematically in Figure 2. Care was taken to make provisions for both air and refrigerant media. In the special case of R-22, it should be pointed out that at normal room temperature and atmospheric pressure, the speed of sound (approximately

600 ft/s, or 180 m/s) is very near the R-22 sonic speed at typical air conditioning compressor discharge conditions, which is the application area of particular interest to the authors. Therefore, flow conditioning equipment has been selected such that it brings R-22 vapor temperature to the room temperature (±3°F) with a flow velocity of 60 ft/s ($M = 0{\cdot}1$), for standard refrigerating piping and system pressures within 7% of that of the atmosphere. Thus the compressor discharge lines, which often require muffling, are simulated on the experimental rig. If the test is to be conducted without flow, then flow is maintained only long enough to purge the system.

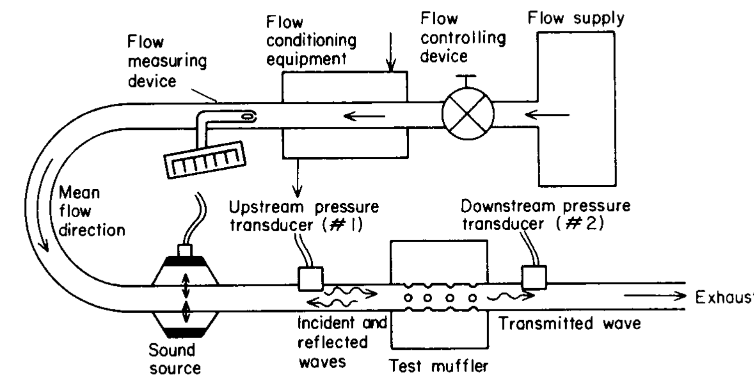

Figure 2. Schematic of the measurement rig.

A horn driver is used to produce an acoustic impulse in the pipe. High resolution, low noise dynamic pressure transducers are flush-mounted in the piping walls for the measurement of traveling sound waves.

5. MEASUREMENT CONSIDERATIONS

A two-channel minicomputer based digital data acquisition and processing system is shown, along with other instrumentation, in Figure 3. Both auxiliary hardware and software processing operations are depicted, in block diagram format. (Details of a single channel discrete instrumentation facility for the impulse technique are described in Appendix A.)

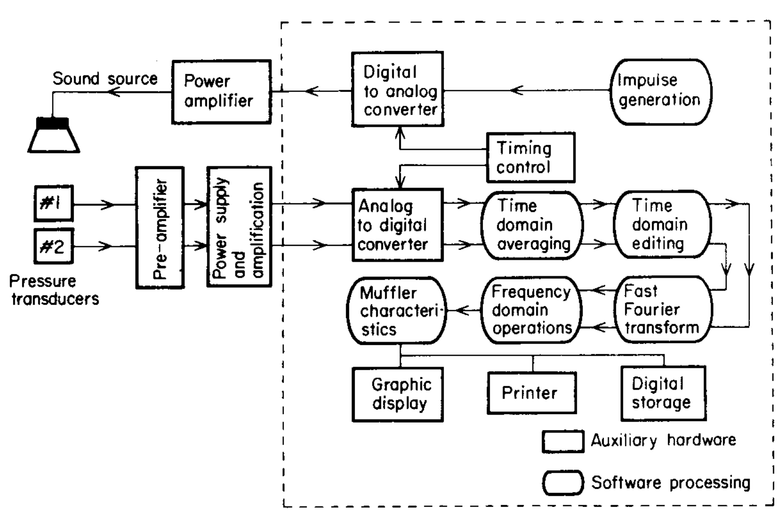

Figure 3. Block diagram of instrumentation with minicomputer based digital data acquisition and processing system.

5.1. IMPULSE GENERATION

The criteria for choosing an impulse function or an impulse-like transient are the following: (i) the excitation must be adequately uniform over the desired frequency range, (ii) it must have a relatively short time span to avoid excessive pipe lengths or excessively long time window (5)–(8), (iii) it should be wide enough to contain sufficient power spectral density for a good signal to noise ratio, and (iv) (most importantly) it must be a physically realizable signal. One must also not forget the imperfect response of the sound source, whose output should really be impulse-like. All of these considerations necessitate a compromise. The discussion on the generation of an acoustic impulse is a subject in itself, and will be the focus of another paper. But it can be stated that a wide variety of symbolic impulse transients can be used, depending primarily on the frequency range of interest.

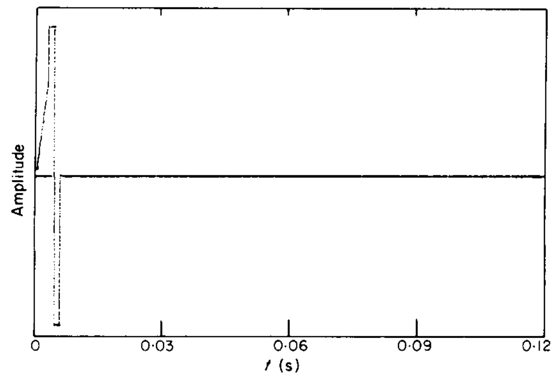

Figure 4. Symbolic impulse function shape, shown over the entire time window B. This function is generated mathematically and sent to sound source through a digital to analog converter.

In the present technique, the shape of the input excitation function is generated mathematically. Figure 4 shows an illustration of a typical symbolic impulse function shape, over the time window B, used by the authors. A digital to analog (D/A) converter is used to change this mathematical function to a voltage suitable for driving the power amplifier and hence the acoustic horn driver.

Care has been taken to insure that the impulse signal amplitude does not possess any non-linear propagation characteristic problems.

5.2. DATA ACQUISITION

To allow time domain averaging and preserve phase information (see section 5.3), it is vital to have time synchronization between data acquisition from both pressure transducers. This is achieved by locking data acquisition to impulse generation. A common timing generator controls and synchronizes the operations of the digital to analog (D/A) and analog to digital (A/D) converters. It also establishes the reference time, i.e., $t = 0$ (initial point of the time window), at the moment of source excitation. This has the additional advantage of pre-triggering the acquisition of data, so that the leading edge information of the pressure wave signals is not lost.

This technique utilizes digital data acquisition and processing techniques. These are widely reported in the literature [29, 30]. The following sampling parameters require proper and careful selection: (1) time window or record length ($B = N\Delta t$), (2) time resolution (Δt), or its reciprocal, sampling frequency (f_s), (3) maximum frequency of interest ($f_{max} = f_s/2$), (4) frequency resolution ($\Delta f = 1/B$), and (5) number of sampling points (N). The mathematical interrelationships among these reveal that only two are independent [29, 30]. Thus, one must select an optimum set, depending upon the evaluation criteria and available instrumentation.

Each time variant pressure ($p(t)$) sample, for example the nth, is

$$p_n(t) = p(n\Delta t), \qquad n = 0, 1, 2, \ldots, N-1. \tag{9}$$

Its Fourier transform $P(f)$ is computed for a finite time interval $(0, B)$. The discrete frequency value (f_k) of the kth spectral pressure is

$$P(f_k, B) = \Delta t \sum_{n=0}^{N-1} p(n\Delta t) \exp(-\mathrm{j}2\pi(k\Delta f)(n\Delta t)), \qquad k = 0, 1, 2, \ldots, \tfrac{1}{2}N - 1. \tag{10}$$

In order to illustrate the intermediate steps involved in the impulse technique, a data set has been taken for the muffler configuration shown in Figure 5. The test parameters for this muffler, with an R-22 medium are listed in Table 1. In sections 5.3–5.5, this data set will be used.

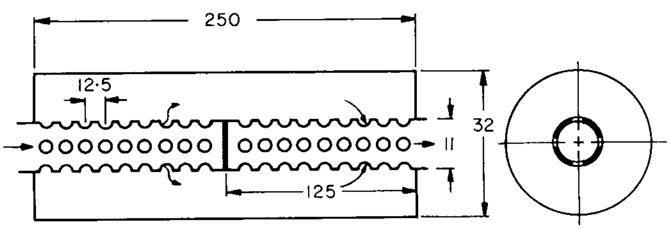

Figure 5. Muffler used for the illustration of intermediate steps. All dimensions are in mm. The diameter of the tube perforation is 3 mm. The steady flow is from the left and the flow direction is indicated by the arrows.

TABLE 1

Test parameters for the muffler shown in Figure 5

Medium	R-22
Flow velocity	30 ft/s (9 m/s)
Mach number, M	0·05
Time window, B	0·125 s
Time resolution, Δt	0·25 ms
Sampling frequency, f_s	4096 Hz
Maximum analysis frequency, f_{max}	2048 Hz
Frequency resolution, Δf	8 Hz
Number of samples, N	512
Number of averages, a	100

Transducer signals consist of the incident wave, system response, subsequent reflections and spurious noise signals. The upstream pressure transducer signal, $p_1(t)$, is

$$p_1(t) = p_{1I}(t) + p_{1R}(t) + \sum p_{1-\mathrm{ref}}(t) + y_1(t), \tag{11a}$$

where $p_{1I}(t)$ is the incident wave, $p_{1R}(t)$ is the reflected wave from the muffler, and $\sum p_{1-\mathrm{ref}}(t)$ denotes all subsequent reflections captured by transducer #1 in the time window B. $y_1(t)$ is the noise signal and consists of both electrical and flow noise signals. Similarly, for transducer #2 the signal $p_2(t)$ is

$$p_2(t) = p_{2T}(t) + \sum p_{2-\mathrm{ref}}(t) + y_2(t), \tag{11b}$$

where $p_2(t)$ is the transmitted wave, $\sum p_{2-\mathrm{ref}}(t)$ signifies all subsequent reflections in B, and $y_2(t)$ is the noise signal.

5.3. TIME DOMAIN AVERAGING

The sound pressure signals $p_1(t)$ and $p_2(t)$ are contaminated with the electrical and, in the case of superimposed fluid flow, flow noise signals. Figure 6(a) shows a sample of the pressure transducer #2 signal, $p_2(t)$. It is for the data set shown in Table 1 except that here the medium is air, with a flow velocity of 130 ft/s (40 m/s), or $M = 0{\cdot}11$. The masking of pressure signal detail by the flow noise is apparent. While the desired pressure signal is deterministic, the noise signal is random in nature. Time domain averaging can be used to decrease noise errors in signal waveforms if these signals can be caused to occur in a known manner. Signal components, being in phase with each other, add linearly with each successive accumulation.

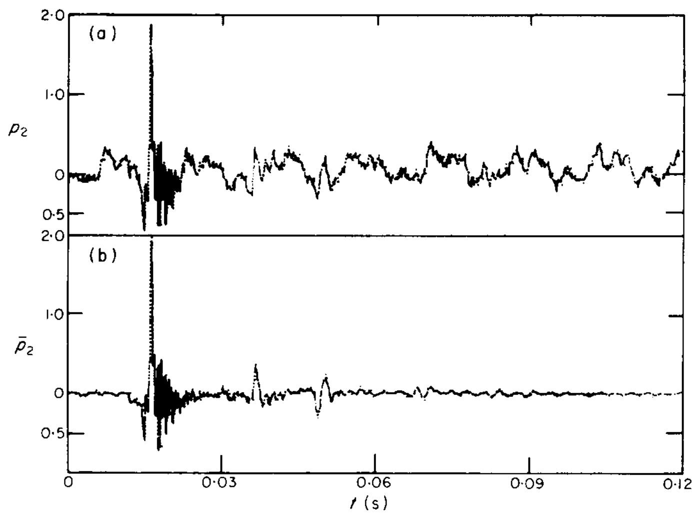

Figure 6. Effect of time domain averaging on pressure transducer #2 signal $p_2(t)$. The medium is air with a flow velocity of $M = 0{\cdot}11$. (a) $p_2(t)$ for one sample; (b) $\bar{p}_2(t)$ after time domain averaging (100 averages).

The noise components, being random, tend to cancel each other and should average out to an acceptably small value after a sufficient number of averages. If the errors are random about a mean value and have a Gaussian amplitude distribution, then the average of these errors after a measurement, will have a value within the range of ± (initial root mean square magnitude)/$(a)^{1/2}$. It should be emphasized that there must always be accurate synchronization between the pulse generation and data acquisition, as already discussed.

The time domain ensemble averaging provides the averaged pressures, $\bar{p}_1(t)$ and $\bar{p}_2(t)$, as

$$\bar{p}_1(t) = \frac{1}{a}\sum_{i=1}^{a} [p_1(t)]_i = \bar{p}_{1I}(t) + \bar{p}_{1R}(t) + \sum \bar{p}_{1-\mathrm{ref}}(t), \tag{12a}$$

$$\bar{p}_2(t) = \frac{1}{a}\sum_{i=1}^{a} [p_2(t)]_i = \bar{p}_{2T}(t) + \sum \bar{p}_{2-\mathrm{ref}}(t), \tag{12b}$$

since

$$\sum_{i=1}^{a} [y_1(t)]_i \approx 0 \qquad \text{and} \qquad \sum_{i=1}^{a} [y_2(t)]_i \approx 0. \tag{13a, b}$$

Figure 6(b) shows the effect of time domain averaging (note that the spurious noise signals have been greatly reduced after 100 averages) and reveals the signal details.

Figures 7 and 8 show the averaged pressure signals $\bar{p}_1(t)$ and $\bar{p}_2(t)$ for the illustration example data set listed in Table 1.

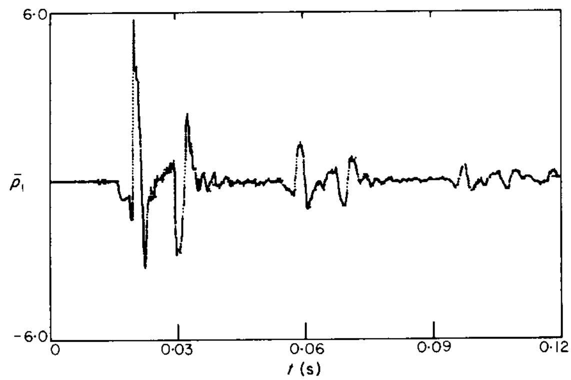

Figure 7. Averaged pressure transducer #1 signal $\bar{p}_1(t)$ for the values listed in Table 1 and muffler shown in Figure 5. The first spike is the incident wave, the second spike is the reflected wave, and the rest of the time window is filled with the subsequent reflections.

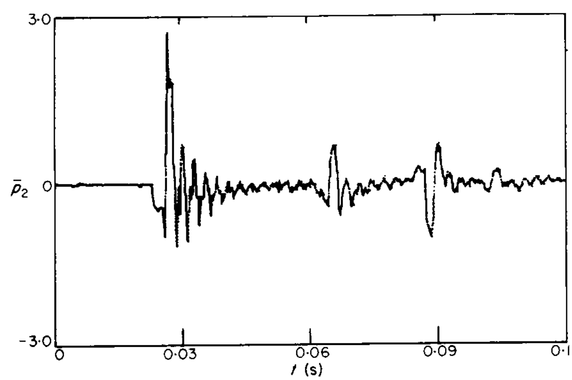

Figure 8. Averaged pressure transducer #2 signal $\bar{p}_2(t)$ for the values listed in Table 1 and muffler shown in Figure 5. The first wave form is the transmitted wave, and then the rest of the time window is filled with the subsequent reflections.

It should be noted that an adequate settling time delay must be allowed between successive ensembles to let spurious wave reactions decay to ambient noise levels.

5.4. TIME DOMAIN EDITING

Ideally the time window (B) should be such that it contains only the incident and reflected waves in the $p_1(t)$ signal, and only the transmitted wave in the $p_2(t)$ signal (see equations (7) and (8)). But for a small B the spacing (Δf) between discrete frequency values is large. Thus, for a continuous frequency spectrum, Δf should be as small, and the corresponding B should be as large, as possible. This would correspond to the inequality conditions in expressions (7) and (8). Therefore, the enlarged time window will contain unwanted subsequent wave reflections, and these must be edited out prior to any frequency domain operations.

In the present technique incident, reflected and transmitted waves are separated mathematically. A simple way, of course, is to set the unwanted data values equal to zero, but this can create some computational problems, as the trailing edge of the wave (including dc offsets and very low frequency components) would be cut off abruptly, generating artificial higher frequency components. An exponential smoothing function is used to correct this problem.

Isolated incident ($p_I(t)$), reflected ($p_R(t)$) and transmitted ($p_T(t)$) waves are obtained by multiplying the pressure signals by appropriate editing functions:

$$p_I(t) = \bar{p}_1(t)\,E_I(t), \qquad p_R(t) = \bar{p}_1(t)\,E_R(t), \qquad p_T(t) = \bar{p}_2(t)\,E_T(t), \tag{14a–c}$$

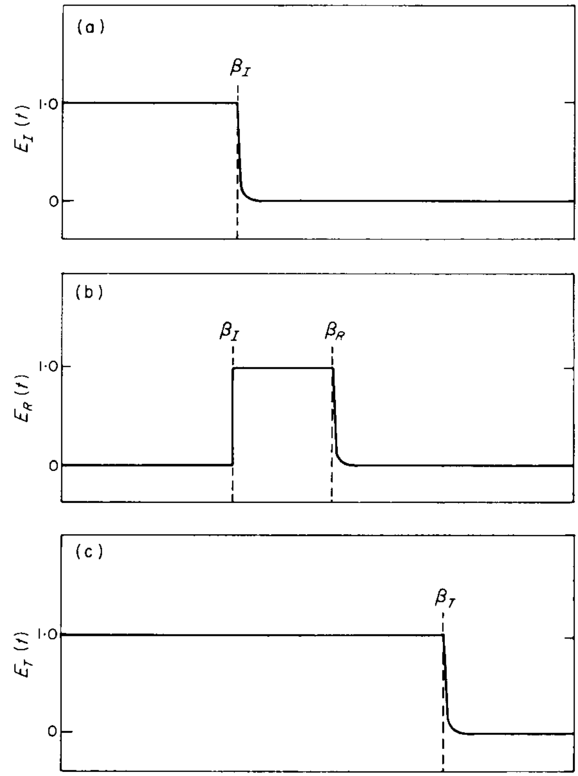

Figure 9. Time domain editing functions. (a) For incident wave, $E_I(t)$; (b) for reflected wave, $E_R(t)$; (c) for transmitted wave, $E_T(t)$.

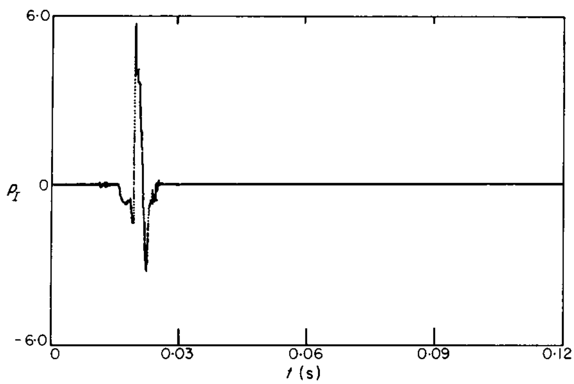

Figure 10. Isolated incident wave pressure signal, $p_I(t)$.

where the editing functions are given by the following expressions and are shown in Figure 9:

$$E_I(t) = U(t-\beta_I)\,[\mathrm{e}^{-(t-\beta_I)/\tau} - 1] + U(t), \tag{15a}$$

$$E_R(t) = U(t-\beta_I) + U(t-\beta_R)\,[\mathrm{e}^{-(t-\beta_R)/\tau} - 1], \tag{15b}$$

$$E_T(t) = U(t-\beta_T)\,[\mathrm{e}^{-(t-\beta_T)/\tau} - 1] + U(t), \tag{15c}$$

where $U(t)$ is the unit step function; β is the cut-off time (see Figure 9); and τ is the time constant for the exponential smoothing function. The value of τ should be chosen properly, as

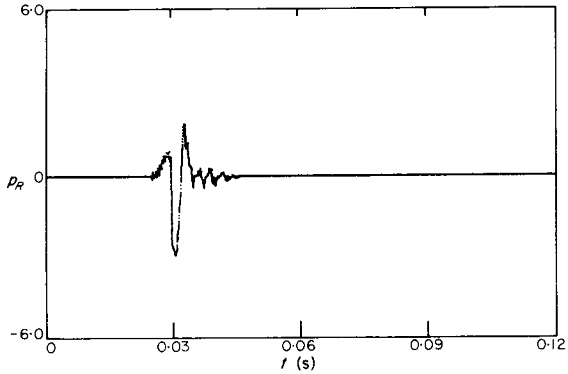

Figure 11. Isolated reflected wave pressure signal, $p_R(t)$.

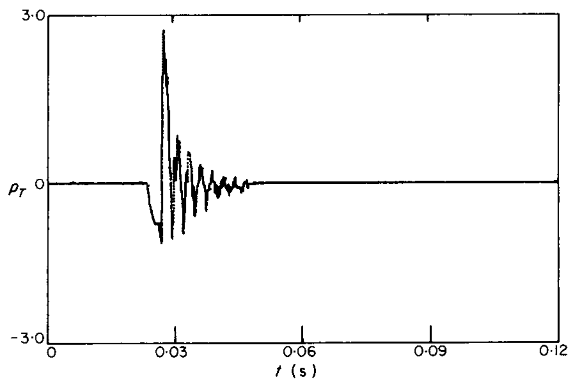

Figure 12. Isolated transmitted wave pressure signal, $p_T(t)$.

very small τ corresponds to an abrupt cut-off and large τ will include some portion of the subsequent wave.

Isolated incident ($p_I(t)$), reflected ($p_R(t)$) and transmitted ($p_T(t)$) waves are shown in Figures 10–12.

5.5. FREQUENCY DOMAIN OPERATIONS

The finite time interval pressure data ($p(t)$) is converted into spectral pressure $P(f)$ (see expression (10)) by using a Fast Fourier Transform (FFT) algorithm [29, 30]. This FFT capability allows true Fourier spectral estimates, and greatly reduces the standard computation time. The procedure for calculating transmission loss (TL of equation (1)) is based on the computation of auto- and cross-energy spectra of incident and transmitted waves, as follows:

(a) auto-spectral incident wave pressure energy, $P_{II}(f)$:

$$P_{II}(f) = P_I(f)P_I^*(f), \tag{16a}$$

where * denotes the complex conjugate;

(b) auto-spectral transmitted wave pressure energy, $P_{TT}(f)$:

$$P_{TT}(f) = P_T(f)P_T^*(f); \tag{16b}$$

(c) cross-spectral (incident-transmitted waves) pressure energy, $P_{IT}(f)$:

$$P_{IT}(f) = P_I(f)P_T^*(f). \tag{16c}$$

Auto- and cross-spectra, $P_{II}(f)$, $P_{TT}(f)$ and $P_{IT}(f)$, are shown, for same scale, in Figures 13–15 for the frequency range $(0 - f_{max})$. Although anti-aliasing filters (96 dB/octave) are used, the last 10% of the frequency spectrum data is generally suspect because of potential aliasing errors. It is shown in each spectra as the "cross-hatched region".

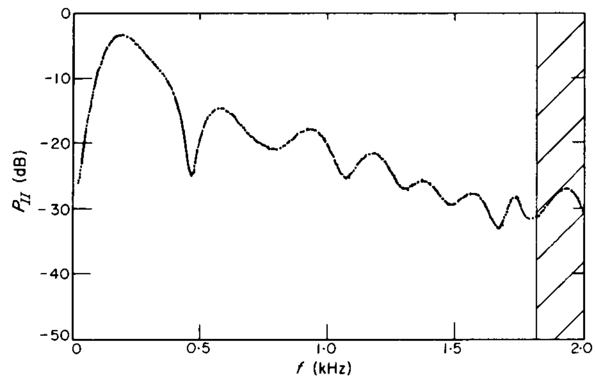

Figure 13. Auto-spectral incident pressure wave energy, $P_{II}(f)$. The cross-hatched region contains aliasing errors.

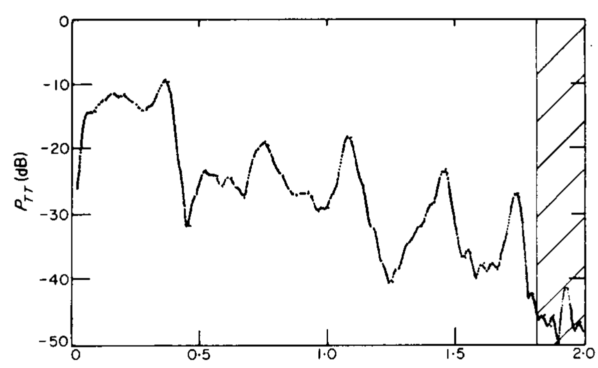

Figure 14. Auto-spectral transmitted pressure wave energy, $P_{TT}(f)$.

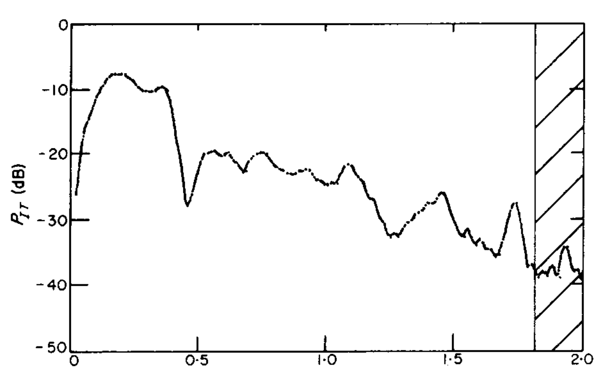

Figure 15. Cross-spectral (incident-transmitted waves) pressure energy, $P_{IT}(f)$.

Figure 13 shows the spectral pressure energy content of the input excitation to the muffler (arbitrary energy reference). The spectrum is relatively smooth and contains substantial energy over the entire frequency range. Thus, the input mathematical function shape is not only impulse-like in the time domain, but also has the inherent property of the impulse function in the frequency domain, even though the signal has been filtered by the sound source response.

The transmission loss (TL) is given as

$$TL(f) = 10\log_{10}\{(P_{IT}(f)/P_{TT}(f))(P_{IT}(f)/P_{TT}(f))^*\}. \qquad (17)$$

The transmission loss (TL) for the illustrative example is shown in Figure 16. Since the example was with flow, a no-flow type data set was taken and its transmission loss is also plotted in Figure 16. It should be noted that for this muffler, the magnitude differences

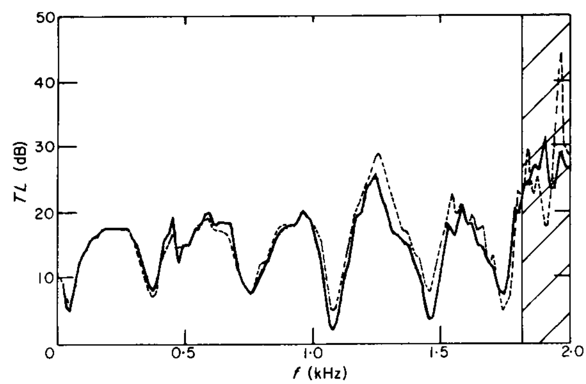

Figure 16. Transmission loss of the muffler shown in Figure 5. Medium is R-22, ----, without flow case; ——, with flow ($M = 0{\cdot}05$). The cross-hatched region contains errors due to aliasing.

are not very significant, and, as expected, the transmission loss is slightly higher without flow than with the low flow velocity ($M = 0{\cdot}05$) used.

The phase ($\psi_{IT}(f)$) between incident and transmitted waves is

$$\psi_{IT}(f) = \psi_{P_{IT}}(f) - \psi_{P_{TT}}(f) = \psi_{P_{IT}}(f). \qquad (18)$$

5.6. DISCUSSION

The phase relationship between the incident and transmitted waves, $\psi_{IT}(f)$, as expressed by equation (18) requires an adjustment dependent upon the set-up design, since the transducers are not placed exactly at the muffler ends. An illustration of the phase spectrum is not shown here because its present format has little physical significance. But when phase is expressed in the impedance form it can be visualized, and could be used to study interference mechanisms between the muffler and source and/or load impedance effects [25]. The information required to compute both magnitudes and phases of the input and transfer impedances given by equations (2) and (3) has been shown here to be available, from expression (14), and can be readily used.

In the field of digital data processing it is common practice to compute the coherence function when determining frequency response functions, to ascertain the degree of correlation between the excitation and response. However, the random data processing algorithms for spectral density estimates are written such that the absence of any frequency domain averaging produces a coherence function estimate of unity, even for the case of totally incoherent data [29]. Therefore, because only time domain averaging is used in this technique, unity coherence would result even in the presence of significant statistical errors. Since the impulse excitation is the only significant system input which is deterministic relative to the data acquisition process, and time domain averaging, in the limit, should remove all non-deterministic contributions, there should be a high degree of correlation among the incident, reflected, and transmitted signals. However, the authors have neither devised nor found in the literature a procedure/technique for verification of this correlation, and have instead relied on comparison of transmission loss results, as outlined in the next section.

6. RESULTS

Through the application of conventional plane wave propagation and signal processing theories, the technique has been shown to be generally valid for linear systems, and to be valid with appropriate attention to excitation amplitudes for investigating non-linear effects. Perhaps more satisfying, however, is a heuristic verification by a comparison with the results obtained by classical experimental and theoretical methods. This is also the accepted norm for any new technique.

Figures 17 and 18 show the comparative results for three typical expansion chamber type muffler configurations (these simple configurations were chosen so as to allow prediction by

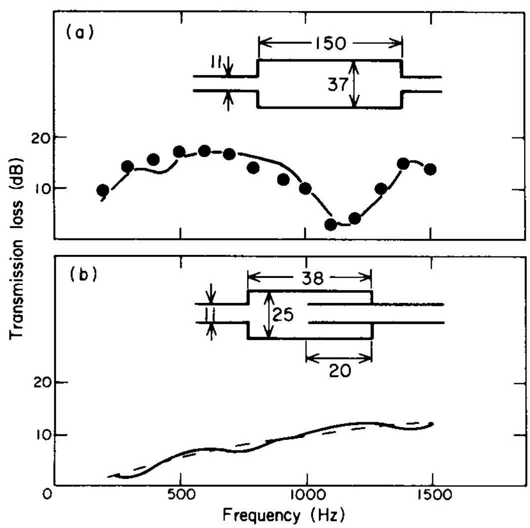

Figure 17. Verification of the impulse technique (a) ——, Impulse technique; ●, standing wave tube method. The medium is air. (b) ——, Impulse technique; ----, theory. The medium is R-22. All muffler dimensions are in mm.

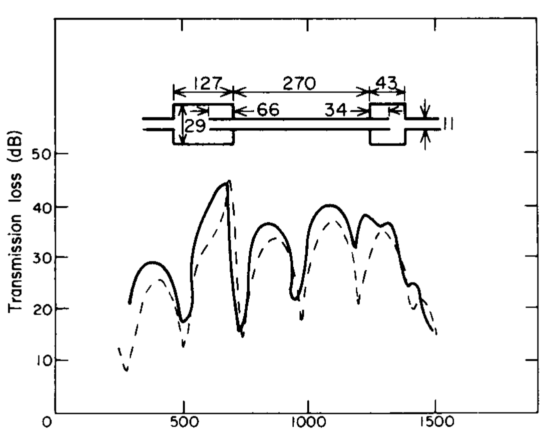

Figure 18. Comparison of impulse technique (——) results with theory (----). The medium is R-22. All dimensions are in mm.

conventional theory). In Figure 17(a), impulse technique results are compared with the standing wave tube method results for a simple expansion chamber; note the excellent agreement. Figure 17(b) shows excellent agreement between the impulse technique and theory for a pipe resonator. Finally in Figure 18, measured and computed spectra are compared for a more complicated muffling system, and again the agreement is very good.

7. CONCLUSIONS

An impulse technique has been presented which measures transmission loss and other acoustical characteristics of a muffler directly and efficiently. The authors feel that it is an accurate and reliable experimental technique, and are not aware of any previous similar efforts.

The impulse technique has several distinct advantages and improvements over some other existing measurement methods. These are as follows. (1) In general, direct measurement of transmission loss has been accomplished. (This has been made possible by isolating waves with mathematical manipulations, and the intermediate measurement steps resemble the analytical derivation steps.) (2) Need for an anechoic termination, which is difficult to construct (especially at low frequencies), has been avoided. (3) There is no inherent limitation on the measurement of low frequencies. (4) A continuous frequency spectrum is measured quickly and easily. Any reasonable frequency resolution and maximum frequency of interest, for a particular muffler, can be achieved. The authors have measured frequency spectra up to 5 kHz and discrete frequency values spaced as closely as 1 Hz. (5) The technique does not require the design and calibration of any probe type microphone, and thus not only has instrumentation been simplified, but also the sound fields in the measurement ducts are undisturbed.

Perhaps of more fundamental importance is the fact that complex acoustical characteristics (i.e., both magnitude and phase), such as impedances, four-pole coefficients, etc., can be determined free from the effects of source and load impedances. This has at least three distinct advantages: (a) basic properties can be better evaluated; (b) the effectiveness of a given muffler in a realistic environment can be mathematically predicted if the source and termination effects are known and (c) a building block type of transfer function approach, wherein muffler elements are evaluated experimentally and combined analytically, becomes feasible. Thus both experimental and theoretical efforts can converge to a unified approach. Work is continuing in this direction and shall be reported in the future.

Dynamic range is the main limitation of the impulse technique, as compared to some other methods in which harmonic excitation is used. The authors feel that there is much room for improvement and refinement in this area. For example, the excitation function could be modified to increase energy density, and thus dynamic range. Other limitations may be that the technique is costly, complicated and requires considerable efforts in the development stage. These should be weighed against its numerous advantages.

Emphasis has been laid on the criteria for proper design of the measurement set-up, especially the transducer locations, which result in a minimum piping length. But it is considerably shorter than those used by other investigators in their pulse methods [8, 9, 16]. Also, dissipation along the piping has not been found to be a problem, primarily because the short excitation pulse allows transducer location quite close to the test specimen.

The measurement facility has been completely automated. After choosing necessary data parameters, the computer selects mathematical functions, starts the experiment, collects and processes data, and finally displays and plots it. It has been used for the design and optimization of several complex mufflers for which analytical evaluation was not practical.

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APPENDIX A: IMPULSE TECHNIQUE WITH USE OF CONVENTIONAL DISCRETE INSTRUMENTS

In the text (section 5) of the paper, a two-channel digital instrumentation system is described. However, a single channel discrete instrumentation system scheme has been tried successfully. It is shown in Figure A1. Although it is inferior to the digital system in several respects (i.e., no phase information is acquired and dynamic range is more limited) it requires less costly and perhaps more readily available equipment. Although the concepts of the impulse technique remain the same, a few physical and measurement procedures are somewhat different, as described below.

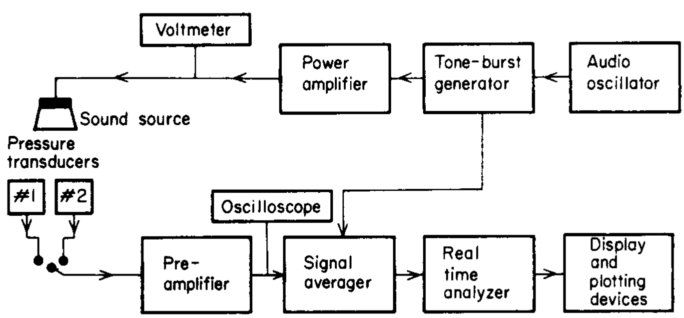

Figure A1. Conventional discrete instrumentation system schematic for the impulse technique.

The time window during which the signal is detected (controlled by the real time analyzer) is very critical. It should be such that only one complete isolated wave, i.e., incident or transmitted is captured, thus requiring no time domain editing. That is why the locations of transducers #1 and #2 (see Figure 1) are very important. With reference to Figure 1 and section 4, the time window (B) and transducer location limitations can be described as

(i) $$t_I + \delta_I \leqslant B \leqslant t_R, \tag{A1}$$

or $$\{x_1/c(1+M)\} + \delta_I \leqslant B \leqslant \{2x_m - x_1(1-M)\}/c(1-M^2); \tag{A2}$$

(ii) $$t_T + \delta_T \leqslant B \leqslant t_e, \tag{A3}$$

or $$\{x_2/c(1+M)\} + \delta_T \leqslant B \leqslant \{2x_e - x_2(1+M)\}/c(1-M^2). \tag{A4}$$

Since this system is only capable of single channel processing, a switching device is utilized to select one transducer at a time. To assure the validity of obtaining incident and transmitted wave data from separate runs, the test conditions are maintained such that repeatable pressure signals are obtained.

An input pulse has been generated by gating the signal from an audio oscillator with a tone-burst generator. One cycle of a 1000 Hz sine wave (i.e., pulse duration of 1 ms) has been found to be adequate for measurements up to 2 kHz. Although the signal is somewhat distorted due to the imperfect response of the driver, the resulting transients are repeatable and possess the required spectral content. The signal synchronization for time domain averaging purposes is achieved by triggering the signal averager with the tone-burst generator, and thus data acquisition is locked with the generation of pulses. The real time analyzer in transient mode measures the magnitude of the Fourier transform of its input signal. Thus, both incident and transmitted auto-power spectra, $P_{II}(f)$ and $P_{TT}(f)$, are measured separately and plotted. The transmission loss (TL) is

$$TL(f) = 10 \log_{10}\{P_{II}(f)/P_{TT}(f)\}. \tag{A5}$$

The dynamic range achieved with the analog system is about 30 dB maximum.

APPENDIX B: NOMENCLATURE

a	number of averages
A	pressure amplitude
B	time window or time record length
c	speed of sound
E	editing function
f	frequency
Δf	frequency resolution
j	imaginary number
k	discrete frequency index
M	flow velocity in Mach number
n	time sample index
N	number of samples
p	time variant pressure
P	spectral pressure (with one subscript); spectral pressure energy (with two subscripts)
t	time
Δt	time resolution
TL	transmission loss
U	unit step function
x	longitudinal co-ordinate
y	noise signal
Z	acoustic impedance
δ	impulse duration
τ	time constant
ρ	density
ω	radian frequency
ψ	phase
β	cut-off time

Subscripts

+	positive direction, (right traveling wave)
−	negative direction, (left traveling wave)
1	upstream pressure transducer (#1)
2	downstream pressure transducer (#2)
e	exhaust, open end
i	index
I	incident
m	muffler
max	maximum
R	reflected
ref	subsequent reflections
s	sampling
T	transmitted