Case History:
The Effect of Radial Slots on the Noise of Idling Circular Saws*

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The effect of milled and laser-cut radial slots on the radiated noise from idling circular saws was investigated experimentally. The geometric discontinuities caused by such slots seem to control the intense tonal sound radiation by disrupting the whistling instability mechanism. Several slot patterns, all of which can easily be cut with a computer-aided laser, have been found to be suitable. Some recommendations are included for further work to promote understanding of the physical mechanisms involved.

The primary purpose of a radial edge slot in a circular saw is to allow thermal expansion during the cutting process without the development of circumferential stresses. Often, in practice, a hole also is cut at the end of the slot in order to relieve the radial stress introduced by the slot which can cause blade cracking. This hole is then plugged with a copper insert, as the general notion in the industry has been that this practice helps in reducing the radiated noise.[1] The effects of slots on noise, vibration, and dynamic stability are yet to be demonstrated conclusively, as indicated by the conflicting reports in the available literature.[2-13]

The first objective of this study is to check whether the common industrial practice of cutting radial slots and inserting copper plugs in holes is a good engineering solution to the circular saw idling noise problem; it should be pointed out that the copper plug may come off during saw operation, causing a safety problem. Further, since the modern trend is to manufacture saw blades with a computer-aided laser, the second objective is to examine the usefulness of various laser-cut slot patterns, including a simulation of the milled-copper plugged radial slot.

The organization of this paper is as follows. First, measured noise data for the case history is reported and discussed. Second, limited structural modal analysis results are given. Third, possible explanations regarding the physical mechanism(s) associated with the slots are offered along with a critical review of the literature. Finally, recommendations for further research are included.

Sample Cases

A number of saw blades as listed in Table 1 were evaluated experimentally. These consist of one benchmark blade without any slots (#B1), two blades with conventional milled slots (#C1 and #C2), 11 blades with various types of laser-cut slots (#D1-9, E1, and E2) and one blade with a combination of milled and laser-cut slots (#E3). Figure 1 shows typical milled (#C1) and laser-cut (#D3) slots. It should be noted that the spiral termination used in blades #D3-D9 is essentially a simulation of a radial slot terminated in a hole plugged with a copper insert. Two blanks (blades without teeth) with laser-cut slots (#A2) and without any slots (#A1) were also evaluated for comparative purposes.

All saw blades were run, with no protective cover, on a Dewalt Standard radial arm saw at the rated rotation speed of $\Omega = 57.5$ Hz. The free-field sound pressure level (L_p) was measured in the far field at $r = 1.52$ m to yield the following database: (1) octave band spectra and overall (A-weighted) levels L_{pA}, (2) narrowband (unweighted) frequency spectra over 0 to 10 kHz with 25 Hz frequency resolution, and (3) directivity patterns $D(\theta)$, where θ is measured from the plane of the blade. A modal analysis experiment was also conducted on a blade without any slots (#B1) and on a blade with radial laser-cut slots (#D3) in order to examine the problem from a viewpoint of structural dynamics.

TABLE 1
IDENTIFICATION OF SAW BLADES TESTED

Saw Blade #	Description
A1	Blank without any teeth and without any slots
A2	Blank without any teeth and with 4 radial, equally-spaced laser-cut slots (L = 19.05 mm) terminated in a spiral manner like #D3
B1	Saw blade – without any slots
C1	Saw blade with 4 radial, equally-spaced milled slots (length L = 19.05 mm) and open holes
C2	With 4 radial, equally-spaced milled slots (L = 19.05 mm) and holes plugged with copper inserts
D1	Saw blade with 4 radial, equally-spaced laser-cut slots (L = 19.05 mm) and milled, open holes
D2	With 4 radial, equally-spaced laser-cut slots (L = 19.05 mm) and holes plugged with copper inserts
D3	With 4 radial, equally-spaced laser-cut slots (L = 26.98 mm), terminated in a spiral manner
D4	Like D3 except L = 9.52 mm
D5	Like D3 except L = 38.1 mm
D6	Like D3 except 6 slots
D7	Like D3 except 8 slots
D8	Like D7 except unequally-spaced slots
D9	With 4 long (L = 38.1 mm) and 4 short (L = 19.05 mm) radial slots terminated in a spiral manner like D3. Long slots are 90° apart and 45° from a short slot
E1	With 8 radial, equally-spaced laser-cut slots (L.28.57 mm), like D3, and 8 inside laser cut slots 87.31 mm long).
E2	With laser-cut slots spread over the entire saw body depicting manufacturer's logo
E3	With a combination of cuts – 8 radial equally-spaced laser-cut slots (L = 19.05 mm) and 8 wide milled slots spread over the saw body

Specifications: Carbide saw body, zero hook on the teeth, teeth slant depth = 19 mm, without carbide tips, outside diameter (2R) = 303.2 mm, inside diameter = 15.87 mm, clamp diameter = 34 mm, blade thickness (h) = 3.4 mm and number of teeth = 80. (See Figure 1)

Radiated Noise Results

Saw without Slots. Figure 2 shows the narrowband L_p spectrum for the benchmark blade without any slots (#B1). The radiated noise is essentially of the whistling type—very intense pure tones which are extremely annoying and possibly damaging to the ear. The characteristics of idling saw aerodynamic noise sources and especially the whistling-type tonal radiation phenomenon are complex and not well-defined in the available literature.[1,4,9,11,14-23] However, many investigators have reported that the dominant source seems to be of the point dipole type, and the classic theory of periodic shedding of vortices from the teeth edges appears to explain the idling saw noise data partially—even though several acoustic issues remain unresolved. So, we will attempt to identify the dominant peaks in Fig. 2 using this theory.[22-27]

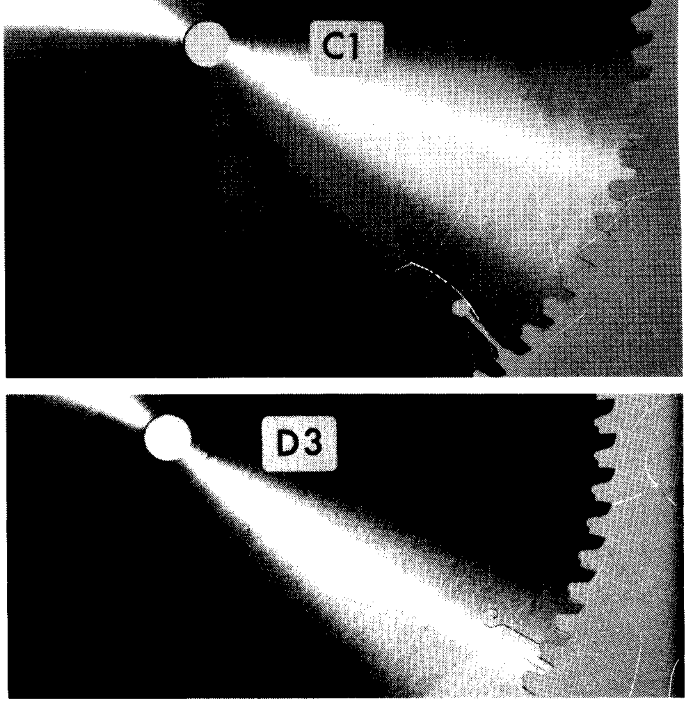

Figure 1—Saw blade segment with a radial slot. a) milled slot, without copper plug (#C1); b) laser-cut slot (#D3); refer to Table 1 for more details

First, we calculate the Reynolds number $Re = 2\pi R^2\Omega/\nu$ at the edges where R is the saw radius, Ω is the speed in Hz, and ν is the kinematic viscosity. For our sample case, we find $Re \simeq 1.17 \times 10^6$, which indicates that the disk surface boundary layer is turbulent. Hence, the frequency of the periodic vortex shedding f_e in Hz is given by the Strouhal number S as: $f_e = 2\pi R\Omega S/h$, where h is the blade thickness. For idling circular saws, the value of S is typically between 0.1 and 0.2, depending on blade kinematics and flow conditions. For our sample case, S is likely to be about 0.16 to 0.18 (i.e., f_e is expected to be within the 2572 to 2894 Hz range). Figure 2 confirms this as we note two major frequencies in the range

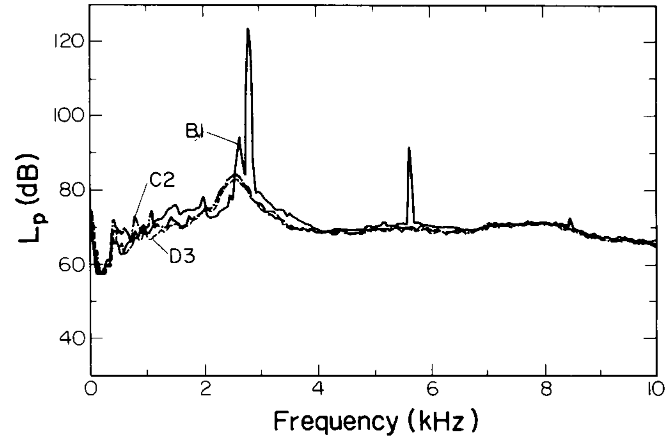

Figure 2—Narrow band frequency spectra of free-field sound pressure level L_p (unweighted) at $r = 1.52$ m, $\theta = 30°$ where θ is measured from the plane of the blade: a) Saw blade #B1 (without slots ———); b) Saw blade #C2 (with milled slots and copper plugs —·—); and c) Saw blade #D3 (with laser-cut slots -------)

TABLE 2
COMPARISON OF SAW BLADES FOR RADIATED NOISE

L_{pA}, dB at r = 1.52 m				
Blade #	Averaged overall $\bar{L}_{pA}$	Octave band levels, at θ = 30°		
		1 kHz	2 kHz	4 kHz
A1	52	50	44	41
A2	59	53.5	46.5	41.5
B1	107	67.5	103	104
C1	99	67.5	99	90
C2	78.5	61	78	72
D1	81	59.5	77	72
D2	79	50	78	72
D3	78	61	76	71
D4	95	68.5	99	91.5
D5	78.5	61.5	78.5	71.5
D6	78	61.5	79.5	71.5
D7	78.5	61	79.5	72
D8	78	62.5	78.5	71
D9	78.5	61.5	77.5	71.5
E1	78	61.5	77.5	71.5
E2	78	61	78.5	72
E3	78	66	77.5	71.5

dictated by the vortex shedding mechanism: f_{e1} = 2825 Hz (dominant tone) and f_{e2} = 2650 Hz. Further, the second and third harmonics of f_{e1} can be noticed at 5650 and 8475 Hz, respectively. This demonstrates that the whistling noise generation mechanism is essentially in a nonlinear regime.

Table 2 presents L_{pA} levels for the dominant octave band levels. For blade #B1, most of the energy is concentrated in the 2 and 4 kHz bands. When this blade with teeth is compared to the disk, i.e., blank without any teeth (#A1), we note that the 2 and 4 kHz bands for #A1 are almost 60 dB lower than the levels measured for #B1. This demonstrates that the whistling type of intense sound is due primarily to the acoustic sources on and near the teeth around the disk; similar results have been reported in the literature.[3,18-22,27]

The sound pressure level directivity pattern D(θ) is shown in Fig. 3, where θ is measured from the plane of the saw blade. For an ideal normal dipole source oriented along the symmetry axis (θ = 90°), the directivity is D(θ) = $\sin^2\theta$. We note that the measured D(θ) for #B1 shows considerable deviation from the ideal dipole expression. Similar results have been reported in the literature, and the precise nature of the source directivity is yet to be characterized.[14,21,27]

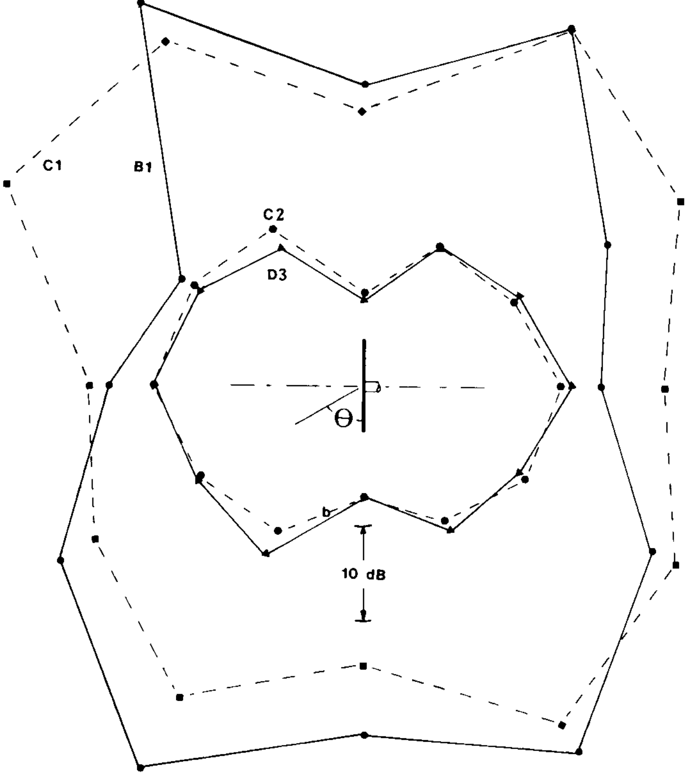

Figure 3—Comparison of directivity patterns: D(θ) at r = 1.52 m, and θ at 30° increment for Blade #B1 (–●–), #C1 (–■–), #C2 (–●–), and #D3 (–▲–)

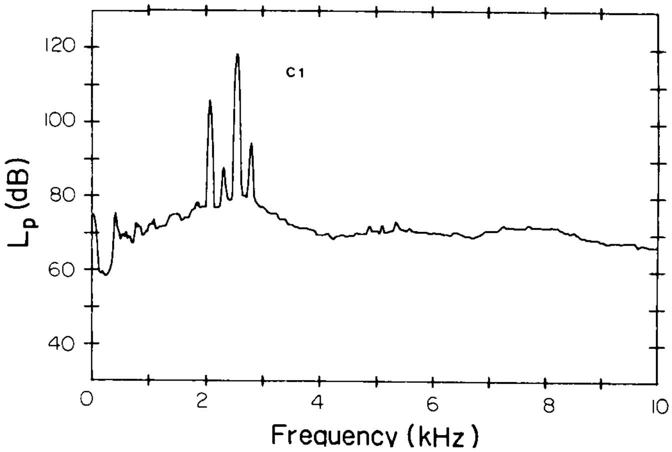

Figure 4—Narrowband spectrum of saw blade #C1 (with milled slots and unplugged holes); see Figure 2 caption and Table 1 for other details

Effect of Milled Slots. Figures 3 and 4 and Table 2 show the effect of 4 equally-spaced, radial-milled slots which terminate in holes [#C1, as shown in Fig. 1(a)]. We note a reduction in the overall L_{pA} and in the 2 kHz and 4 kHz octave band levels; and the directivity pattern is similar to that of #B1 except at θ = 120°. The narrowband spectrum as shown in Fig. 4, however, shows that the pure tones in the vortex shedding controlled frequency regime are now shifted from 2650 and 2825 Hz as seen for #B1 to 2075, 2325, 2525, and 2800 Hz. The results here can probably be explained as follows: since the radial slots split some of the repeated eigenvalues of the saw structural dynamics, as shown by Yu,[12] four peaks instead of two should be evident; see the section on structural modal results for more discussion. When these holes are plugged with copper inserts (#C2), the sound level drops drastically, as seen in Table 2, and all pure tones in Fig. 2 essentially vanish. The directivity pattern D(θ) shown in Fig. 3 is now more like the $\sin^2\theta$ shape of a point dipole.

Effect of Laser Cut Slots. Table 2 compares the effect of laser-cut slots (#D1-D9) with milled slots (#C1 and C2). Based on the measured data, it is evident that laser-cut slots,

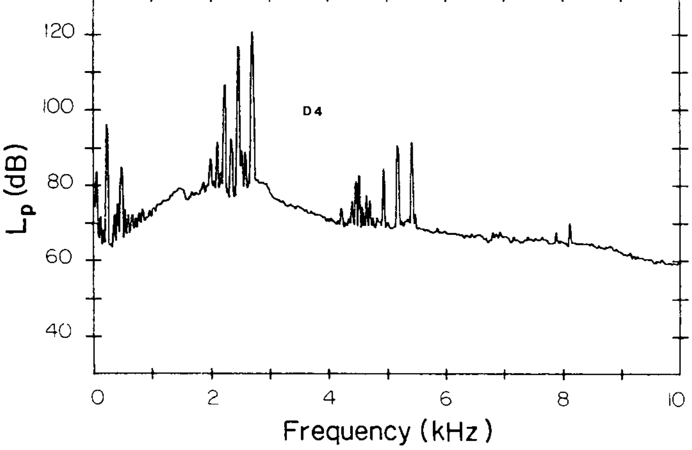

Figure 5—Narrowband spectrum of saw blade #D4; see Figure 2 and Table 1 for other details

with the exception of #D4, constitute a good engineering solution to the noise problem, as the low sound levels produced by radial, milled-copper plugged slots (#C2) are essentially duplicated. For instance, the spectra produced by #D3 and #C2 are virtually identical over the dominant 2 kHz range and beyond, as seen in Fig. 2 and Table 2. The results of #D4 illustrate that the length of the radial slots must be more than some critical length to achieve an adequate noise reduction. The narrowband spectrum in Fig. 5 shows that #D4 results are somewhat similar to the spectrum produced by #C1 in Fig. 4. We also note strong second harmonic components of the fundamental pure tones. It seems that in this case, the geometric discontinuities split vibration modes considerably without stabilizing the whistling tonal radiation mechanism which is still in the nonlinear regime.

An investigation of the number of radial slots, unequal spacing of slots, and a combination of short and long slots reveals that the noise results are almost identical to those seen for blade #D3. Thus, the geometric imperfections beyond those required to stabilize the whistling instability mechanism do not yield any further reduction in the radiated noise (see Table 2). This result is very encouraging since it suggests that many different slot patterns are suitable from the viewpoint of noise control. These slot patterns, however, may not be optimal with regard to cutting stability and vibration control.

Finally, it should be noted that the effect of adding radial slots to the blank (#A1) is to increase the sound level slightly, (see Table 2)—almost in the broadband manner. However, this blank (#A2) is still quiet and does not radiate any intense pure tones.

Structural Modal Results

The stationary circular saw without any slots can be analyzed theoretically as a thin circular plate clamped inside and free at the rim.[12,28,29] The natural frequencies (f_{mn}) and modes (ψ_{mn}) of undamped transverse vibrations can be determined for any integer combination of m and n, where m and n represent the number of nodal circles and nodal diameters, respectively. The mode shape form is given as: $\psi_{mn}(r,\theta) = R_{mn}(r)\cos[n(\theta - \phi_{mn})]$, where R_{mn} is given by the Bessel functions, r is the radial coordinate, θ is the circumferential coordinate, and ϕ_{mn} is the phase constant which locates the position of a nodal diameter. Because of the axisymmetric nature of the circular saw, repeated roots (with modes $\cos n\theta$ and $\sin n\theta$) are witnessed. According to Yu, the introduction of k equally-spaced, identical radial slots splits the modes with $n = kj/2$ nodal diameters into two distinct modes with fixed nodal lines where k is even, and $j = 1,2,\ldots$.[12] All other modes are repeated with ϕ_{mn} being arbitrary. When the saw rotates with ΩHz, the forward and the backward flexural wave natural frequencies are $f_{mn} \pm n\Omega$ in Hz. The distribution of vibratory response between these two waves depends on the excitation spectral density. However, in practice, either the forward or the backward component is excited.[12,17]

Since modal damping cannot be predicted, experimental modal analyses on two saw blades (#B1 and #D3) were conducted. The scope of the modal testing was deliberately limited, as the intent was to explore structural behavior for a clue regarding the damping introduced by radial slots. Initially, forty measurement locations were chosen for Blade #B1 on the outer circumference in order to identify the circumferential mode number n given by the number of nodal diameters. Then, a quarter segment of the blade ($\theta = 0$ to 90°) was examined with 16 points described along an arc and a radial (r) line in order to determine the radial pattern $R_{mn}(r)$. All measurements were conducted on a stationary blade mounted on the radial arm saw. An instrumented hammer and an accelerometer were used to acquire transfer functions at all locations, and a computer-based modal extraction program was used. Over the frequency range of interest (0 to 4 kHz), natural frequencies (f_{mn}), modal damping ratios (ζ_{mn}), and mode shapes of order (m,n) were determined. The same experiment was repeated for Blade #D3. The modes which are excited in the frequency range of interest (2500 to 2850 Hz) seem to be as follows: (m,n) = (0,7), (1,7), (0,8) and (1,8); i.e., there are seven or eight nodal diameters along with a possible nodal circle. The exact pinpointing was not possible with our coarse measurement grid.

Measured modal damping ratios ζ_{mn} are compared in Fig. 6 for stationary blades #B1 and #D3. The damping ratios as-

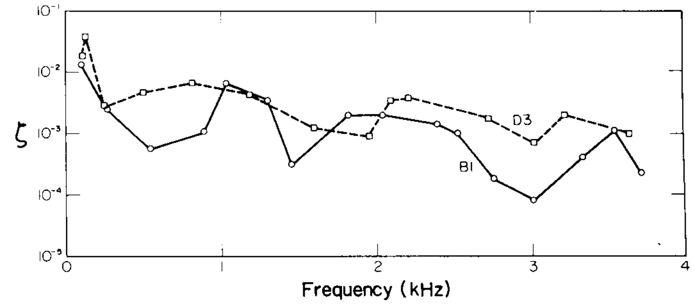

Figure 6—Modal damping ratios of saw blade #B1 (without slots —○—) and #D3 (with laser-cut slots -- □ --)

sociated with #D3 are higher, except over the 1 to 2 kHz range, than those observed for #B1. And, over the whistling noise frequency range, i.e., around 2800 to 3200 Hz, laser-cut slots increased ζ_{mn} by a factor of 7 to 10. Further, a qualitative estimate of the blade radiation was noted by striking the stationary blade with a hammer and then listening to the nature of the sound. The blade without any slots (#B1) emits essentially a distinct pure tone, and the blade with slots (#D3) radiates a not too distinct tone. These subjective observations are in agreement with measured damping data.

Discussion of Physical Mechanisms

Whistling Phenomenon. The generation of a discrete intense tone at the resonant frequency of an idling saw blade is usually referred to as the whistling phenomenon. A mathematical description of this phenomenon and of the precise conditions under which it occurs is not available in the literature.[3,4,14-22] However, the phenomenon is certainly due to a self-excited structure-acoustic instability mechanism of aerodynamic origin.

Many aerodynamic noise sources with discrete pure tones (aeolian tones) have been explained using the vortex shedding noise theory.[23,24,30] According to this, an aeolian tone is emitted when the frequency f_e of periodic vortices shed from the edges excites the blade resonance through a nonlinear locking on of f_e with $f_{mn} \pm n\Omega$. Several saw noise researchers believe this to be the case, even though all of the empirical data does not support this theory.[3,4,14-20] The characteristics of our measured noise data for #B1 exhibiting the whistling phenomena can be related to the vortex-shedding noise theory since: (1) the discrete tone frequency is close to f_e and the measured f_{mn}; (2) the magnitude of the pure tone is very large, suggesting a nonlinear feedback mechanism which is also evident from the harmonics seen in the data; and (3) the directivity $D(\theta)$ is due to the combination of the dipole source and the structural modal radiation characteristics.

The possibility of the laminar boundary layer instability in the form of Tollmien-Schlichting (T-S) waves[24] has been suggested by a few investigators to explain the emission of pure tones from airfoils and low speed fans.[32,33] This instability mechanism has not been explored for idling saws, as this phenomenon need not involve any blade resonances. Nonetheless, this concept can clarify the observation that the surface roughness introduced at the tooth base can suppress the whistling phenomenon by introducing turbulence.[3]

Mote and Leu believe that the actual instability mechanism in idling saws may be due to vortex shedding and galloping type vibrations, with possibly sustained wake oscillations.[17] Although the phenomenon is not well understood, the experimental evidence shows that a sufficiently damped saw blade does not radiate whistling sounds.[3,14,17]

Effect of Slots. While the need for radial slots from the viewpoint of thermal stress is well understood,[6,7] there is no consensus in the literature regarding their effects on structural vibrations, damping, dynamic stability, and radiated noise.[2-13] For instance, Barz finds milled radial slots ineffective and instead has recommended annular slots near the hub.[2] Stakhev is in agreement with Barz, as he has reported that the annular slots can raise the critical speed significantly.[5] Conversely, McKenzie states that uniformly-spaced narrow radial slots improve stability.[6] But Matsuhisa and Sato believe that the short rim slots are better than long slots.[11] Several investigators have found that long, narrow radial slots reduce whistling type noise.[4,7,9] However, Dugdale considers this effect to be uncertain unless a specific vibration mode is suppressed.[3] Recently, Yu has reported that radial slots split only some of the modes and that the slots could degrade the dynamic performance if the parametric resonances are excited.[12] Further, Mote has stated that the measured damping ratio range of saws with and without slots is about the same.[13]

The experimental results reported in Table 2 and in Figs. 2 and 3 clearly show that a number of slot patterns (milled or laser-cut) suppress the intense whistling sounds. Since an analytical treatment of this problem is very cumbersome, explanations based on the physics are offered; we propose a hypothesis to explain the effect of slots as follows: the geometric discontinuities disrupt the inherently unstable, nonlinear feedback mechanism by introducing some positive damping over the whistling phenomenon frequency range. Figures 2, 3, and 6 indicate that this may be the case. Further, Tobias and Arnold have claimed that the geometric imperfections in a rotating disk lead to higher aerodynamic damping.[34] Based on the measured damping values for #B1 and #D3, and considering the resonant vibrations of a harmonically excited linear system, one should expect a noise radiation of from 15 to 20 dB associated with radial slots. But we find a reduction of about 40 dB around the resonant peak in the noise data given in Fig. 2, which suggests that the whistling mechanism is of the non-linear feedback type. While the structural damping mechanism is not clear, it is possible that the modal damping ratio ζ_{mn} is higher due to the localized vibration effects (which may increase the modal damping coefficient c_{mn} or reduce the modal stiffness K_{mn}). Further work is required to clarify this issue.

Since the whistling mechanism itself is not well understood, one can speculate further regarding the effect of slots: (1) the slots inhibit the formation of T-S waves if these are, indeed, responsible; (2) the acoustic radiation efficiency of the saw is reduced if the slots lessen the formation of circumferential bending waves and their coupling with the acoustic medium; and (3) the slots induce a spatial incoherence between radial shear stress dipoles distributed over the surface and possibly between normal dipoles distributed over the edges. The third possibility can explain the spectral changes seen, since the uncorrelated sources should generate a lower mean spectral value with a large spectral variance. Further research is needed to resolve these issues.

Concluding Remarks

Our experimental study has shown that milled or laser-cut radial slots can suppress intense whistling sounds radiated from idling blades. One can choose a configuration from a variety of possible slot patterns for essentially the same noise reduction. This aspect is especially attractive, as one must also optimize slots for vibration control and stability. While the results have been given for one sample case, these could be extended to other rotary cutters. However, one must exercise caution since, in some cases, the slots may enhance rather than suppress the whistling mechanism. Nevertheless, the engineering solution presented here for noise control is very promising, as any desirable slot pattern can be completely cut along with the blade by using the computer-aided laser machine—which obviously is an extremely efficient and cost-effective manufacturing solution.

Possible explanations of the effect of slots on idling saw noise have been given. These should be treated as conjectures since the whistling sound emission mechanism itself is not well understood. Thus, further research is required in order to clarify the various issues raised in this study.

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