MODELING OF FLUID TRANSIENTS IN MACHINES
Part I: Basic Considerations

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Abstract - This paper presents a state-of-the-art literature review of the mathematical modeling of fluid transients in machines. Part I deals with the basic equations, assumptions, and various factors involved with the models. Also discussed are typical boundary conditions, source descriptions, and solution methods as pertinent to the machines. Part II deals with applications and advanced considerations.

Fluid transients in machines can be induced in a number of ways: (a) machinery periodicity, in which inherent fluid oscillations are excited at the fundamental and higher harmonics of machinery running speeds; (b) unstable operation, in which a change in operating characteristics can cause fluid transients; (c) structural excitation, in which vibrating structures and boundaries around the fluid can produce a dynamic coupling between solid parts and fluid flow; (d) abrupt change in initial/boundary conditions, for example, sudden changes in running speed and load, or in a valve in the piping; and (e) self-excited oscillation mechanisms, for example, when fluid flows around a solid object or in and past cavities. Of primary interest in machines is periodic or pulsating flow.

Fluid transients can affect the thermodynamic and dynamic performance characteristics of a machine. Fluid transients are generally undesirable because they cause structural vibrations, noise radiation, fatigue, and failure problems. However, in special cases flow pulsations can be used to obtain better mass flow requirements.

Because the design of fluidmachines is often based on steady flow analysis, the problems associated with transients and pulsations must be solved after the design process. A mathematical model of the machine and analysis of fluid transients are necessary in order to characterize instabilities, resonances, and pressure variations. Increasing thermodynamic efficiencies and higher speeds, as well as smaller size and lower costs, dictate that adequate attention be given to the dynamic aspects of fluid transients during the machine design process.

OBJECTIVES

The problems involved in analyzing and controlling fluid transients in machines have always been recognized. But the solution of basic equations in general form was difficult unless certain simplifying assumptions were made. The advent of high-speed digital computers has made numerical solutions possible.

This paper reviews literature since 1970; advances before 1970 are well documented [1-10]. Some research is included in books [11, 12], even though they focus on the analysis of transients in hydraulic piping systems. The literature reviewed in this article is pertinent to machines in which gas and liquid serve as the working media. Piping and other components and their dynamic interrelationships are important; models are therefore required.

BASIC EQUATIONS AND ASSUMPTIONS

Consider one-dimensional ideal flow with negligible friction losses and heat transfer effects. The Navier-Stokes equation, continuity equation, and the equation of state are as follows.

$$\frac{\partial u_t}{\partial t} + u_t \frac{\partial u_t}{\partial x} + \frac{1}{\rho_t} \frac{\partial p_t}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial \rho_t}{\partial t} + u_t \frac{\partial \rho_t}{\partial x} + \rho_t \frac{\partial u_t}{\partial x} = 0 \qquad (2)$$

$$c^2 = \frac{\partial p_t}{\partial \rho_t} \qquad (3)$$

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P, ρ, u, and c are pressure, density, velocity, and speed of wave propagation respectively; subscript t implies total instantaneous values. Time and longitudinal coordinate are denoted by t and x.

Method of characteristics. The partial differential equations can be solved by the method of characteristics; these three equations can be reduced to two differential equations known as the characteristic equations. Subscripts + and − refer to the wave propagation in the +x and −x directions respectively.

$$dp_{t_{\pm}} \pm \rho_t c\, du_{t_{\pm}} = 0 \qquad (4\text{-a})$$

$$\frac{dx}{dt} = \lambda_{\pm} = u_t \pm c \qquad (4\text{-b})$$

These wave equations can be solved in the x − t plane using finite difference techniques [11-16]. Initial and boundary conditions can easily be specified, and the pressure distribution and time history can be calculated. The model is potentially very accurate because it can handle nonlinear effects; however, it might be limited by the cost of analysis. The model can incorporate any variation of c with p_t and ρ_t. But the velocity of wave propagation c could be considered a constant c_o to simplify the analysis. Subscript o implies mean or steady values.

$$\text{Liquids:}\ c^2 \simeq c_o^2 = \left(\frac{\partial p_t}{\partial \rho_t}\right)_o = B_\ell/\rho_o \qquad (5)$$

B_ℓ is the bulk modulus of the liquid.

$$\text{Gases:}\ c_o = \left(\frac{\partial p_t}{\partial \rho_t}\right)_o = k\,\frac{P_o}{\rho_o}; \qquad (6)$$

$$c^2 = c_o^2\,(\rho_t/\rho_o)^{k-1} \qquad (7)$$

where k is the adiabatic constant. The assumption of a perfect gas helps in simplifications; consequently c (as opposed to c_o) can be retained for the analysis [4, 11, 14, 15, 19-21]. Computer programs based on the method of characteristics are available [12, 14, 21].

Linearized wave equation. Equations (1) and (2) are nonlinear; thus their general solution presents a difficult problem. The equations, which can be linearized by assuming small amplitudes for the perturbations, are given below without any subscript.

$$P_t(x,t) = p_o + p(x,t) \quad , \quad p(x,t) \ll p_o \qquad (8)$$

$$\rho_t(x,t) = \rho_o + \rho(x,t) \quad , \quad \rho(x,t) \ll \rho_o \qquad (9)$$

$$u_t(x,t) = u_o + u(x,t) \quad , \quad \text{assuming } u_o = o \qquad (10)$$

$$c^2 = c_o^2 \qquad (11)$$

Equations (1-3) and equations (8-11) yield the following classical wave equation.

$$\frac{\partial^2 p}{\partial t^2} = c_o^2\,\frac{\partial^2 p}{\partial x^2}\text{, and } p_{\pm} = \pm \rho_o c_o\, u_{\pm} \qquad (12)$$

The general harmonic solution is

$$p(x,t) = [P_+ e^{-\gamma x} + P_- e^{\gamma x}]\, e^{j\omega t}\ ; \gamma \simeq j\omega/c_o \qquad (13)$$

P is the amplitude, γ is the propagation constant, and ω is the circular frequency. The linearized wave model is reasonably valid up to $p/p_o = 0.1$ and can be extended depending upon the application, nature of the fluid, and the objective of the analysis [11, 12, 22, 23]. The linear model has such attractive features as ease of developing a mathematical model for machinery components and initial/boundary conditions, a building block approach, and combining theoretical models and experimental test data [12, 23-36]. Linear models are often described using fluid impedance (Z) and/or matrix methods [5-12, 23-36].

$$Z_x = p_x/Su_x = p_x/Q_x \qquad (14)$$

S is the cross-sectional area and Q is the volume velocity. For a piping of length ℓ, upstream variables (u) can be described as a function of downstream variables (d) and inherent dynamic characteristics of the piping fluid as given by the propagation constant γ and characteristic impedance Z_c (= $\rho_o c_o/S$).

$$\begin{Bmatrix} p \\ Q \end{Bmatrix}_u = \begin{bmatrix} \cosh \gamma \ell & Z_c \sinh \gamma \ell \\ \frac{1}{Z_c} \sinh \gamma \ell & \cosh \gamma \ell \end{bmatrix} \begin{Bmatrix} p \\ Q \end{Bmatrix}_d \quad (15)$$

When the component dimensions are small compared to the wavelength of sound, fluid systems can be discretized and a lumped parameter model of the following type can be established.

$$I \frac{dQ}{dt} + K \int Q dt = o \quad (16\text{-a})$$

or

$$Z(\omega) = j\omega I + (K/j\omega) \quad (16\text{-b})$$

$$\text{fluid inertia} = I = \rho_o \ell_I / S_I$$

$$\text{fluid stiffness} = K = \rho_o c_o^2 / V_k \quad (17)$$

when ℓ_I and S_I are the length and cross-sectional area of the inertial element, and V_k is the volume of the stiffness or compliance (1/K) element.

Factors to be considered. Factors include convective effect, gas bubbles, friction, turbulence, heat transfer effects, and non-circular cross sections. The convective effect can be expressed as the effective speeds of wave propagation (c_o') in the direction of flow (+) and opposite to flow (−)

$$(c_o')_{\pm} = c_o \pm u_o \quad (18)$$

Gas bubbles in liquids and pipe elasticity are expressed in equation (19)

$$c_o' = [B_m/\rho_m) / (B_m/Ed)]^{1/2} \quad (19)$$

B_m and ρ_m are the mixture bulk modulus and density; and E, h, and d are the modulus of elasticity, thickness, and diameter respectively of the piping wall [12].

Energy dissipation losses (friction) can be considered proportional to the u(x, t); when included in the wave equation, equation (12), the losses become

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} - \frac{R}{\rho_o} \frac{\partial p}{\partial t} \quad (20)$$

R is the fluid resistance per unit area and length. This can be combined with γ (as a part of the attenuation factor α) to keep the same harmonic solution

$$\gamma = \alpha + j \frac{\omega}{c_o} = \frac{R}{2 p_o c_o} + j \frac{\omega}{c_o} \quad (21)$$

R depends on the application; e.g., for laminar flow at low frequencies, $R \simeq 32 \mu/d^2$ (Poiseuille's law). For higher frequencies $R \simeq (8\rho_o \mu\omega/d^2)^{1/2}$ (Stokes's law) where μ is the fluid viscosity and d is the piping diameter. For laminar flows, R can also be computed using Darcy-Weisbach/Fanning friction formulations or experimentally derived attenuation factors [37-41].

An eddy viscosity model for high frequencies is similar to the viscosity model for laminar flows [6, 41-44]. However, over the mid-frequency or transitional range, reliable models are not available. Turbulence can similarly interact with the transient flow and fluid perturbations may be amplified [6].

When thermal conductivity and heat transfer effects are important, an energy equation is necessary in addition to equations (1) and (2). Models for both the method of characteristics [11, 14, 15] and the linearized wave equation [5, 11, 12] are available. Empirical expressions relating viscosity, eddy viscosity, and thermal conductivity to the attenuation factor α are also available [6].

In general all formulations assume that the cross-sectional area is circular. For non-circular cross sections an equivalent radius can be defined that would give the same dynamic response [45-50].

BOUNDARY CONDITIONS

Some typical boundary conditions generally encountered in machines and associated piping system are described in this section [5-12, 51-53]. The boundary conditions for the linearized wave equation model, equations (12-14), are described. For the method of characteristics similar models for the boundary conditions have been documented [11-21, 54].

- Closed end: $Z = \infty$ (22)
- Open end or large tank: $Z \simeq o$ (23)

- Very long pipe or anechoic termination:

$$Z \simeq Z_c = \rho_0 c_0 / S \qquad (24)$$

- Sudden geometry change: Z is continuous at the boundary or interface
- Branching: at the interface of the main line and n branches, the impedance condition is

$$\frac{1}{Z_{line}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n} \qquad (25)$$

- Series elements: Z is continuous at the interface of two elements in series
- Parallel elements: at both interfaces, branching condition as given above needs to be satisfied
- Partial restrictions such as valves, ports, orifices, nozzles: flow and pressure conditions must be satisfied [12, 54].

The mathematical models of boundary conditions when the energy equation is included are available [54, 55].

DESCRIPTION OF MACHINERY SOURCES

The most difficult aspect is modeling machinery sources and characteristics. Some of the common models follow.

- Pressure source with source impedance; if the source impedance is negligible, it is a constant pressure source.
- Flow source with source impedance; if the source impedance is infinite, it is a constant flow source.
- Empirical models of source impedances are based largely on measurements [56-59]. One such model suggests that the internal combustion engine is a constant incidence amplitude source at high frequencies [57].
- Mathematical modeling of machinery processes is a direct method in which the source is not only desired completely and realistically but the interactions between the machinery processes and transients are also accounted for [11, 12, 60-66, 68]. In such models all equations are solved simultaneously using digital or analog computers.

SOLUTION TECHNIQUES

Ease of computation sometimes determines the selection of a particular approach. Early analyses were either graphical or were carried on analog computers [1-4, 9-12, 66]; the recent thrust is to digital computations [7, 11, 23], and a hybrid computer might prove to be more attractive [67, 68]. Solution techniques range from finite differences [13-21], transfer matrices or equations [25-33, 55], and the impedance concept [23, 37, 56, 61] to the bond-graph method [62] and Green's function approach [23].

The solution can be obtained in either the time domain or frequency domain. Truly transient problems are usually handled in the time domain. Moreover, fluid pulsations interacting with machinery processes are also visualized in the time or cyclic domain. In general, steady-state fluid oscillations, especially at higher frequencies, should be analyzed in the frequency domain [5, 30].

Although it is preferable to analyze machines using a distributed parameter analysis, the lumped parameter approach is sometimes very attractive [23, 69], especially in cases in which either the geometry is very irregular and complex or various components of a machine do in fact possess only one dynamic attribute in the frequency range of interest. Lumped parameter analysis has been used successfully in a number of cases [12, 23, 30, 59, 60, 66, 69].

CONCLUDING REMARKS

Part II of this paper discusses applications and such advanced considerations as modeling of turbomachines, coupling of fluid machines, multi-dimensional transients, and two-phase flows [70].

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