Measurement of acoustic intensity in the presence of one-dimensional fluid flow

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This paper presents the results of an analytical and experimental investigation of the measurement of acustic intensity in the presence of mean fluid flow. The investigation was necessitated by the fact that an acoustic intensity measurement technique with mean fluid flow is not available. Munro and Ingard [J. Acoust. Soc. Am. **65**, 1402–1406 (1979)] have developed a formulation for this problem; but no experimental data or verification is given. In this paper, two independent formulations for acoustic intensity in a duct with mean fluid flow using the two-microphone cross-sectral density method are developed. In order to evaluate and compare our formulations with the Munro and Ingard expression, we have conducted an experiment for the case of one-dimensional, irrotational, isentropic mean flow in a duct over 0–0.15 Mach range and for frequencies from 300 to 800 Hz. All these formulations yield similar values for acoustic intensity, within experimental errors. Although it is still premature to draw definite conclusions regarding the significance of additive terms to the nonflow acoustic intensity, some insight based on our preliminary experimental results has been gained.

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LIST OF SYMBOLS

c_0	speed of sound in still fluid
C_{12}	real part of $\tilde{G}_{12}$
f	frequency (Hz)
G_{11}	auto-spectral density of microphone 1
G_{22}	auto-spectral density of microphone 2
G_{12}	cross-spectral density between microphones 1 and 2
I	acoustic intensity in the absence of mean fluid flow
I_f	acoustic intensity in the presence of mean fluid flow
j	imaginary unit
k	wavenumber
M	Mach number
p	acoustic pressure
p_1 and p_2	pressures at microphone locations 1 and 2, respectively
P_1 and P_2	pressure amplitudes at microphone locations 1 and 2, respectively
Q_{12}	imaginary part of $\tilde{G}_{12}$
t	time
u	acoustic particle velocity
u_0	mean fluid velocity
x	longitudinal coordinate
x_1 and x_2	positions of microphones 1 and 2, respectively
α and β	additive terms for I_f expression
δL	intensity correction (dB)
Δ	microphone spacing
θ	phase angle between microphones 1 and 2, i.e., $\theta = \theta_1 - \theta_2$
θ_1 and θ_2	phase angles at microphone locations 1 and 2, respectively
ρ_0	mean fluid density
ω	angular frequency (rad/s)
$\langle\ \rangle_t$	time-averaged quantity
	boldface character means vector
	over tilde means complex quantity

INTRODUCTION

Recently there has been considerable interest in the measurement of acoustic intensity using the two-microphone technique.[1–4] Based on a finite difference approximation, acoustic intensity can be estimated by

$$I(\omega) \simeq Q_{12}/\omega\rho_0\Delta. \tag{1}$$

Note that Eq. (1) relies on the measurement of the cross-spectral density between two closely spaced microphones, which can be easily computed with a two-channel frequency analyzer. This technique has been shown to be valid for sound radiation and propagation in the absence of mean fluid flow.[1–4] The acoustic intensity technique in the presence of mean fluid flow has yet to be developed. Munro and Ingard[5] have claimed that this technique, in general, cannot be extended to mean ambient flow situations. However, they have developed an analytical formulation for acoustic intensity in a duct with mean fluid flow over the plane-wave frequency regime; the validity of this formulation has not been verified experimentally. Chung and Blaser[6] have proposed a new method of measuring acoustic intensity with flow; this method is somewhat different from conventional intensity techniques[1–5] and is similar to other duct impedence measurement methods.[7–9]

The purpose of this paper is to develop the measurement methodology of acoustic intensity in the presence of one-dimensional, isentropic and irrotational fluid flow. Two independent intensity formulations will be developed. These formulations will be evaluated experimentally, and compared with the Munro and Ingard[5] formulation. Finally, we will examine the significance of various additive terms to the nonflow acoustic intensity expression, within the range of our experimentation.

I. THEORY

A. Acoustic intensity with flow

The acoustic medium is assumed to be a homogeneous, isotropic, and inviscid fluid. For the case of constant, irrotational, and isentropic mean background flow, the following two expressions for acoustic intensity with flow can be derived.

I. Based on a linearized acoustic energy equation along the lines suggested by Blokhintsev,[10] Ryshov and Shefter,[11] Eversman,[12] etc.

$$\mathbf{I}_{f\mathrm{I}} = \langle p\mathbf{u} + \tfrac{1}{2}\mathbf{u}_0(\rho_0 u^2 + p^2/\rho_0 c_0^2)\rangle_t . \quad (2)$$

II. Based on the control volume-bulk fluid properties approach to the continuity and energy equations, along the lines suggested by Cantrell and Hart,[13] Morfey,[14] etc.

$$\mathbf{I}_{f\mathrm{II}} = \langle (1 + \mathbf{u}_0^2/c_0^2)p\mathbf{u} + \mathbf{u}_0(\rho_0 u^2 + p^2/\rho_0 c_0^2)\rangle_t . \quad (3)$$

The derivations of Eqs. (2) and (3) along with critical comparisons and discussions are available in the literature.[15–17] Munro and Ingard[5] used Eq. (3) in their formulation for acoustic intensity with flow. There is, however, some question as to which equation should be used. Therefore, rather than choosing one "valid" expression, we will use both Eqs. (2) and (3) and then evaluate them experimentally.

B. Two-microphone technique

To estimate acoustic intensity using the two-microphone cross-spectral density technique, consider the schematic shown in Fig. 1. Assuming one-dimensional fluid flow and unidirectional plane-wave propagation, the harmonically varying acoustic pressure and velocity at x can be estimated as:

$$\tilde{p}(x,t) \simeq \tfrac{1}{2}[\tilde{p}_1(x,t) + \tilde{p}_2(x,t)] \simeq \tfrac{1}{2}(p_1 e^{j\theta_1} + p_2 e^{j\theta_2})e^{j\omega t}, \quad (4)$$

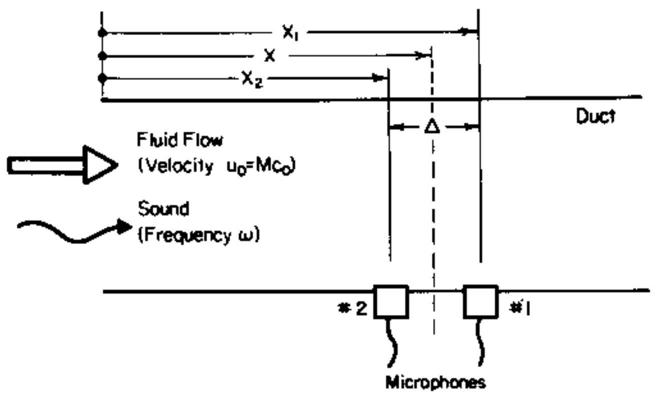

FIG. 1. Acoustic intensity measurement concept in a duct with fluid flow.

$$\tilde{u}(x,t) \simeq -\left(\frac{1}{j\rho_0\omega'}\right)\frac{\partial}{\partial x}\tilde{p}(x,t), \quad (5)$$

where

$$\omega' = \omega/(1 \pm M). \quad (6)$$

Equation (6) accounts for the convective wave effect; a positive sign (+) is used when the flow and sound are in the same direction and a negative sign (−) is used for opposite directions. For the case where $\Delta \ll (2\pi c_0/\omega')$, a finite-difference approximation can be used for the pressure gradient,

$$\frac{\partial \tilde{p}}{\partial x} \simeq \frac{\Delta \tilde{p}}{\Delta x} = \frac{(P_2 e^{j\theta_2} - P_1 e^{j\theta_1})e^{j\omega t}}{\Delta} \quad (7a)$$

and then the expression for acoustic velocity becomes

$$\tilde{u}(x,t) \simeq -[(P_2 e^{j\theta_2} - p_1 e^{j\theta_1})/j\omega'\rho_0\Delta]e^{j\omega t}. \quad (7b)$$

Using Eqs. (4) and (7b), the expressions for acoustic intensity [Eqs. (2) and (3)] are

$$I_{f\mathrm{I}} \simeq (1 + M/2)(1 \pm M)(P_1 P_2 \sin\theta)/2\omega\rho_0\Delta + M(1 \pm M)^2 \times (c_0^2/\omega^2)(P_1^2 + P_2^2 - 2P_1P_2\cos\theta)/4\rho_0 c_0\Delta^2, \quad (8)$$

$$I_{f\mathrm{II}} \simeq (1 + M + M^2)(1 \pm M)(P_1P_2\sin\theta)/2\omega\rho_0\Delta + M(1 \pm M^2)(c_0^2/\omega^2) \times (P_1^2 + P_2^2 - 2P_1P_2\cos\theta)/2\rho_0 c_0\Delta^2. \quad (9)$$

Using one-sided Fourier transform definition of the auto and cross-spectral densities,[18] we can develop expres-

TABLE I. Acoustic intensity formulations.

Formulation	$I_f(\omega) = \alpha I(\omega) + \beta$ α	β
I	$(1 + M/2)(1 \pm M)$	$\frac{c_0 M(1 \pm M)^2}{2\rho_0\omega^2\Delta^2}(G_{11} + G_{22} - 2C_{12})$
II	$(1 + M + M^2)(1 \pm M)$	$\frac{c_0 M(1 \pm M)^2}{\rho_0\omega^2\Delta^2}(G_{11} + G_{22} - 2C_{12})$
III	$(1 - M^2)(1 + 3M^2)$	$\frac{M(1 + M^2)}{2\rho_0 c_0}(G_{11} + G_{22} + 2C_{12})$
Munro and Ingard[5]		$+\frac{c_0 M(1 - M^2)^2}{\rho_0\omega^2\Delta^2}\left(\frac{(G_{11} - G_{22})^2 + 4Q_{12}^2}{G_{11} + G_{22} + 2C_{12}}\right)$

sions for acoustic intensity with flow in the following form:

$$I_f(\omega) = \alpha I(\omega) + \beta = \alpha[Q_{12}(\omega)/\omega\rho_0\Delta] + \beta,$$

where

$$\alpha = \alpha(M),$$
$$\beta = \beta(M, c_0, \omega, \rho_0, \Delta, G_{11}, G_{22}, C_{12}, Q_{12}). \quad (10)$$

We can consider α and β as the additive terms to the acoustic intensity expression without flow. Our formulations along with Munro and Ingard's expression[5] are given in Table I. Formulations I and II are essentially similar and therefore should predict somewhat identical results. Formulation III is quite different, especially the expression for β which, unlike I and II, consists of two terms: note that the first term does not contain $1/\omega^2\Delta^2$. At this point it is difficult to predict the comparison between formulations III and formulation I or II.

II. EXPERIMENT

A general purpose laboratory wind tunnel has been used to achieve the required one-dimensional, irrotational, and isentropic mean air flow. The wind tunnel, overall length of 14 m, has a horn-type inlet nozzle, rectangular test section, and a horn-type outlet diffuser. It is shown schematically in Fig. 2 along with the instrumentation used in this study. The test section consists of three parts each of dimensions (1.2×0.45×0.6 m); the first part, the entry section, connects with the inlet nozzle; the second part, the main test section, contains the microphones; and, the third part contains the damping doors which control the speed of the air flow in the test section. The flow speed can be varied in the Mach range 0–0.15 and is measured with a calibrated water manometer. The velocity profiles for the main test section are measured using a hot-wire anemometer and are shown in Figs. 3(a) and (b). The mean turbulence level, as defined by Bayley and Wood,[19] is also measured using a hot-wire anemometer, and is found to be 0.32%.

Two ceramic micorphones are fitted into a holder with $\Delta = 10.9$ mm. This holder is located in the main test section such that the microphones are flush with the floor of the wind tunnel. The microphones are calibrated for absolute magnitudes and relative phase. The dynamic transfer function between the two-micorphone measurement channels is measured, as given below, in an anechoic room for a known source over 125–2000-Hz frequency range.

Amplitude 0.0 ± 0.3 dB,
Phase $0° \pm 2°$.

Since the amplitude and phase mismatches between two microphones are minimal and within the accuracy limits of our

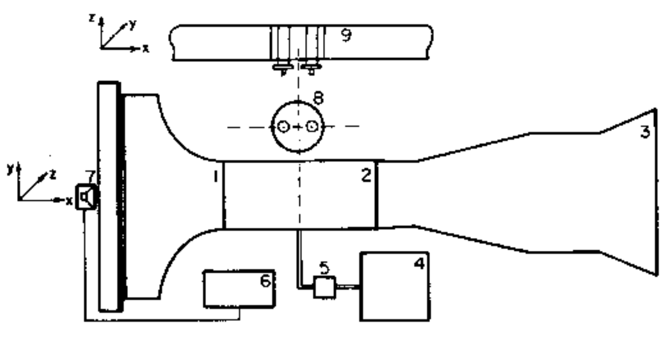

FIG. 2. Schematic of the experimental stand and instrumentation. 1: Wind tunnel inlet nozzle and damping screen, 2: test section, 3: exit diffusers section, 4: two-channel frequency analyzer, 5: signal conditioners for microphones, 6: audio signal generator and power amplifier, 7: sound source, 8: microphone holder, 9: microphone assembly.

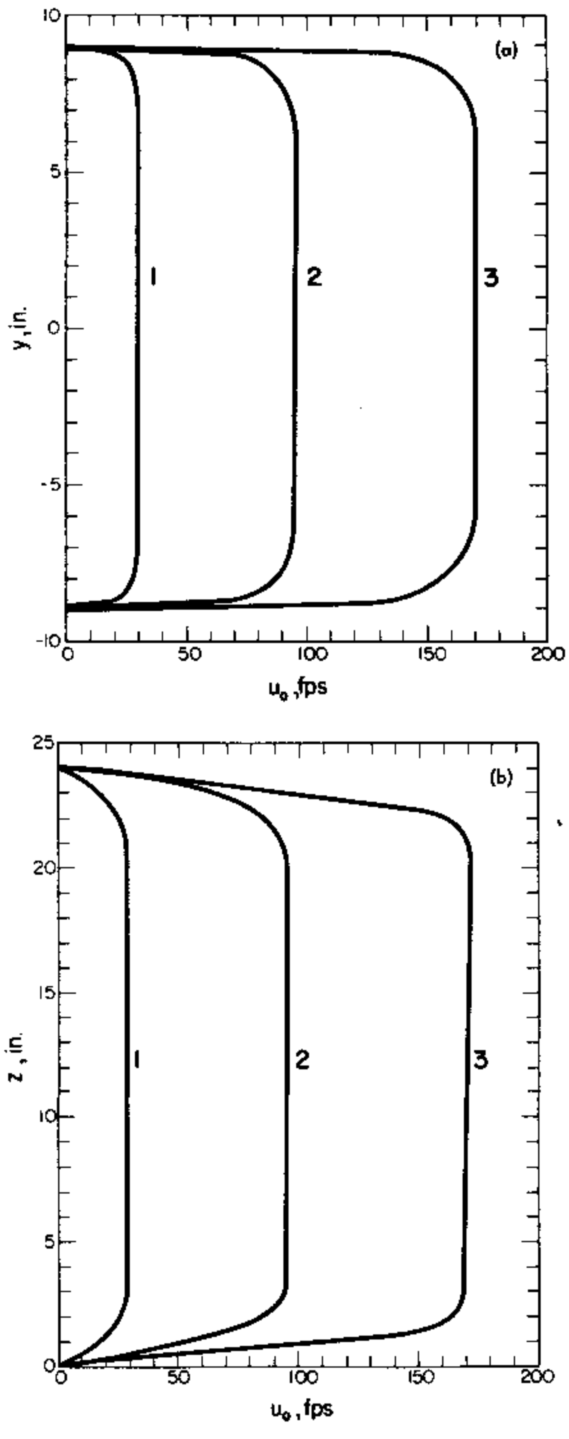

FIG. 3. Velocity profiles in the test section. (a) along the height (y), (b) across the width (z). Velocity profiles are shown for typical flow cases: low (1), medium (2), and high (3). The microphones are located at $y = 0.0$ and $z = 0.0$.

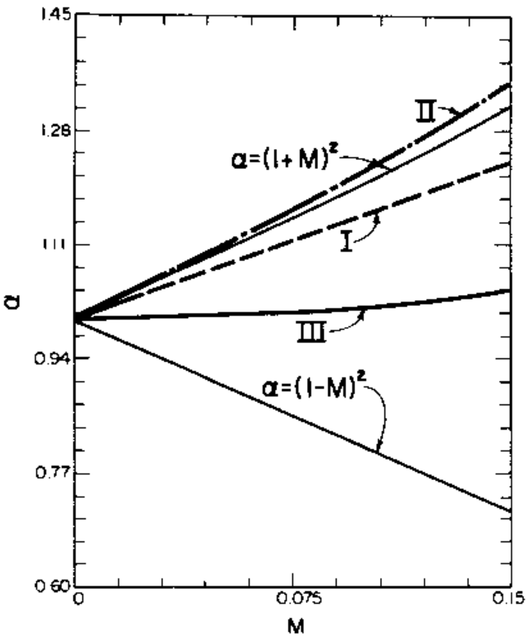

FIG. 4. Comparison of formulations for α. Formulation I- - -, formulation II——, and formulation III—. Also plotted are $\alpha = (1 \pm M)^2$ curves.

instrumentation, no corrections are made to the microphone channels.

A 60-W laboratory sound source is placed approximately 4.5 m from the microphones, and is driven by an audio-oscillator over a frequency range of 300-800 Hz. The pure-tone signal level is at least 10-20 dB above the uncorrelated flow noise level of the wind tunnel. It is ensured by adjusting the power level of the sound source; also, frequency-domain averaging (number of average = 100) is performed to eliminate uncorrelated noise from the signal.

An experimental check on the existence of cross modes and standing wave patterns has not been made. However, a simple analysis of the test section provides the following frequencies over our range of interest.

Cross-mode frequencies: 286, 381, 476, 572, 687, and 762 Hz.

Standing-wave frequencies: 143, 286, 429, 572, and 715 Hz.

Although our excitation frequencies do not coincide with these frequencies, in some cases they are fairly close. Overall, we believe that the wind tunnel geometry, and the locations of sound source and microphones ensure one-dimensional plane-wave propagation. Nonetheless, there is an element of uncertainty in our experimental data.

III. RESULTS AND DISCUSSIONS

Figures 4–6 compare α and β values for all three formulations, based on the experimental results. In fact, no experimental data is required for comparing α as it only depends on M. Figure 4 shows that all three formulations predict an increase in α with M for sound wave propagation in the direction of mean fluid flow. While formulation III allows for a small increase in α over 0–0.15 M flow velocity range, formulations I and II allow for a somewhat larger increase in α. We can approximate α by the following simple equation:

$$\alpha \simeq (1 \pm M)^2. \tag{11}$$

We can now recognize α to be due to the convective effect because of the $(1 \pm M)$ term. We should point out that in the literature[20] we often find $I_f = (1 \pm M)^2 I$; note that in

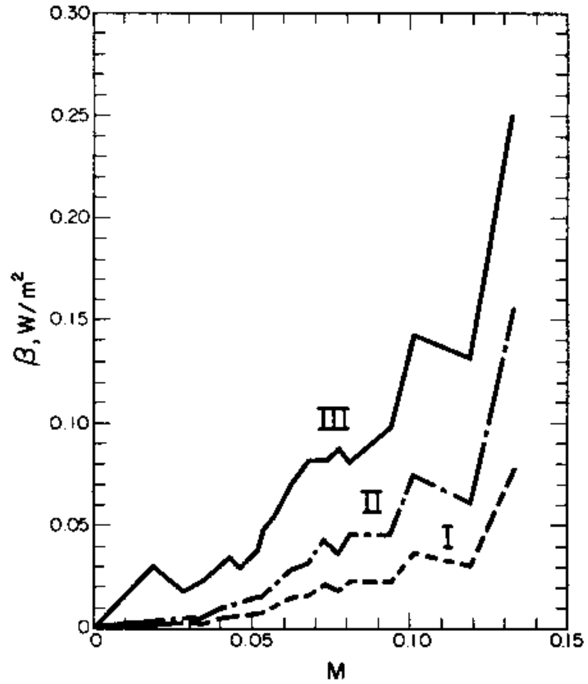

FIG. 5. Comparison of formulations for β vs M at $\omega/2\pi = 800$ Hz. Formulation I- - -, formulation II——, and formulation III—.

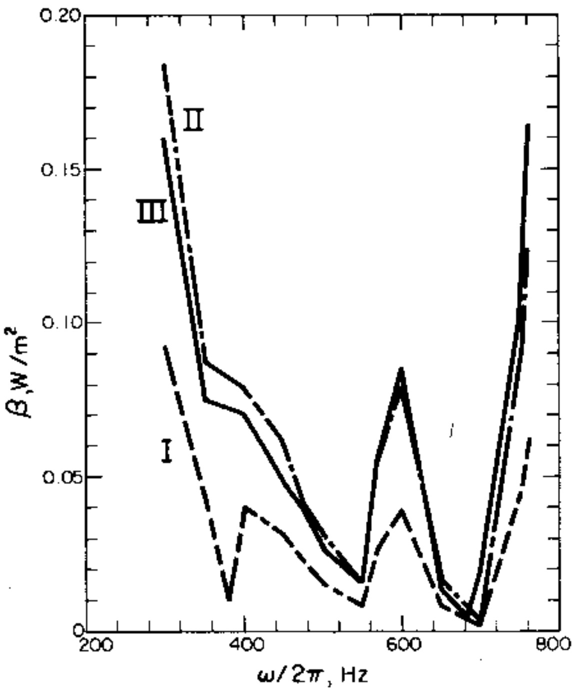

FIG. 6. Comparison of formulations for β vs ω at $M = 0.119$. Formulation I- - -, formulation II——, and formulation III—.

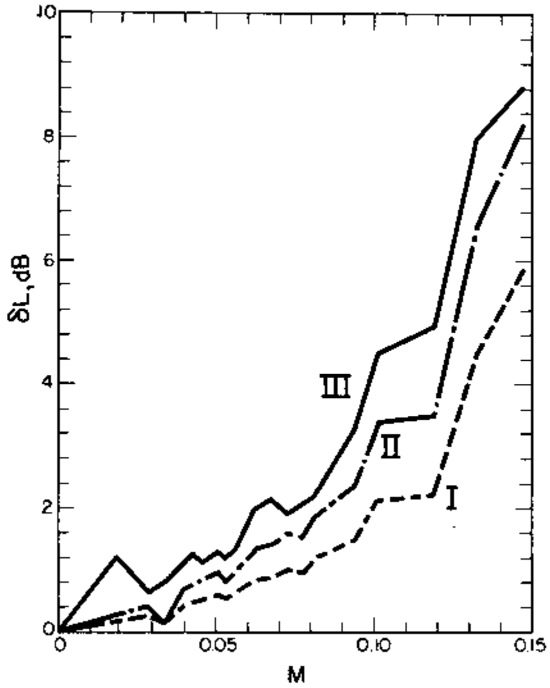

FIG. 7. δL vs M at $\omega/2\pi = 800$ Hz. Formulation I- - -, formulation II——, and formulation III—.

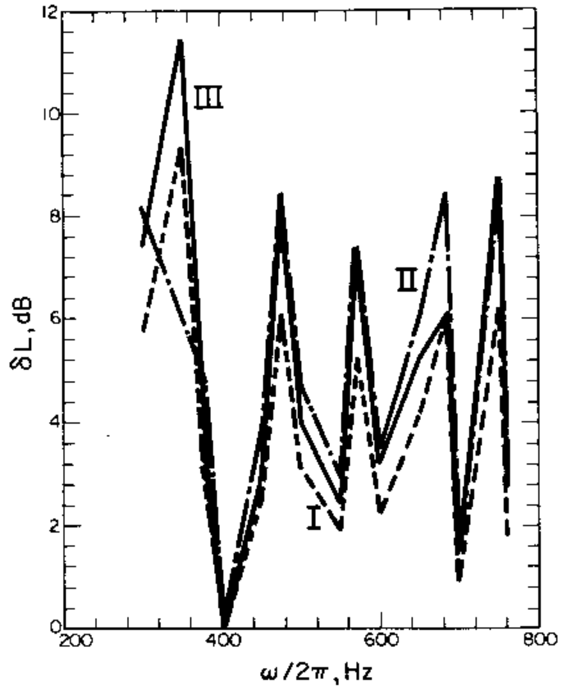

FIG. 8. δL vs ω at $M = 0.119$. formulation I- - -, formulation II——, and formulation III—.

this expression $\alpha = (1 \pm M)^2$ and $\beta = 0$. Our experimental results show, however, that finite values of β are actually obtained. These results are shown in Figs. 5 and 6. Figure 5 compares β values at 800 Hz over 0–0.13 M range; all formulations show identical trends with formulation III giving highest β and formulation I lowest. Figure 6 shows a comparison at $M = 0.119$ over 300–800 Hz range; again, all these formulations predict identical trends.

Now we can examine the combined effect of α and β. Since acoustic intensity is often expressed in intensity level (L_I), we compute the following:

$$\begin{aligned}\delta L &= L_{I_f} - L_I \\ &= 10\log_{10} I_f - 10\log_{10} I, \quad \text{dB } re\text{: } 10^{-12}\text{W/m}^2, \\ &= 10[\log_{10}(\alpha I + \beta) - \log_{10} I], \quad \text{dB } re\text{: } 10^{-12}\text{W/m}^2. \end{aligned} \tag{12}$$

We can view δL as the "correction" term which needs to be applied to the measured intensity L_I in order to compute the "correct" intensity L_{I_f}. Figure 7 compares this term for the three formulations over 0–0.15 M and at 800 Hz; note that all three formulations predict identical values and trends, within experimental errors. Similar comparisons are obtained when δL is plotted as a function of ω as shown in Fig. 8. Based on Figs. 7 and 8, we observe the following:

(i) δL vs ω curves in Fig. 8 show certain "peaks" and "valleys." At "peak" frequencies (350, 475, 570, 685, and 750 Hz) there is strong coupling between fluid and sound modes; consequently, some energy transfer (7–12 dB) takes place from fluid mode to the sound mode. At "valley" frequencies (400, 550, 600, and 700 Hz) the coupling seems to be weak, and therefore at most 2 dB increase takes place. It is difficult to correlate these frequencies with the cross-mode and standing-wave frequencies.

(ii) At a given frequency, δL increases with an increase in M; this suggests that the fluid-sound coupling is stronger at higher flow velocities.

Analytical prediction or justification of the frequencies where maximum or minimum energy transfer takes place is obviously beyond the scope of this paper. It should be, however, noted that acoustic intensity technique can be used for identification of these frequencies.

IV. CONCLUDING REMARKS

Based on our analytical study we can draw the following conclusions: (i) Measurement of I_f requires more spectral density terms (C_{12}, Q_{12}, G_{11}, and G_{22}) than only Q_{12} required by I, (ii) Eqs. (2) and (3) lead to virtually identical I_f expressions and values, (iii) Our I_f formulations are simpler than the one proposed by Munro and Ingard,[5] and (iv) If the expression $I_f = \alpha I + \beta$ is to be considered true, α is an indication of convective effect and β signifies both convective and fluid-sound interaction effects. While we can approximate α by $(1 \pm M)^2$, it is very difficult to simplify β.

Our experimentation has been limited to a single location measurement since our primary objective has been to compare the three formulations. Based on our experimental data, we find that all three formulations match very well within experimental errors. Thus our limited study does not sort out the validity of Eq. (2) versus Eq. (3).

The true determination of additive terms α and β would require a substantial experimental effort as intensity measurements for a known sound source have to be conducted over a spatial region. Nevertheless, our preliminary results from a single location intensity measurement of an unknown source do provide us some insight. Over our experimentation range ($M = 0$–0.15, $f = 300$–800 Hz), the effect of α and

β could be 0–12 dB depending on M and f. However, the effect is primarily due to β as α at most causes a change of 1.5 dB over our range of interest.

We believe that the work reported in this paper is informative, even though some experimental results are still premature to draw a definite conclusion. We plan to extend this work through exhaustive experimentation and develop a suitable measurement methodology. This should help in evaluating and understanding fluid-sound interaction effects.

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