Pneumatic Chamber Nonlinearities

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1 Introduction

This note examines the nature of nonlinearities associated with a closed pneumatic chamber coupled to a linear mechanical system as shown in Fig. 1. This simple model could represent several practical applications dealing with passive vibration isolators, shock absorbers, and cushioning type actuators. The feasibility of finding an approximate analytical solution for such systems using perturbation techniques has not been investigated, with the exception of a paper by Chen (1977), who analyzed a symmetric, double-sided closed pneumatic chamber system coupled to a cam-actuated mechanism. His study considered only the nonlinearity induced by the gas compressibility; the dynmic response was obtained by the Krylov-Bogoliubov method of slowly varying parameters. Even though no numerical or experimental validation was given, his analysis found that the resonant peak shifted toward a lower frequency as the excitation amplitude was increased. However, he did not examine some of the critical issues dealing with singularities, mean value shifting,

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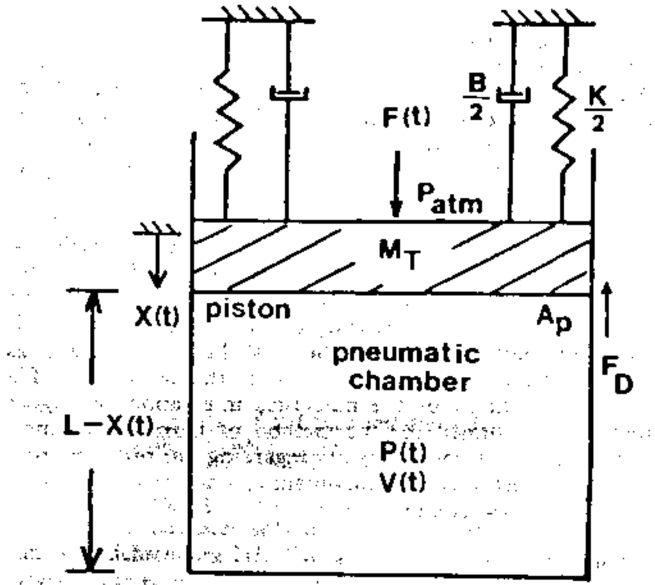

Fig. 1 Schematic of an example case: closed pneumatic chamber

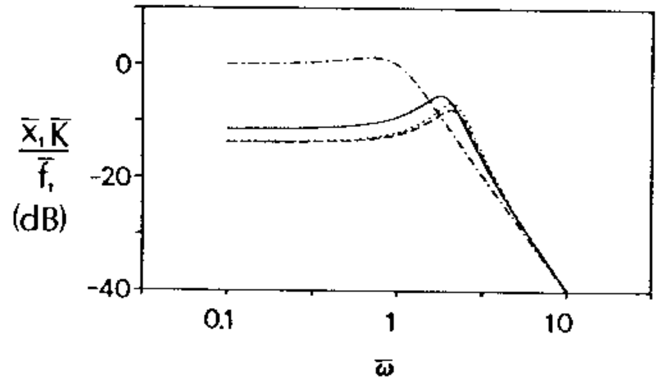

Fig. 2 Magnitude frequency response for various nonlinearities at $\bar{f}_1 = 0.6$: –·–· pure mechanical system ($H = 0$); ——— compressibility; · · · · compressibility and sliding friction effects; ---- all nonlinearities included

role of nonlinear damping (only the viscous damping was considered), and the justification of the selection of his perturbation method. Some of these issues will be discussed here.

2 Mathematical Formulation

For the thermodynamic compression process, we assume a polytropic model

$$P(t)[V(t)]^n = P(0)[V(0)]^n;\ V(0) = A_pL \text{ and } V(t) = A_p(L - X(t)) \quad (1)$$

where n denotes the polytropic constant, P is the absolute pressure, V is the gas volume, A_p is the chamber or piston area, L is the initial height of the chamber, and X indicates the piston displacement from the initial point. The equation of motion is given as

$$M_T\ddot{X}(t) + B\dot{X}(t) + KX(t) = F(t) + M_Tg + (P_{atm} - P(t))A_p - F_D(t) \quad (2)$$

where M_T is the total mass, B is the linear mechanical damping coefficient, K is the linear mechanical spring stiffness, $F(t)$ is the external force, P_{atm} is the atmosphere pressure, and $F_D(t)$ is the total damping force which is assumed to be of the following form: $F_D = \mu_{\dot{x}}\dot{X} + \mu_pP^q + \mu_g[\text{sign}(\dot{X})]$, where

Table 1 Comparison of results at $\bar{f}_1 = 0.6$ and $\bar{\omega} = 2.09$

Harmonic coefficient	Numerical integration		The method of harmonic balance	
	$\bar{x}_m/\bar{x}_1$	$\bar{p}_k/\bar{p}_1$	$\bar{x}_m/\bar{x}_1$	$\bar{p}_k/\bar{p}_1$
os	−0.261	0.019	−2.60	0.021
1	1	1	1	1
2	0.088	0.356	0.094	0.371
3	0.019	0.135	0	0
4	0.005	0.052	0	0
5	0.002	0.019	0	0
Reference values	$\bar{x}_1 = 0.464$	$\bar{p}_1 = 0.637$	$\bar{x}_1 = 0.468$	$\bar{p}_1 = 0.656$

$\text{sign}(\dot{X}) = 1$ for $\dot{X} > 0$, $\text{sign}(\dot{X}) = 0$ for $\dot{X} = 0$ and $\text{sign}(\dot{X}) = -1$ for $\dot{X} < 0$, and $\mu_{\dot{x}}$ is the viscous friction coefficient (in force/velocity unit), μ_p is the scaled sliding friction coefficient (in force/pressure unit) to account for P_{atm}, q is the friction exponent, and μ_g is the dry (Coulomb) friction coefficient (in force unit). Note that $\mu_{\dot{x}}$, μ_p, q, and μ_g are unique to the physical system chosen.

From the initial point ($t = 0$), we define the excitation $F(t)$ and responses $X(t)$ and $P(t)$ as $F(t) = f_o + f(t)$, $X(t) = x_o + x(t)$, and $P(t) = p_o + p(t)$, where f_o is the time-averaged value of $F(t)$. Now, define response operating points x_o and p_o corresponding to f_o. Using equations (1) and (2), we get

$$K(L - x_o)^{nq+1} + (f_o + P_{atm}A_p + M_Tg - KL)(L - x_o)^{nq} - P(0)A_pL^n(L - x_o)^{nq-n} - \mu_pP(0)^qL^{nq} = 0 \quad (3)$$

We find that equation (3) has unique solution at any f_o, i.e., $x_o = x_o(f_o)$ and $p_o = p_o(f_o)$, provided $x_o < L$. Note that $L = x_o$ indicates that the piston will compress the gas down to zero volume—an impossible condition to achieve. However, x_o could approach L which is somewhat realistic for the cushioning type actuation and isolation cases.

3 Nature of Nonlinearity

First, define dimensionless parameters and variables as follows.

$$\bar{f} = f/[p_oA_p],\ \bar{p} = p/p_o,\ \bar{x} = x/[L - x_o],\ \bar{\mu}_p = \mu_pp_o{}^q/[p_oA_p],$$
$$\bar{\mu}_g = \mu_g/[p_oA_p],\ \bar{K} = K(L - x_o)/[p_oA_p],$$
$$\omega_n = \sqrt{K/M_T},\ \bar{\omega} = \omega/\omega_n,\ \xi = [B + \mu_{\dot{x}}]/[2\sqrt{KM_T}]$$

where ω is the excitation frequency and ω_n is the undamped natural frequency of the mechanical system. The governing equations, equations (1) and (2), are reduced to the following dimensionless form:

$$\bar{p}(t) = (1 - \bar{x}(t))^{-n} - 1 \quad (4)$$

$$\ddot{\bar{x}}(t) + 2\xi\omega_n\dot{\bar{x}}(t) + \omega_n^2\bar{x}(t) = \frac{\omega_n^2}{\bar{K}}\bar{f}(t) - H(\bar{x}(t), \dot{\bar{x}}(t)), \quad (5)$$

where $H(\bar{x}, \dot{\bar{x}}) = \frac{\omega_n^2}{\bar{K}}\{\bar{p} + \bar{\mu}_p[(1 + \bar{p})^q - 1] + \bar{\mu}_g[\text{sign}(\dot{\bar{x}})]\}$

$$= \frac{\omega_n^2}{\bar{K}}\{[(1 - \bar{x})^{-n} - 1] + \bar{\mu}_p[(1 - \bar{x})^{-nq} - 1] + \bar{\mu}_g[\text{sign}(\dot{\bar{x}})]\}$$

Note that the system is completely described by equation (5) in terms of $\bar{x}$ which is related to $\bar{p}$ through equation (4). The nonlinear function $H(\bar{x}, \dot{\bar{x}})$ consists of the following terms: (*i*) nonlinearity induced by the gas compressibility; (*ii*) nonlinearity induced by the sliding friction model; and (*iii*) nonlinearity induced by the dry (Coulomb) friction. For both compressibility and sliding friction effects, we note that $H \to \infty$ as $\bar{x} \to 1.0$, $H = 0$ at $\bar{x} = 0$ and H is unsymmetrical about $\bar{x} = 0$. The unbounded behavior of H close to $\bar{x} - 1.0$ indicates that the nonlinearities are very large. The nature of such a singularity can be termed as "hard"—as defined by Bota and Mickens (1984). The unsymmetrical behavior about $\bar{x}=0$ is somewhat similar to Mahaffey's problem (1976) on plasma oscillations—he called such oscillations "anharmonic".

For weakly nonlinear systems, many perturbation methods (Mickens, 1981; Nayfeh and Mook, 1979; and Siljak, 1969) may work. But for those cases where nonlinear effects are large, it is not clear which method will work. However, Mickens (1984) claims that the method of harmonic balance can be applied to such problems. Also, Bota and Mickens (1984) claim that only the method of harmonic balance will work for "hard" singularity type, one-dimensional oscillatory problems. Since our example case fits into such a description, the method of harmonic balance seems to be the most logical technique that can be applied.

4 Results

Now, we apply the method of harmonic balance to evaluate the frequency response. Since the nonlinear function H is not symmetric about $\bar{x}=0$, the mean value shift must be considered even though the excitation $\bar{f}(t)$ may have a zero mean value. The excitation $\bar{f}(t)$ and responses $\bar{x}(t)$ and $\bar{p}(t)$ are assumed to be given as follows:

$$\bar{f}(t)=\bar{f}_1\cos(\omega t),$$

$$\bar{x}(t)=\bar{x}_{os}+\bar{x}_1\cos(\omega t+\theta_{x1})+\bar{x}_2\cos(2\omega t+\theta_{x2}) \text{ and}$$

$$\bar{p}(t)=\bar{p}_{os}+\bar{p}_1\cos(\omega t+\theta_{p1})+\bar{p}_2\cos(2\omega t+\theta_{p2})$$

where $\bar{x}_{os}/\bar{x}_1=\bar{x}_2/\bar{x}_1=\bar{x}_1=0(\epsilon)$, $\bar{p}_{os}/\bar{p}_1=\bar{p}_2/\bar{p}_1=\bar{p}_1=0(\epsilon)$ and the subscript *os* denotes the mean value shift or zeroth harmonic. Since lower frequencies are of interest in practical pneumatic systems, the series solution is limited here up to the second harmonic. Numerical parameters used to illustrate this example case are: $\bar{K}=0.511$, $\bar{\mu}_p=0.2$, $q=2$, $\xi=0.5$, $\bar{\mu}_g=0.07$, and $\omega_n=0.5$ (see Wang, 1986, for more details).

Figure 2 shows frequency response curves, the magnitude of the first harmonic versus the dimensionless frequency $\bar{\omega}=\omega/\omega_n$, for various values of $H(\bar{x},\dot{\bar{x}})$. Note that the overall system with all nonlinearities included is still a second order system, but the resonant frequency is shifted to $\bar{\omega}=2.09$ as shown in Fig. 2. It is apparent that the compressibility term is the most dominant, followed by the sliding friction term which is also related to the pressure (per our assumption). Both of these nonlinear effects increase stiffness as well as damping. Conversely, the dry friction is influential only at the resonance, thereby adding only to the damping, as one would expect. We must caution that in a practical system the real damping mechanism could dictate the response.

In order to validate the results obtained by the method of harmonic balance, we compare these with the results yielded by the numerical integration of equations (4) and (5). In Table 1, the zeroth and first five harmonics are examined for both $\bar{x}$ and $\bar{p}$ at the resonant frequency ($\bar{\omega}=2.09$); an order analysis has been performed using the first harmonic results as reference values. We find that some of the harmonic coefficients as predicted by the numerical integration are higher than those assumed for the method of harmonic balance. Hence, there is some ambiguity regarding our order approximation! Nonetheless, we still obtain very accurate results with considerably less computer time.

Acknowledgment

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