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MODELING OF AN IMPULSE-ABSORBING PNEUMATIC CYLINDER

1. INTRODUCTION

Empirical or dimensionless formulations are often used to analyze shock-absorbing devices for design purposes [1]. Discrete pneumatic elements such as springs and dampers have been examined by a number of investigators [2–4], but pneumatic systems such as cylinders have not been properly investigated. Adams *et al.* [5] have however shown that a computer simulation model is required to study a pnuematic cylinder because of the non-linear equations of motion; this study was a demonstrative modeling exercise as a constant forcing function was assumed for determining response. The purpose of this letter is to present a mathematical model of a reciprocating cushioning cylinder which is designed to absorb an impulsive load. The model is validated by comparison with results of an experiment.

2. MATHEMATICAL MODEL

Figure 1 shows schematically a cushioning cylinder; note that the pneumatic cushioning effect starts when the piston stub blocks the exit port (at time $t = 0$). An impulsive load

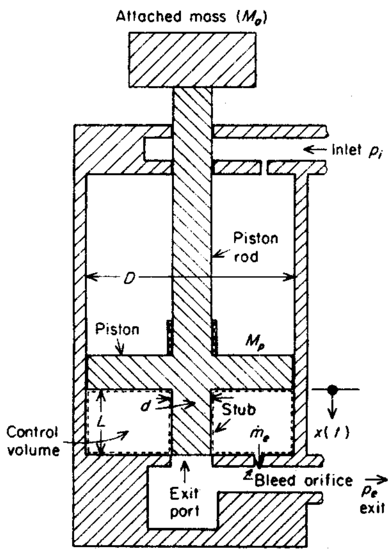

Figure 1. Schematic of a cushioning-type pneumatic cylinder.

problem can be converted into an equivalent initial value problem by considering $x(0) = 0$ and $\dot{x}(0) = I/M$, where x is the piston displacement, I is an impulse load and M is the sum of the piston assembly mass (M_p) and the attached mass (M_a). A single degree of freedom system model can be devised as follows for the piston and pneumatic system shown by the control volume in Figure 1. Piston–cylinder wall frictional effects are assumed to be negligible and the piston is assumed to come to a stop at $x = L$ without impacting the exit port wall. Also, the forcing function due to the pressure difference

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across the piston is assumed to be negligible. Then

$$M\ddot{x}(t)+K(t)x(t)=0, \qquad 0\leqslant x\leqslant L, \tag{1}$$

where $K(t)$ is the pneumatic stiffness. The gas is assumed to be perfectly elastic and homogeneous, so that

$$K(t)=\gamma p(t)A_p^2/V(t), \qquad A_p=(\pi/4)(D^2-d^2), \tag{2, 3}$$

where γ is the isentropic constant, $p(t)$ is the instantaneous cylinder pressure, and $V(t)$ is the instantaneous cylinder volume,

$$V(t)=V(0)-A_px(t)=A_p[L-x(t)]. \tag{4}$$

One can now focus attention on the control volume and make the following assumptions: (i) the gas is perfect, of gas constant R; (ii) there is no leakage past the piston seals; (iii) the compression process is isentropic; (iv) the exit reservoir pressure p_e is constant. Then

$$p(t)V(t)=m(t)RT(t), \qquad p(t)[m(t)/V(t)]\gamma=p_0[m(0)/V(0)]^\gamma, \tag{5, 6}$$

$$m(t)=m(0)-\int \dot{m}_e(t)\,\mathrm{d}t, \tag{7}$$

$$\dot{m}_e(t)=C_dA_ep(t)\left[\frac{2\gamma g_c}{(\gamma-1)RT(t)}\right]^{1/2}\left[\left(\frac{p_e}{p(t)}\right)^{2/\gamma}-\left(\frac{p_e}{p(t)}\right)^{(\gamma-1)/\gamma}\right]^{1/2}. \tag{8}$$

TABLE 1

Comparison of results for Example Case I: $\dot{x}(0)=1143$ mm/s, $D=50{\cdot}8$ mm, $d=15{\cdot}8$ mm, $L=25{\cdot}4$ mm, $M_p=1{\cdot}0$ kg, $A_e=5{\cdot}8$ mm^2, $C_d=0{\cdot}6$, $p_i=p_e=6{\cdot}5\times10^5$*Pa*, $A_p/A_e=315$, *and medium is air*

	Peak deceleration, $\ddot{x}_m$				
		Predicted, $\dot{m}_e(t)=0$		Predicted, $\dot{m}_e(t)=\dot{m}_{ec}$	
(M_a(kg))	(Measured) (g)	$\ddot{x}_m$(g)	Error† (%)	$\ddot{x}_m$ (g)	Error† (%)
1·1	14·0	15·4	−10·0	16·5	−17·8
6·6	15·0	14·9	0·7	10·3	31·3

† Error = 100 (measured–predicted)/measured.

TABLE 2

Comparison of results for Example Case II: $\dot{x}(0)=635$ mm/s, $D=101{\cdot}6$ mm, $d=25{\cdot}4$ mm, $L=30{\cdot}5$ mm, $M_p=1{\cdot}0$ Kg, $A_e=12{\cdot}9$ mm^2, $C_d=0{\cdot}6$, $p_i=p_e=6{\cdot}5\times10^5$*Pa*, $A_p/A_e=589$, *and medium is air*

	Peak deceleration, $\ddot{x}_m$				
		Predicted, $\dot{m}_e(t)=0$		Predicted, $\dot{m}_e(t)=\dot{m}_{ec}$	
(M_a(kg))	(Measured) (g)	$\ddot{x}_m$(g)	Error† (%)	$\ddot{x}_m$ (g)	Error† (%)
8·2	6·0	5·8	3·3	7·4	−23·3
40·9	4·0	3·9	2·5	3·8	5·0

Here m is the instantaneous mass in the cylinder, T is the instantaneous gas temperature, $\dot{m}_e$ is the mass flow rate out of the bleed orifice (of geometric area A_e and coefficient of discharge C_d), and g_c is the gravitational constant. One can now consider the following two limiting cases of $\dot{m}_e(t)$. (i) $\dot{m}_e(t)=0$: i.e., the minimum mass flow rate situation; this is a valid case since generally $A_p \gg A_e$; for this case, the control volume can be regarded as a closed system. (ii) $\dot{m}_e(t)=\dot{m}_{ec}$, where $\dot{m}_{ec}$ is the maximum mass flow rate associated with the critical pressure ratio $[p_e/p(t)]_c=[2/(\gamma+1)]^{\gamma/(\gamma-1)}$; dimensionless formulations currently used for sizing pneumatic cylinders are based on the assumption of critical mass flow rate.

3. RESULTS

Since the equation of motion is non-linear, a computer model has been developed to solve this problem. In the simulation program, only the limiting cases of $\dot{m}_e(t)$, are considered, and predictions based on these are compared with measured results in order to establish the validity of the mathematical model [6]. Since the peak deceleration ($\ddot{x}_m$) is often treated as an index of the shock-absorbing capability of a cylinder, we have compared $\ddot{x}_m$ for two example cases as shown in Tables 1 and 2. The predictions based on the zero mass flow rate assumption are in excellent agreement with the experiment; thus the control volume can be treated as a closed system. Conversely, the predictions based on the critical mass flow rate assumption are in poor agreement with the experiment. Our computations have shown that the minimum pressure ratio for these example cases is about 0·65 which is much higher than the critical pressure ratio of 0·53. Therefore, the critical mass flow rate assumption is crude, and hence formulations based on this assumption should not be used for design purposes.

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REFERENCES

1. F. D. YEAPLE (Editor) 1968 *Hydraulic and Pneumatic Power and Control*. New York: McGraw-Hill. See Chapter 10.
2. G. L. FOX and E. STEINER 1972 *Shock and Vibration Bulletin* **4**, 85–91. Transient response of passive pneumatic isolators.
3. M. S. HUNDAL 1978 *Journal of Mechanical Design* **100**, 236–241. Analysis of performance of pneumatic impact absorbers.
4. C. M. HARRIS and C. E. CREDE (Editors) 1976 *Shock and Vibration Handbook*. New York: McGraw-Hill. See Chapter 33.
5. G. O. ADAMS, R. D. BONNELL and J. E. FUNK 1968 *National Conference on Fluid Power* 169–182. Computer simulation of fluid power systems.
6. D. BECK and K. L. JOHNSON 1974 *Fluid Power Institute of Milwaukee, Report No.* 50194. Mosier air cylinder cushion comparison test results.