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EFFECT OF FRICTION ON THE DYNAMIC
RESPONSE OF AN ACTUATOR

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ABSTRACT

Friction at the piston-cylinder interface is fairly complex and poorly understood. Consequently, macroscopic friction models such as the Coulomb friction and/or empirical models are generally used for the dynamic analysis of fluid power control systems. In this paper, a new semi-empirical mathematical model for friction at the piston-cylinder interface is presented. The model is composed of elastic, viscous, and sliding components which are related to geometry and mechanical properties, and fluid conditions. Using this model, various cases of friction behavior are established and compared to reported behavior in the literature.

The proposed model is also used to examine the effect of various friction parameters on the static and dynamic response of a fluid power (hydraulic) actuator. The behavior is examined numerically for step and periodic pressure excitations.

1. INTRODUCTION

Recently we studied the mechanical friction at the piston-cylinder interface (Figure 1), and developed a semi-empirical mathematical model [1]. The friction force F_T is composed of the following three components: (i) elastic component due to the metallic contact, asperity interlocking and elastic deformation of the surface films, $F_e = F_o e^{-\xi|V|}$ sign (V), (ii) viscous component due to the shear of fluid lubricant $F_s = \delta U$, $\delta = -2\pi \nu L/\ln(k)$, and (iii) sliding friction force component due to the gradient of pressure (p), across the clearance space

$$F_p = A_p \Delta p, \ A_p = \pi Ri^2, \ \gamma = k^2 + \frac{1-k^2}{2k\ln(k)}$$

Accordingly the total friction force is

$$F_T = F_e + F_s + F_p = (F_o \text{ sign } (V) + \delta V) e^{-\xi|V|} + \gamma A_p \Delta p \tag{1}$$

2. VARIOUS FRICTION CASES

In practice, the lubrication regime changes continuously and the friction parameters vary accordingly. Consequently F_T versus V relationship is fairly complex; Table 1 and Figure 2 show various cases of friction force behavior. For highly viscous lubricants (U=V), the friction force is

$$F_T = F_o e^{-\xi|V|} \text{ sign}(V) + \delta V + \gamma A_p \Delta p \tag{2}$$

Expanding $e^{-\xi|V|}$ sign (V) using Taylor series expansion, gives a linearized model

$$F_T = F_o \text{ Sign}(V) + (\delta - \xi F_o) V + \gamma A_p \Delta p \tag{3}$$

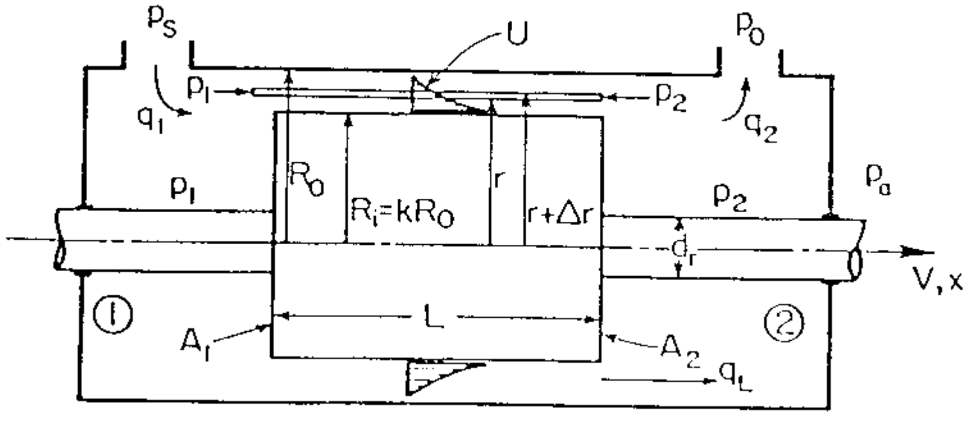

Figure 1. Example case: hydraulic actuator illustrating piston cylinder interface.

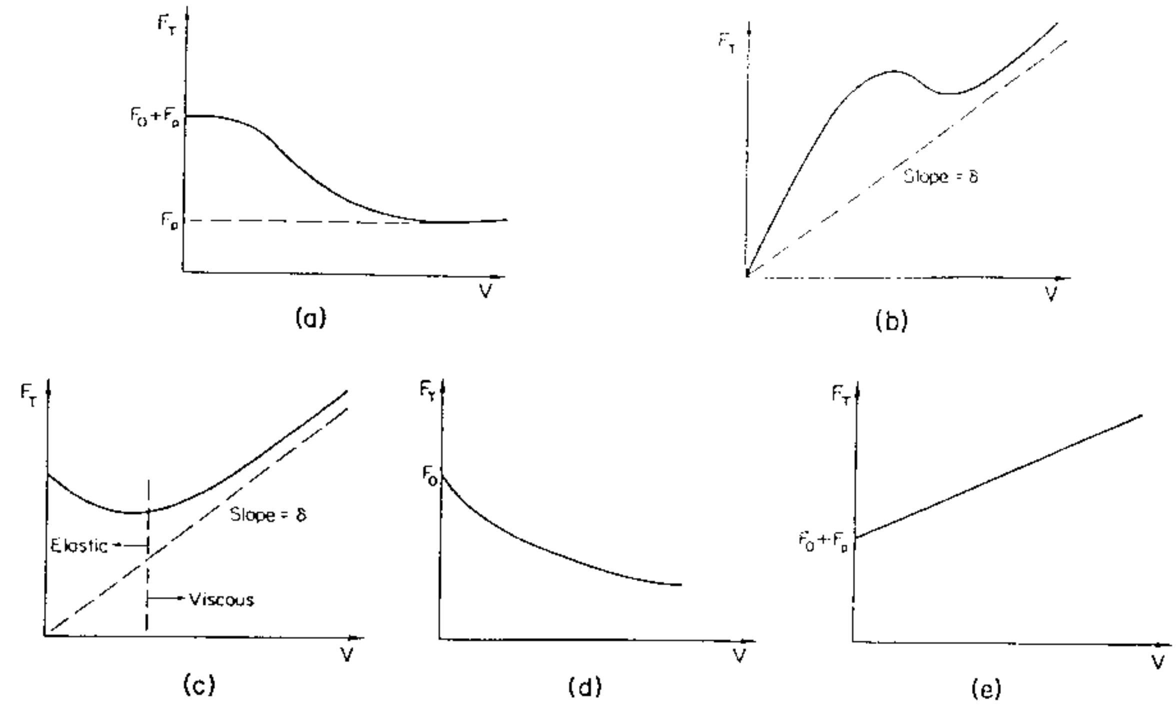

Figure 2. Various cases of friction model, (a) Lag effect on friction. (b) Transformed lubrication conditions. (c) Transfer from elastic to viscous contact. (d) Elastic contact. (e) Viscous contact.

Table 1. Various Cases of Friction Model.

Case	Proposed Model	Reference and Comment
Dry friction contact only Fig 2-d	$F_T \simeq F_e - F_o e^{-\xi\|V\|} \text{sign } V$	[2]
Sliding friction only	$F_T \simeq F_s - \gamma A_p \Delta p$	[3]
Ideal viscous lubricant $U = V$ Fig. 2-c	$F_T = F_o e^{-\xi\|V\|} \text{sign } V + \delta V + \gamma A_p \Delta p$	Experimental data [4]
Visco-elastic contact $U = V e^{-\xi\|V\|}$ Fig. 2-a	$F_T = (F_o \text{ sign } V + \delta V) e^{-\xi\|V\|} + \gamma A_p \Delta p$	[5] Empirical model of the form $e^{-a_3\|V\|}(a_1 + a_2 V) + a_4$
Constrained metallic contact $F_o = 0$ Fig. 2-e	$F_T = \gamma A_p \Delta p + \delta V$	Here $F_T = a + bV$, a form commonly used in fluid power system analysis [3,4]

3. DYNAMIC ANALYSIS USING LINEAR FRICTION MODEL

Consider a piston cylinder assembly with total mechanical stiffness K_o and damping coefficient c; these lamped parameters can be either associated with load and/or may represent the inherent dynamic properties of the piston-rod assembly. The governing equation of motion for the piston mass is

$$A_p \Delta p - F_T = M\ddot{x} + c\dot{x} + K_o x \text{ , } A_p = \pi R_f^{\,2} \tag{4}$$

Taking Laplace transform of Equation (4), setting initial conditions to zero and substituting the Linearized form of F_T given by Equation (3) gives the following steady state equation

$$(Ms^2 + (c+\delta-\xi F_o)s + K_o)x = A_p(1 - \gamma)\Delta p - F_o \text{Sign}(sx), \; A_p = A_e \tag{5}$$

Figure 3 shows the block diagram of this equation, where

$$(x/\Delta p)(s) = K/(s^2/w_n^{\,2} + 2\zeta/w_n + 1) \tag{6}$$

$$(x/F_o)(s) = - K'/(s^2/w_n^{\,2} + 2\zeta/w_n + 1) \tag{7}$$

where $K = A_p(1-\gamma)/K_o$ and $K' = 1/K_o$ are the system gains, $w_n = (K_o/M)^{1/2}$ is the equivalent natural frequency of the system and $\zeta = (c+\delta-\xi F_o)/2(K_oM)$ is the equivalent damping ratio. It is obvious from Figure (3) that the friction force plays the role of a feedback disturbance. The system is stable for $\zeta > 0$; hence one must increase the system damping coefficient c and reduce the parameter F_o, to improve stability. The critical value of the viscous and inertial Lag is $\xi_{cr} = (c+\delta)/F_o$. Also note that the motion of the piston cannot be initiated unless $\Delta pK \geq K'F_o$; hence $\Delta p_{min} = F_o/A_p(1-\gamma)$.

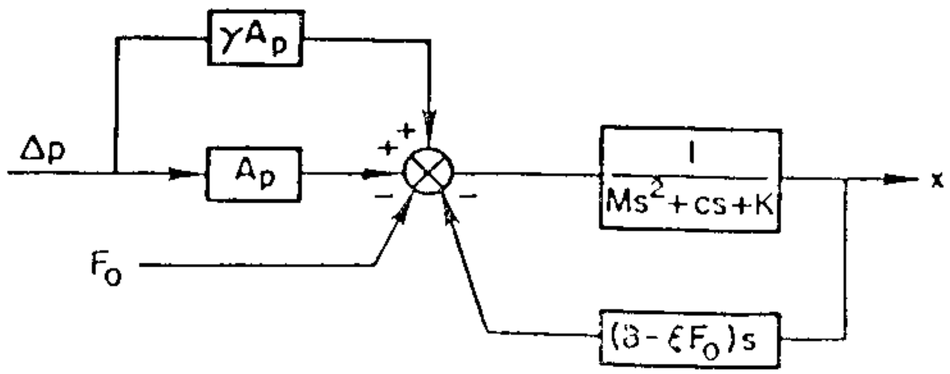

Figure 3. Block diagram of actuator dynamics.

For the hydraulic actuator example case shown in Figure 1, the input pressure differential, $\Delta p_I = p_s - p_o$, is assumed to be the excitation. To develop the governing system equations, several assumptions are made. First, the only significant leakage is the internal leakage from the high pressure chamber 1 to the low pressure chamber 2. Second, the fluid medium is incompressible with infinite effective bulk modulus, and all fluid properties are uniform throughout chambers 1 and 2. Third, only the friction at the piston cylinder interface is considered in the analysis. Linearized flow equations for chambers 1 and 2 and for ports are

$$q_1(t) - q_L(t) = A \frac{dV_1}{dt} = \pi \frac{d^2}{4} \dot{x}(t) \tag{8}$$

$$q_I(t) - q_2(t) = A \frac{dV_2}{dt} = - \pi \frac{d^2}{4} \dot{x}(t) \tag{9}$$

$$p_s(t) - p_1(t) = R_1 q_1(t) \tag{10}$$

$$p_1(t) - p_2(t) = R_L q_L(t) \tag{11}$$

$$p_2(t) - p_o(t) = R_2 q_2(t) \tag{12}$$

where R_1, R_2, and R_L are fluid resistances of the left and right ports and of the clearance space at the interface, respectively. They are assumed to be given by the capillary tube resistance equation. Equation of motion for the piston, assuming very stiff rod, is as follows

$$p_1 A_1 - p_2 A_2 - F_T = M\ddot{x}, \; A_1 = A_2 = A_p - A_r \tag{13}$$

Consider $A_r \ll A_p$, substitute for F_T (Equation (2)) and take Laplace transform and then set initial conditions to zero.

$$p_1 A_p - p_2 A_p - \gamma p_1 A_p + \gamma p_2 A_p = Ms^2 x + \delta s x + F_o e^{-\epsilon |sx|} \quad \text{sign}(sx) \tag{14}$$

Using Equations (8) through (14) and defining $R_1 = R_2$,

$$\frac{R_1 + R_L}{R_L} = \alpha, \; \frac{R_1}{R_L} = \beta \text{ and } A_p \; (1 - \gamma) = B; \text{ gives}$$

$$R_1 A_p s x + \frac{\alpha + \beta}{2} \; (Ms^2 + \delta s x + F_o e^{-\epsilon |sx|} \; \text{sign}(sx)) = (p_s - p_o)/2 \; . \tag{15}$$

Equations (8) through (15) can be solved now for six unknowns of the system.

4.1 Linear Friction Model

To observe the effect of the various friction parameters on the dynamic behavior of the actuator, the linearized Force F_T (Equation (31)) is substituted in Equation (15) and after some manipulation gives

$$(\dot{x}/\Delta p_i)(s) = K_1/(\tau_1 s + 1), \quad (\dot{x}/F_o)(s) = -K_2/(\tau_1 s + 1) \qquad (16)$$

$$p_1(s) = \frac{1}{2}(p_s + p_o) + [K_3(\tau_2 s + 1)/(\tau_1 s + 1)]\Delta p_i + (K_4(\tau_3 s + 1)/(\tau_1 s + 1))F_o \qquad (17)$$

$$p_2(s) = \frac{1}{2}(p_s + p_o) - [K_3(\tau_2 s + 1)/(\tau_1 s + 1)]\Delta p_i - (K_4 \cdot (\tau_3 s + 1)/(\tau_1 s + 1))F_o \qquad (18)$$

where

$$K_1 = 1/[2R_1 A_p + (\alpha + \beta)/2\eta \quad (\delta - \xi F_o)] \qquad (19)$$

$$K_2 = ((\alpha + \beta)/2\eta)/[R_1 A_p + (\alpha + \beta)/2\eta \quad (\delta - \xi F_o)] \qquad (20)$$

$$K_3 = (\delta - \xi F_o)K_1/\eta \qquad (21)$$

$$K_4 = (\tfrac{1}{2}\eta)/[1 - K_2(\delta - \xi F_o)] \qquad (22)$$

$$\tau_1 = ((\alpha + \beta)/2\eta)(M/[R_1 A_p + (\alpha + \beta)/2\eta \quad (\delta - \xi F_o)]) \qquad (23)$$

$$\tau_2 = M(\delta - \xi F_o)/\eta \qquad (24)$$

$$\tau_3 = (\tau_1 - K_2 M)/[1 - K_2(\delta - \xi F_o)] \qquad (25)$$

4.2 Numerical Solution of Nonlinear Friction Model

Assume the resistance ratios as $\alpha = 0.07$, $\beta = 1.07$, the friction-pressure ratio as $|\gamma| = 0.04$ (k = 0.98). Consider an actuator of R_i = 16 mm, L = 20 mm, and operating with mineral oil. CSMP simulation program is used to solve the nonlinear differential equations for the time response of the hydraulic linear actuator for step and periodic excitations. The left chamber pressure response $p_1(t)$, the velocity of the piston $\dot{x}(t)$ and the friction force of the piston cylinder interface $F_T(t)$ are studied.

4.2.1 Step Input. For a step pressure applied at the ports of the actuator, the system response is as shown in Figure 4. The actuator behaves as a first order system, as predicted with a time constant τ_1 = 0.22 sec. The effect of system parameters on the dynamic characteristics are listed in Table 2.

The effect on the speed of response can be explained on the basis of the linearized model. For instance the reduction of ξ increases the effective stiffness of the actuator and thus reduces its speed of response. Similarly, the reduction of δ is accompanied with an increase in the gain K_1 and the time constant τ_1 of the system, as indicated by Equations (19) and (20).

The effect of the various friction parameters on the system characteristics is shown in Table 3. The effect of these parameters is small on the piston velocity and the chamber pressures. However, as these parameters become more significant, their effect on the system gain may be more noticeable.

Despite the nonlinear nature of the system, one may deduce a resultant viscous coefficient of friction, μ, from the friction velocity curve which is almost linear; slope of the curve (μ) is approximately 0.104. Hence, the friction force can be given by a linear equation $F_T = C_1 + C_2V$, where C_1 is the static friction force and $C_2 = \mu$. In British units, for the current example, $C_1 = 0.85$ and $C_2 = 0.104\ lb_f\ sec/in$.

Table 2.
Effect of Friction Parameters on Time Constant τ_1

Case	M=0.22 ξ=0.1	M=0.44 ξ=0.1	M=0.22 ξ=0.001	δ=o	F_o=o	η=o
τ_1(sec)	0.22	0.45	0.47	0.30	0.171	0.188
	Fig. 4	Fig. 5a	Fig. 5b	Fig. 5c	Fig. 5d	Fig. 5e

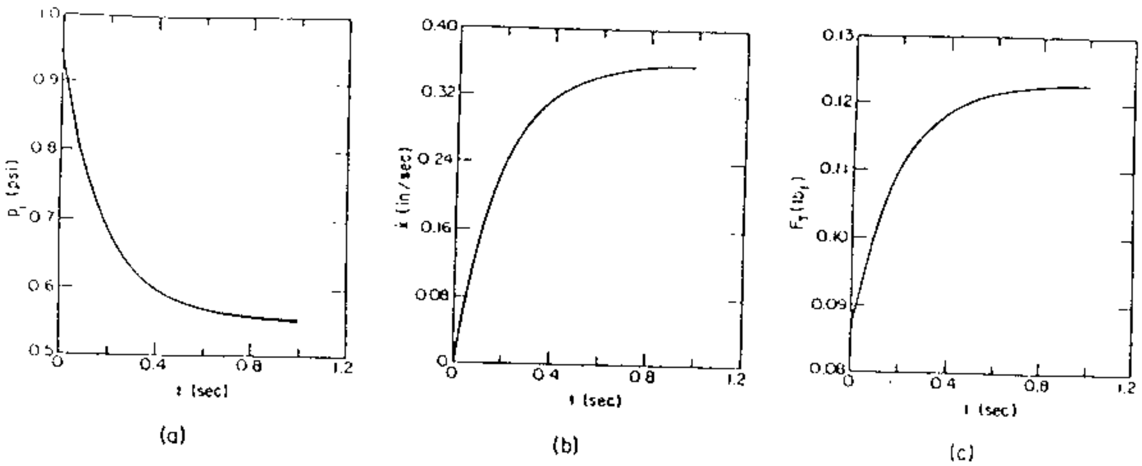

Figure 4. Step input system response (a) $P_1(t)$. (b) $x(t)$. (c) $F_T(t)$.

Table 3
Effect of Friction Parameters on Dynamic Characteristics of Linear Actuator (Based on Step Response Results)

The Increase of	(Time Constant)	Steady State Gain In Friction	Steady State Gain In Velocity
M	Increases	Has No Effect	Has No Effect
ξ	Increases	Decreases	Increases
δ	Decreases	Decreases	Decreases
F_o	Increases	Increases	Decreases
η	Increases	Increases	Has No Effect

4.2.2 Periodic Excitation. When the actuator is excited by a periodic square wave pressure input, the corresponding system response is as shown in Figure 6. The behavior of p_1 during each half cycle shows an exponential decay, typical for a first order system. The friction curve behavior can be explained on the basis of the friction force model and the pressure response in the chambers. By referring to the velocity and pressure response curves in Figure 6, one can see that at time t = 0 $F_T = F + A_p \Delta p$, ($\gamma < 0$). At time equal to the half period, the velocity is again zero but Δp reaches its minimum during the half cycle. This causes an increase in the friction force at the end of each stroke. Such behavior is generally described by the "stick-slip" phenomenon or the "sticking" effect.

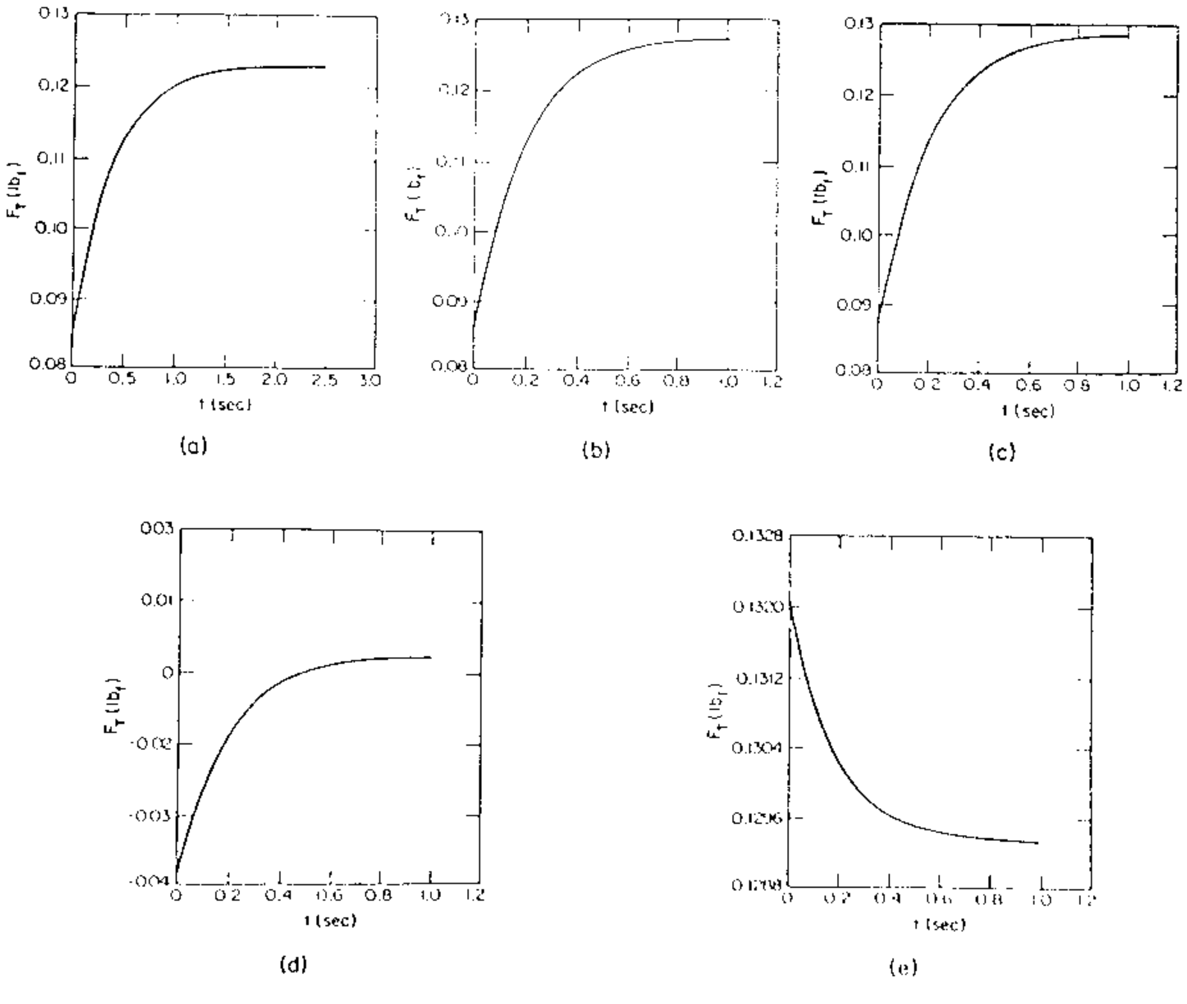

Figure 5. Effect of Parameters on Friction Response (a) (M=.44). (b) (ξ=.001). (c) (δ=01). (d) (F_o=0). (e) (γ=0).

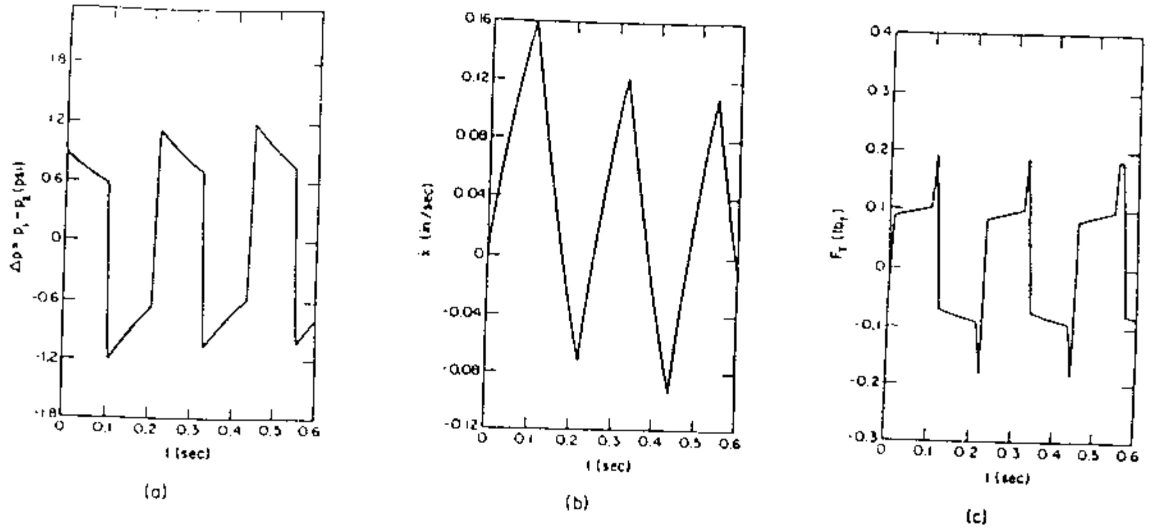

Figure 6. Periodic input system response. (a)$\Delta p(t)$. (b) $x(t)$. (c) $F_T(t)$.

The friction velocity behavior is shown in Figure 7 for two cases of periodic excitations: square wave and sinusoidal. The predicted nonlinearity and hysteresis effect on the dynamic response can be observed from the noncoincidence of the loading and nonloading curves. The effect of hysteresis is physically attributed to the imperfect elasticity of the materials of contacting bodies. It is also physically associated with the nonequilibrium nature of the process of viscoelastic deformation of the medium. This effect depends on the rheological properties of the medium and its viscous and damping properties which are related to the parameter ξ [5].

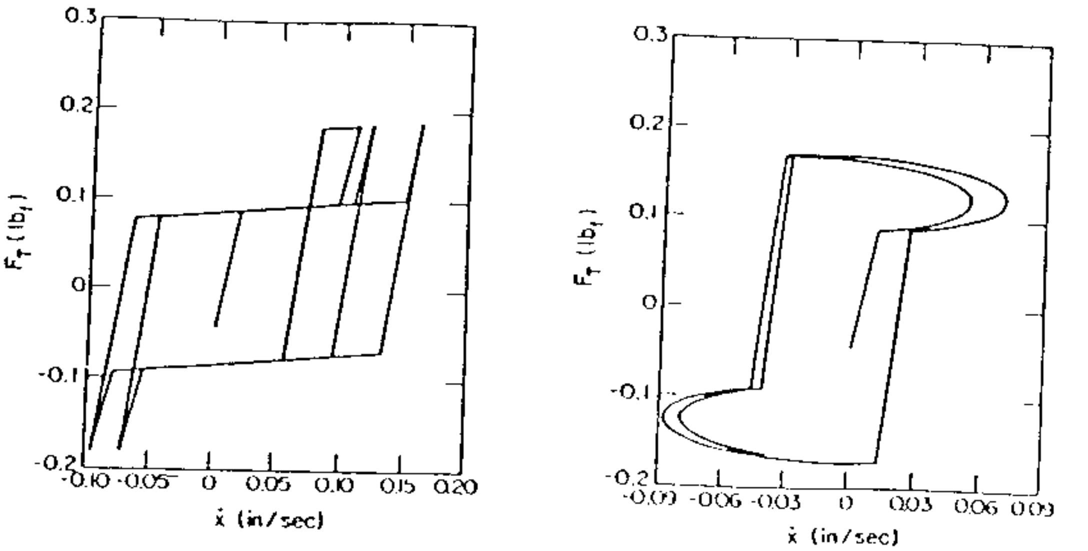

Figure 7. Friction vs. Velocity Curve. (a) Square wave pressure input. (b) Sinusoidal pressure input.

5. CONCLUDING REMARKS

A semi-empirical model for the friction at the piston-cylinder interface was developed based on the mechanism of viscoelastic deformation. The proposed model provides the explanation of the various observed behavior of friction on the basis of friction parameters and contact regime. It also provides more information and explanation as to the effect of friction disturbance on the static and dynamic response. Further work including an experimental study is in progress to validate the proposed model.

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