Simulating the response to a sonic boom

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ABSTRACT

Sonic booms can cause considerable annoyance and damage. Studying the response of a room with an open window can be useful in design and acoustic treatment. In most cases, a one-dimensional lumped parameter model can be used for the acoustics of the room-window system. A linear, energy-equivalent damping model is adequate for shock response calculations at off-resonance conditions. A shock response spectrum showing the maximum response as a function of shock duration is a convenient way to present results for design purposes. Output from a CSMP program suggests that a little damping is usually sufficient when the natural period of the room and window is well above the duration of the sonic boom; heavy damping is necessary when this is not true.

INTRODUCTION

Sonic boom pressures annoy people and cause structural damage. Minimizing these undesirable effects requires the mathematical simulation of acoustic and structural responses to sonic boom pressure signatures. For example, Crocker and Hudson's determined structural response by treating a room with a closed window as a mechanical system having a single degree of freedom. They used this model to examine the possibility of structural damage resulting from overflights of a supersonic aircraft; they investigated the effects of shock rise time and duration, and of structural parameters such as damping.

Here we describe a simulation of the acoustic response (i.e., pressure buildup) of a room with an open window. Several investigators have examined this problem analytically and experimentally. They have used the following mathematical formulations:

1) Helmholtz resonator approach. A Helmholtz resonator is essentially a cavity of large volume connected to the outside space through a narrow opening. The most familiar examples are partially-filled milk or soft-drink bottles. A Helmholtz resonator is a single degree-of-freedom system consisting of lumped acoustic ele-
ments. Lin, Vaidya, and Wabha et al. have used this model to determine transient acoustic responses that agree reasonably well with measured acoustic pressures. Reddy and Lowery have modeled interconnected rooms as coupled resonators. According to Vaidya, the Helmholz resonator approach is useful for predicting diffuse field conditions.

(2) Modal expansion approach. The room is regarded as a duct terminated by the end wall and its acoustic characteristics are described in terms of acoustic modes and natural frequencies. Then a Green's function technique is used to compute response in the time domain for a sonic boom. According to Vaidya, this approach can predict accurately the initial part of the pressure time history.

Although the modal approach is more general and accurate, most investigators have opted for the simpler Helmholtz resonator model. Note, however, that all previous models have considered the room-window system to be linear with small amplitudes of pressure oscillation; finite amplitude effects are yet to be modeled. Lin, Vaidya, and Reddy and Lowery have not formulated mathematical models for energy dissipation in the window and room; rather, they have used arbitrary damping factors in their calculations. Wabha et al. related damping to the geometrical and acoustical properties of the room-window system.

Most previous simulation studies have solved for acoustic pressure time histories of the room. These results should have been presented in the shock response spectrum format which is used extensively in vibration analysis and structural dynamics. A shock response spectrum (to be discussed later) is a non-dimensional plot of the maximum response as a function of the shock duration and the dynamic properties of the system. Since this spectrum condenses the results of many pressure time histories into a single plot, it is useful for design purposes. In this paper, we will first examine the pressure buildup in a room with an open window, and we will then develop shock response spectra.

**MATHEMATICAL MODEL**

**Assumptions**

(1) We assume an idealized time history for sonic boom pressure as shown in Figure 1a: note that the shape resembles the letter N. The duration $T$ is typically from 100 to 300 ms, and maximum amplitude $A$ is generally about 75 to 150 Pa. This idealized shape closely resembles actual sonic boom signatures.

(2) The physical model consists of a room of volume $V$, and a window of cross-sectional area $S$ and length $l$ as shown in Figure 1b. We assume that the room is treated with acoustic materials or resonators.

(3) We assume that the acoustics of the room-window system can be modeled using a one-dimensional lumped parameter approach. This assumption is valid as the smallest wavelength of interest is greater than the typical room dimensions. Since a typical sonic boom signal contains a large amount of energy over the lower frequencies, this criterion is generally satisfied.

(4) We assume that the acoustic system can be represented by a linear energy-equivalent damping model. We believe that this model is adequate for shock response calculations at off-resonance conditions. For the prediction of precise time histories, especially at resonance, a nonlinear model should be employed.

**Window**

To apply lumped parameter acoustic theory to the room-win-
Window system, we must categorize the different acoustic elements. The window density can be considered as an incompressible fluid plug. This element opposes a change in the volume velocity $\dot{V}$; its acoustic mass $M$ is

$$M = \rho l_e / S$$

where $\rho$ is the mean air density and $l_e$ is the effective length which includes end corrections $\Delta l_e$.

$$l_e = l + \Delta l_e$$

These end corrections are tabulated in the literature. Acoustic waves could be attenuated in the window by viscous and heat conduction losses at the boundary, radiation resistance, and finite amplitudes. In fact, the window may behave like an orifice and consequently may exhibit large dissipation characteristics. Because of the complexity of the damping mechanism, experimental methods are generally adopted to assess the damping values. For modeling purposes, however, we treat this nonlinear dissipation by an energy-equivalent linear damping model. Such techniques are often adopted in practice to simplify solutions.

Let us define the damped acoustic mass $M^*$ of the window as

$$M^* = M + \frac{R_m}{D}$$

where $R_m$ is the energy equivalent acoustic resistance associated with the mass, and $D$ is the differential operator $\frac{d}{dt}$.

**Room**

The room can be modeled as an acoustic elastic element because the gas in it is compressible. The pressure inside changes as air moves in and out through the window. We are assuming that the acoustic pressure is uniform throughout the room, i.e., the diffuse field condition is valid. This assumption is valid only if a typical room contains many modes over the frequency range of interest. For example, a room of size 6 x 5 x 3 m has approximately 150 modes below 250 Hz. The acoustic stiffness $K$ of a room is

$$K = \rho c^2 V$$

where $c$ is the speed of sound in air. Note that formulations are available for the shape factor corrections for $K$ and $M$.

Acoustic energy is mostly dissipated at the boundaries of the room; for example, wall treatment with porous materials or resonators will reduce the pressure buildup. Again, for modeling purposes, we will treat this acoustic damping by a linear model. Let us define the damped acoustic stiffness $K^*$ of the room as

$$K^* = K + R_K D$$

where $R_K$ is the energy equivalent acoustic resistance contained within the elastic element. For computations of $R_K$ and $R_m$, see References 8, 12, and 18.

**Room-Window System**

The equations of motion in terms of volume displacement $X$ of the window element is

$$M^* \dddot{X} + K^* \dddot{X} = p_i$$

$$[(M + \frac{R_m}{D}) D^2 + (K + R_K D)]X = p_i$$

Room acoustic pressure $p_0$ is

$$p_0 = KX$$

Thus the operational transfer function from Equation (7) to Equation (8) is

$$\frac{p_0}{p_i} (D) = \frac{1}{MD^2 + D (R_m + R_K) + 1}$$

Defining the natural frequency $\omega_n$ as

$$\omega_n = \frac{K}{M} = \frac{1}{2\sqrt{M K}}$$

and the damping ratio as

$$\xi = \frac{\omega_n}{\omega_n} = \frac{R_m}{2\sqrt{M K}} + \frac{R_K}{2\sqrt{M K}}$$

we obtain

$$\frac{p_0}{p_i} (D) = \frac{1}{\omega_n^2 + \frac{2\xi}{\omega_n} D + 1}$$

This is the transfer function of a second-order system in the standard format. It is also the transfer function of a single degree-of-freedom Helmholtz resonator of natural frequency $\omega_n$, damping ratio $\xi$, and static sensitivity of unity.

**SIMULATION**

We have used IBM's CSMP III simulation program for computations. The sonic boom pressure $p_i(t)$ is defined using the functional statement AFGEN along with the user specified values of $l$, $a$, and $A$. The functional block CMPXPL is used to determine $p_0(t)$; the initial conditions are $p_0(0) = 0$, and $p_0(t) = 0$. To generate the shock response spectrum, let us define the following dimensionless parameters:

$$\gamma = \frac{|p_0|_{max}}{A}$$

$$\tau = T \omega_n = \frac{T c}{2\pi} \left(\frac{S}{I_e V}\right)^{1/2}$$

where $|p_0|_{max}$ is the absolute maximum pressure produced in the room by a sonic boom of amplitude $A$. Although $|p_0|_{max}$ is only a single point value from the $p_0(t)$ curve, it is not considered to be incomplete or inadequate. Rather, the value $|p_0|_{max}$ is treated as an indicator of the severity of the response. Therefore, $\gamma$ is an important parameter in acoustic and structural design as it signifies the dynamic amplification of the room response. It is plotted as a function of dimensionless time $\tau$, obtained by normalizing the excitation period $T$ with the natural frequency $\omega_n$ of the system. Thus, the shock response spectrum is essentially a $\gamma$-$\tau$ plot. Since one simulation run yields only one point on this plot, we run the CSMP program for numerous values of $\tau$, $\xi$, and $a$ to generate the shock response spectra, shown in Figures 2 and 3.

**EXPERIMENTAL VALIDATION**

Our simulation was verified by comparing the computed shock response values with the measured data reported by Wahba et al. as shown in Table 1. Note that the experimental results were obtained by exciting a laboratory room with a horn-type sonic boom simulator. In our analysis, we have considered only an idealized sonic boom shape. Thus, some
Because of some uncertainties in the $\xi$ and $\alpha$ values in the experimental results, a range of $\gamma$ values is compared in Table 1. We note good agreement between simulation and experiment; Wahba et al.\textsuperscript{18} and Vaidya\textsuperscript{17} have reported similar agreements using more complicated mathematical models. We believe that better agreement, especially at the resonance condition ($\tau = 1.0$), can only be obtained with a nonlinear Helmholtz resonator model.

**DESIGN STUDIES**

It is important to examine the effect of the following parameters on the dynamic response of the room: window and room geometry, sonic boom shape and duration, and energy dissipation characteristics. Figures 2 and 3 show all these effects in the form of shock response spectra. These curves are universally valid as they are plotted for dimensionless parameters. Based on Figures 2 and 3, we conclude the following:

1. For an undamped acoustic system, response $\gamma$ peaks with a value of 2.0 or greater at $\tau$ approximately equal to 1.0, 2.0, 3.0, 4.0 and 5.0. These peaks are reduced with the introduction of damping, and the response is greatly diminished for a heavily damped system.

2. A sonic boom of finite rise time ($\alpha T$) produces higher response at the resonance condition ($\tau = 1.0$) than does a sonic boom with very small $\alpha$. However, at the higher values of $\tau$, a sonic boom of higher $\alpha$ may produce a lower response. The explanation for these results is that the sonic boom with high $\alpha$ generally contains more spectral energy at the lower frequencies than does a sonic boom with small $\alpha$.\textsuperscript{18} On the basis of our study, we recommend the following acoustic design criteria for the room-window system:

\begin{equation}
\omega_n \leq \frac{\pi}{T} \text{ or } \frac{S}{I_{f,0}}^{1/2} \leq \frac{\pi}{T}
\end{equation}

Over this range, a little damping generally is sufficient. From Eq. (15), we can establish design criteria for the room and window. For typical sonic booms, $\omega_n$ should be less than 10 rad/s. This value can be
achieved easily by adjusting window geometry ($S$ and $l_o$) for a given room of volume $V$.

(b) $\xi$ should be as large as possible for the $\omega_0 > \pi/\tau$ range; although it is difficult to design heavily damped acoustic systems at lower frequencies, we can improve damping substantially through proper window design and acoustic treatment of the room.

CONCLUDING REMARKS

Our study presents a simple, yet reasonably accurate, simulation model of the room-window system acoustics. For the sonic boom excitation, we have developed shock response spectra which are valuable from the design viewpoint. Based on these spectra, we have also formulated some acoustic design guidelines for reduced room pressure.

We believe that a nonlinear model of the room-window system would be more accurate, and therefore we are currently developing one. Also, a more complicated room-window system should be examined in which connecting doors and other rooms in a building are also included.

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Figure 3. Effect of sonic boom rise time $\alpha$ on the shock response spectra. $\xi = 0.0$ ($\xi$ is the system damping).
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