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State Space Modeling of Refrigeration Compressor Shell Dynamics

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ABSTRACT

A structural synthesis method based on state space mathematics is described. This state space method (SSM) is used to model the free vibrations of a refrigeration compressor shell. Five models that progressively approximate compressor shell geometry are analyzed. The results are compared with experimental data and bounded analytical solutions. The SSM is then employed to investigate the dependence of compressor shell natural frequencies on the location of the welded seam, the shape of the end caps, and the introduction of curvature in the cylindrical body. Some of the claims made by earlier investigators have been critically examined. These studies can lead to a stiffer design of the compressor shell.

INTRODUCTION

Structures composed of shell elements (cylinders, spheres, cones, etc.) find a wide range of application in such areas as refrigeration systems, pressure vessels, nuclear reactors, aircraft, and rockets. One typical example is a welded compressor shell, which is the subject of this paper. These structures offer challenging analysis and design problems, not only due to the degree of complexity of the governing shell equations, but also due to the difficulties associated with matching the boundary conditions between the substructures. Consequently, analytical solutions for such structures have been inadequately investigated, and their dynamic behavior and design techniques for reduced noise and vibration are poorly understood.

An acute need exists for methods of synthesis applicable to compressor shell structures, where recent efforts have concentrated on reducing compressor noise in the middle- to high-frequency range (say, 800 Hz to 3000 Hz) by modifying the shape of the compressor shells. Work in this area has been primarily of the trial-and-error and intuitive-reasoning nature with little analytical work or systematic experimentation. For instance, Lowery (1984) proposed that the stiffness of a compressor shell may be increased simply by reducing the abrupt curvature changes throughout the shell. A spherical shell would be the limiting case of this proposition. Saito et al. (1980) proposed the use of geometric asymmetry to separate those normal modes that would otherwise be amplified effectively in an axisymmetric shell. Tojo et al. (1980) also claimed that making the compressor shell nearly spherical increases its stiffness. Unfortunately, these authors have provided limited experimental data (sometimes in the form of overall radiative sound power) in support of their work.

A review of the literature pertinent to shell structures (Tavakoli 1987; Tavakoli and Singh 1988) reveals that methods based on the classical bending theory of shells (Kalnins 1964; Hu and Raney 1967; Smith and Haft 1967), Rayleigh's inextensional shell theory (Saunders and Paslay 1959), the receptance technique (Faulkner 1969; Soedel 1981), and the Lagrangian formulation (Hirano 1969; Takahashi and Hirano 1970; Takahashi and Hirano 1971; Takahashi et al. 1970; Takahashi et al. 1981; Takahashi et al. 1982; Takahashi et al. 1984; Takahashi et al. 1985; Suzuki et al. 1982a and b; Suzuki et al. 1983; Suzuki et al. 1984; Kosowada et al. 1983; Kosowada et al. 1984) have been used to study the dynamics of various shell structures. More recently, Irie et al. (1982; 1984a and b) used a method based on the transfer matrix approach to analyze cases such as a cone with a variable thickness, and a cone-cylinder combination. In this method, the governing equations for a shell are reformatted into a system of eight first-order differential equations. These equations are then put into a state space formulation, and the transfer matrix relating the state vectors at different locations along the structure is computed. Yamada et al. (1984; 1986) used this method to study the vibrations of cylindrical shells with noncircular cross sections and also a cylindrical double-shell system closed by annular end plates. The state space method (SSM) shows a good degree of versatility and ease of application. Tavakoli (1987) and Tavakoli and Singh (1989) verified the method by applying it to various basic structural elements (cylinder, cone, sphere, etc.) whose solutions are well known. They also demonstrated the applicability of the

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SSM to several more complicated shell structures including a hermetic capsule (hemisphere-cylinder-hemisphere combination) and a refrigeration compressor shell.

In this paper, the SSM is used to model the free undamped vibrations of a typical refrigeration compressor shell. Five models that progressively approximate the geometry of the compressor shell are analyzed and compared with experimental data and with bounded analytical solutions. Conclusions are drawn concerning the effects of the end cap on the natural frequencies of the compressor shell. Further, the dependence of the natural frequencies on the location of the welded seam and on the introduction of curvature in the cylindrical body is investigated. Based on these studies, and within the scope of this work, recommendations for improved design are made.

STATE SPACE METHOD (SSM)

For an axisymmetric shell element, such as the cylinder shown in Figure 1, the governing thin shell equations can be rearranged in a system of eight first-order differential equations:

$$\{\phi(x)\}' - [U]\{\phi(x)\} = 0 \qquad (1)$$

where x is the coordinate that spans the substructure from one boundary to the other, ( )' indicates first derivative with respect to x, $[U]$ is the substructure coefficient matrix containing information about its geometry, material properties, and frequency of vibration ω; and $\{\phi(x)\}$ represents the state vector whose elements are the relevant variables at x boundary:

$$\{\phi(x)\}^T = <W_1, W_2, W_3, \beta_x, M_x, N_x, V_x, S_{x\theta}> \qquad (2)$$

Tavakoli (1987) and Tavakoli and Singh (1989) have derived expressions for the coefficient matrix $[U]$ for various standard shell elements (plate, cylinder, cone, sphere, toroid) using Love's thin shell equations.

A cylinder is a space-invariant substructure because none of the elements of its coefficient matrix depends on the location along the substructure. For such a substructure, the solution to Equation 1 is simply:

$$\{\phi(x)\} = [\exp(\textstyle\int_x [U]\,d\tau)]\,\{\phi(x_0)\} \qquad (3)$$

This solution shows how the state vector at an arbitrary x

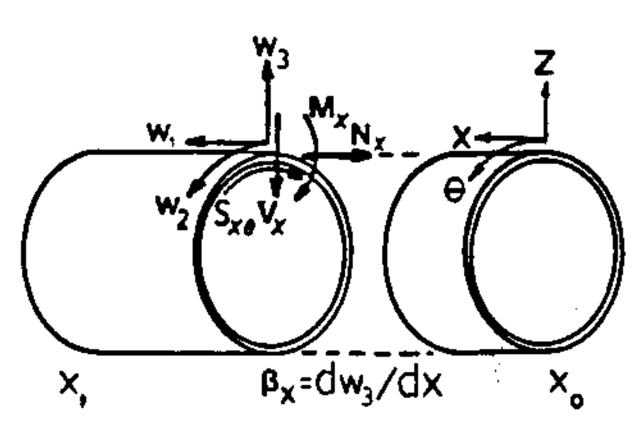


Figure 1 *A circular cylindrical shell*

can be related to the state vector at the boundary x_0 via the transfer matrix $[T(x_0,x)]$ between these two points where:

$$[T(x_0,x)] = [\exp([U](x - x_0))] \qquad (4)$$

The exponentiation method adopted here is based on the eighth order Pade' approximation (Trujillo 1975) inspired by its usage in the commercially available software CTRL-C (CTRL-C User's Guide). Discussing the advantages of using the Pade' method instead of a numerical integration method, such as the Runge-Kutta technique, is beyond the scope of this paper and hence will be discussed in a future paper.

A substructure such as a spherical dome is space-variant because most of the elements of its coefficient matrix depend on the location along the substructure. Under such a condition, the solution to Equation 1 is no longer given by Equation 3 except for the rare case where [U] is commutative (Derusso et al. 1967). However, in order to still be able to employ the convenient Pade' method, it is proposed to numerically discretize a space-variant substructure into the intervals of size h over the domain x_0 to x. Then, over each step, it is assumed that Equation 3 would still hold true as the solution to Equation 1. Mathematically:

$$\{\phi(x)\} = [T(x-h,x)]\,\{\phi(x-h)\} \qquad (5)$$

where the transfer matrix for each step h is defined as:

$$[T(x-h,x)] = [\exp(\textstyle\int_x^x - h[U(\tau)]\,d\tau)] \qquad (6)$$

and the overall transfer matrix between x_0 and x is given by:

$$[T(x_0,x)] = [T(x-h,x)]\,[T(x-2h,x-h)]\ldots[T(x_0,x_0+h)] \qquad (7)$$

Substructures are joined together by appropriately matching their common boundary state vectors. For instance, at the junction of a hemisphere and a cylinder, one can relate the initial boundary vector of the cylinder $\{\phi(x_{0c})\}$ to the final boundary of the hemisphere $\{\phi(x_{1s})\}$ as:

$$\{\phi(x_{0c})\} = [C]_{c,s}\,\{\phi(x_{1s})\} \qquad (8)$$

where subscripts s and c denote hemisphere and cylinder, respectively. The nonzero normalized elements of the substructure coupling matrix $[C]_{c,s}$ are (Tavakoli 1987; Tavakoli and Singh 1988):

$$C_{11} = C_{22} = C_{33} = C_{44} = 1;$$
$$C_{55} = C_{66} = C_{77} = C_{88} = D_s/D_c \qquad (9)$$

Finally, the overall transfer matrix of a model is obtained by properly multiplying the transfer matrices of the individual substructures and their respective coupling matrices. The eigenproblem is then arrived at by appropriate partitioning of the overall transfer matrix (Pestel and Leckie 1963; Tavakoli 1987).

COMPRESSOR SHELL ANALYSIS

Experimental Analysis

Figure 2 shows the cross-sectional view of the refrigeration compressor shell of interest with properties:

TABLE 1
Experimentally Identified Natural Frequencies and Modes

f (Hz)	mode (m,n)
867, 880	mode (0,3)
1090, 1106	mode (0,2)
1143, 1158	mode (0,4)
1551, 1558, 1649	bottom modes
1765, 1838, 1888	m = 1 modes

E = 207 GPa, ϱ = 7800 kg/m^3, and ν = 0.3. While various shell elements are identified in this figure, some of the local attachments (pipe, tube, valve, etc.) are not shown. Also, the internal pump-motor assembly was taken out for the experimental modal analysis. The measured data were collected by exciting the structure (with air inside) with a shaker fed with a random signal. Over the frequency range of 0 to 2000 Hz, 12 resonant frequencies were detected, which are given in Table 1. Further investigation established that the first six frequencies represent only three circumferential modes, since the well-known modal splitting phenomenon for nearly axisymmetric structures (Soedel 1980) was observed. The correlation between the natural modes and frequencies is also given in Table 1.

Natural frequencies identified as "bottom modes" correspond to modes that showed significant amplitudes in the bottom half of the compressor below the welded seam and showed very little motion in the top half. Also, the end caps did not seem to have any significant vibrations for any of the resonances detected in this range.

State Space Analysis

For comparative purposes, five models of the compressor shell are analyzed using the SSM. These models progressively approximate the compressor shell geometry. Model A is simply a shear diaphragmed circular cylinder, as shown in Figure 3a. This represents the simplest and most popular approximation of the compressor shell (Arnold and Warburton 1953; Brookbank 1978; Faulkner and Hamilton 1971). Model B is a circular cylinder closed with circular end plates, as shown in Figure 3b. This represents the simplest hermetic model of the compressor shell. The circular cylinder closed at its ends with spherical domes, shown in Figure 3c, is Model C, which represents a more realistic model of the compressor shell. In Model D, the transition between the cylinder and the spherical dome of Model C is modified by inserting a conical section between the respective substructures, as shown in Figure 3d. This is done to observe the effect of a more gradual transition between the cylinder and the spherical end cap on the natural frequencies and mode shapes. Finally, in Model E, the end caps of the compressor shell are modeled exactly by inserting a toroidal segment between the cylinder and the spherical dome of Model C, as shown in Figure 2.

In its present status, the SSM requires the overall structure to have two independent boundaries so that the global coordinate x_g can span the structure from one boundary to the other in an uninterrupted fashion. However, hermetic models B through E lack any boundaries. Hence, a boundary is created in the form of a free "pinhole" at the center of each end cap/plate. The authors (Tavakoli 1987; Tavakoli and Singh 1988) have shown that the numerical stability of the SSM allows pinholes small enough that the generated eigenvalues are within 1% of the theoretical values for the complete geometry. Nevertheless, the pinhole is believed to introduce error in the moment/force distribution results in the proximity of the pinhole.

Further, the computational task may be reduced if only a half-model is analyzed, and proper boundary conditions are enforced at the midsection of the model according to:

$\{\phi(x_{0s})\}^T = <0,0,0,0,M_x,N_x,V_x,S_{x\theta}>$ at the pinhole;

$\{\phi(x_{1w})\}^T = <W_1,0,0,\beta_x,0,0,V_x,S_{x\theta}>$ for antisymmetric modes; and

$\{\phi(x_{1w})\}^T = <0,W_2,W_3,0,M_x,N_x,0,0>$ for symmetric modes (10)

where $\{\phi(x_{1w})\}$ represents the state vector at the middle of the welded seam.

Results

In Table 2, SSM results for the lowest natural frequency corresponding to the first five circumferential mode numbers of these models are compared with the experimental results presented earlier. Since each measured circumferential mode corresponds to a pair of natural frequencies, the average of each pair of these split frequencies is listed as the experimental value. These results are compared graphically in Figure 4a. In Figure 4b, natural frequencies corresponding to the n = 0 and 1 modes, which represent the predominantly end plate/cap modes, are excluded for better comparison of the five models and the experimental results. Also included in these figures are

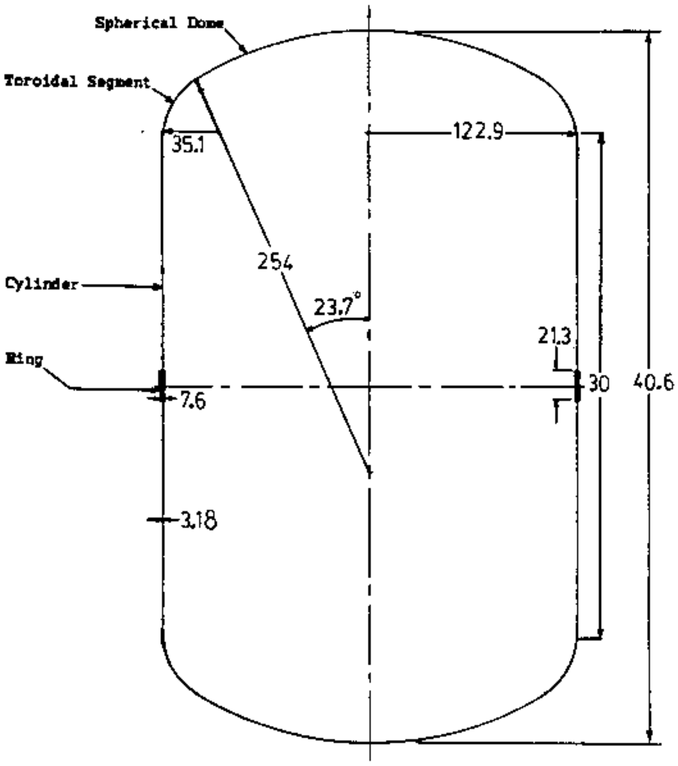
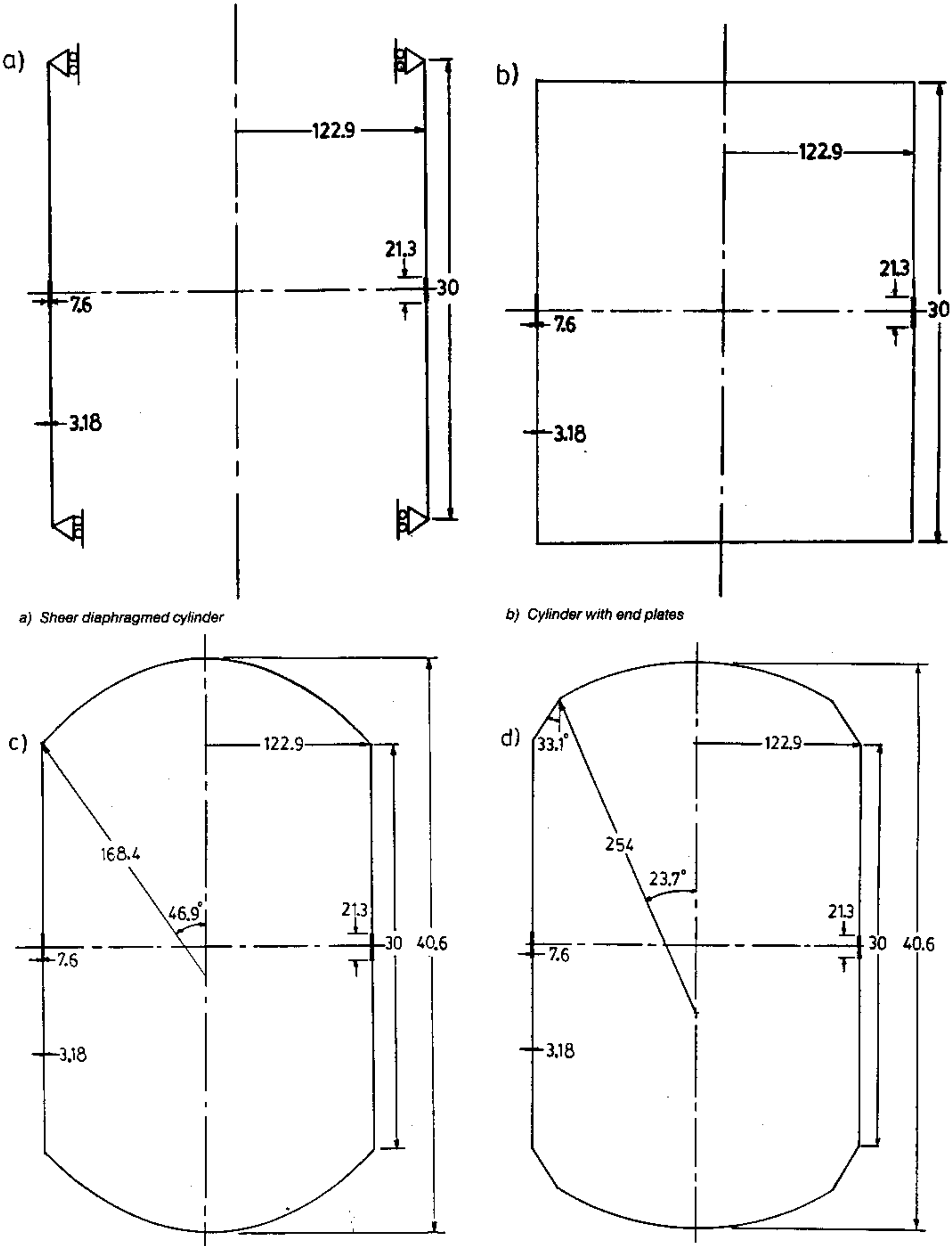


Figure 2 *Hermetic refrigeration compressor shell (Model E) — (all dimensions in mm)*



a) Sheer diaphragmed cylinder

b) Cylinder with end plates

c) Cylinder with spherical end caps,

d) Cylinder with modified end caps — (all dimensions in mm)

Figure 3 *Refrigeration Compressor shell Models*

TABLE 2
Compressor Shell Models:
Comparison of Natural Frequencies (Hz) between SSM and Experiment (EXP)

n	EXP	MODEL A	B	C	D	E	Δ_A%	Δ_B%	Δ_C%	Δ_D%	Δ_E%
0	—	6394	477	3192	2675	2529	—	—	—	—	—
1	—	3050	1039	3539	3315	3132	—	—	—	—	—
2	1098	1644	1447	1330	1291	1236	50	32	21	18	12
3	874	1121	1098	1041	1012	964	28	26	19	16	10
4	1150	1289	1295	1269	1254	1220	12	13	10	9	6
5	—	1798	1813	1793	1781	1744	—	—	—	—	—

$\Delta_x\% = 100X$ (Model x-EXP)/EXP

the results for a free-free cylinder and a clamped-clamped cylinder, which represent the lower and upper bounds of the natural frequencies, respectively.

The predicted natural frequencies for all five models seem to converge at higher values of the circumferential mode number, n. It seems that the cylindrical body becomes the primary vibrating substructure for the lower natural frequencies at higher values of n. In addition, these frequencies show minor sensitivity to the boundary configuration. At lower values of n, however, the more popular Models A and B fail to approximate the compressor shell properly.

For $n > 1$, Models C, D, and E predict the same trend as the experimental results, while the closest approximation of compressor shell natural frequencies is generated by Model E. As mentioned previously, none of the end cap frequencies of the compressor shell were detected experimentally in the range of observed frequencies (0 to 2000 Hz). This is reinforced by Figure 4a, which shows the end cap frequencies to be beyond 2000 Hz. It is speculated here that Model E is the most appropriate one to estimate the end cap vibrations.

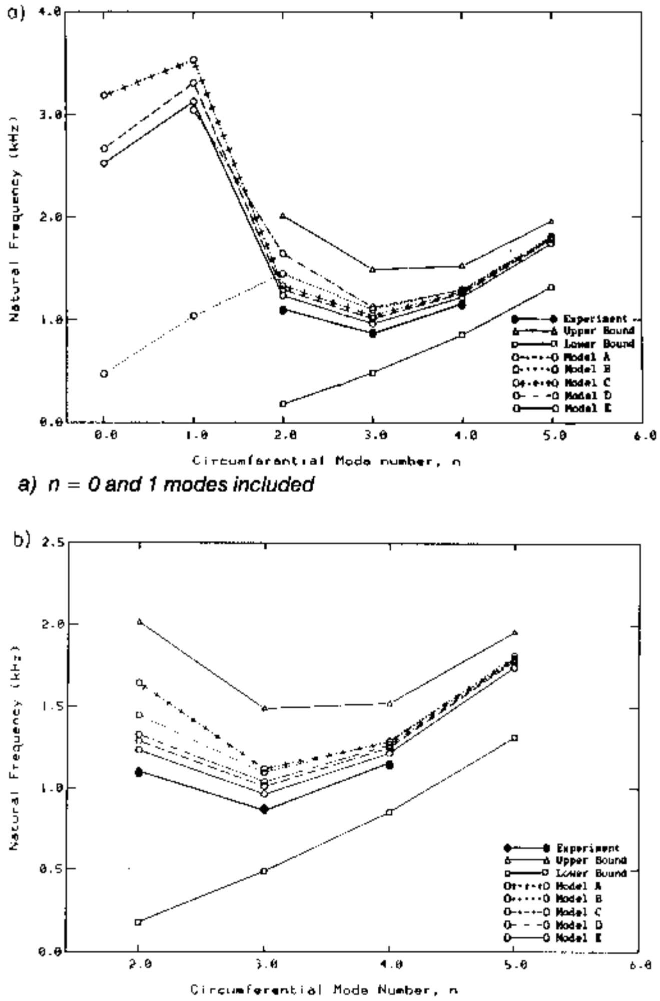
DESIGN STUDIES

Welded Seam Effects

The design issue here is the optimum location of the welded seam on the cylindrical body of the compressor shell. For the five compressor shell models, Tavakoli (1987) compared the SSM results for the case with the welded seam with the case without the welded seam. He concluded that the welded seam generally affects the natural frequencies of those modes for which the cylindrical body has significant participation. Hence, the natural frequencies of the shear diaphragmed cylinder (Model A) are observed while varying the location of the welded seam from one end toward the middle of the cylinder.

In Figure 5, variations of the natural frequencies for several circumferential modes are shown for the case where the welded seam is located in the middle of the cylinder. In Figure 6, the dependence of the natural frequencies on the locations of the welded seam is observed. As a general rule, it is believed (Soedel 1981; Wah and Hu 1968; Al-Najafi and Warburton 1970) that for a given circumferential mode number n, the natural frequencies of the ring-stiffened cylinder lie between those of the stiffening ring and the cylinder itself. Figure 5 shows, however, that the natural frequency of mode ($m = 0$, $n = 3$) is increased due to the welded seam. Furthermore, Figure 6a shows that the natural frequency of mode ($m = 0$, $n = 2$) is increased if the weld is located near the boundary of the cylinder. It is, therefore, concluded that the generality of this rule is questionable and subject to more careful inspection.

It is also seen from Figure 5 that, when the welded seam is located in the middle of the cylinder, its effect on $m = 1$ modes is insignificant. This is intuitively correct given the nature of $m = 1$ modes, which have a nodal circle in the middle of the cylinder. Figure 6 shows that the location of the welded seam affects the $m = 0$ modes more appreciably than, and in a manner different from, the $m = 1$



a) n = 0 and 1 modes included

b) n = 0 and 1 modes excluded

Figure 4 *Comparison of compressor shell models and experiment* modes. In both cases, however, one can conclude that the maximum effect is achieved if the welded seam is placed where the mode is likely to have an antinode. Again, this observation seems intuitively logical.

Curvature Effects

Soedel (1981) has analytically shown that the natural frequencies of a shear diaphragmed cylinder are monotonically increased if positive curvature (S) is introduced in the flat side of the cylinder, as shown in Figure 7. He has also shown that the effect of negative curvature is an initial increase in the natural frequencies followed by a monotonic decrease after the curvature surpasses a critical amount. Furthermore, based on measured data, Lowery (1984) concluded that the lowest natural frequency of a compressor shell can be increased by as much as one octave if discontinuities in both the surface and the curvature are removed. According to Lowery (1984), the most effective method of adding curvature to a cylinder is to positively curve the flat side, resulting in a "barrel" structure. The stiffness of the barrel shape can be further increased by eliminating the abrupt changes in the radii of curvature that occur at the "blend-points" between the cylinder and the end caps.

The SSM is used here to examine the effect of adding various degrees of positive curvature to the unwelded shear diaphragmed cylinder (Model A). The formulation of the problem is based on the equations of the toroidal shell already utilized in the analysis of the compressor shell (Tavakoli 1987). The results for the lowest natural frequencies corresponding to the circumferential modes n = 2 through 5 are presented in Table 3. This table shows that an octave increase in the lowest natural frequency (m = 0, n = 4) of this cylinder requires that approximately 10% (S/L = 0.1) positive curvature be added to the flat side of the cylinder. Even though Lowery (1984) has not given the precise dimensions of his compressor shells, the provided diagrams indicate that he also employed roughly the same percentage of curvature in order to obtain an octave increase in the lowest natural frequency.

In Table 4, SSM results are compared with the results of the closed-form approximate solution derived by Soedel (1981). SSM results are generally in gross disagreement with the results of Soedel (1981), especially as the amount of curvature grows. This is because Soedel's derivation (1981) is based on a curvature that is not too "pronounced." Although it is not quantified in Soedel (1981), it is believed that the amount of curvature used here is too large for the assumption of Soedel (1981) to be still valid.

As for the effect of eliminating abrupt curvature changes at the junctions of the end caps and cylinder, Figure 4a should be examined once again. Careful comparison of Models B, C, D, and E shows that the introduction of positive curvature in the end plates of Model B increases the end plate frequencies drastically. However, contrary to Lowery's claim (1984), joining the end caps to the cylindrical body in a more continuous fashion reduces the natural frequencies of the overall structure. This effect is intuitively supported when one recognizes that a circular plate provides a more rigid boundary condition for the cylindrical body than any other end cap studied here. Also, one can argue that the more continuous attachment of the end caps to the cylinder seems to add to the "effective length" of the cylinder, hence reducing its natural frequencies.

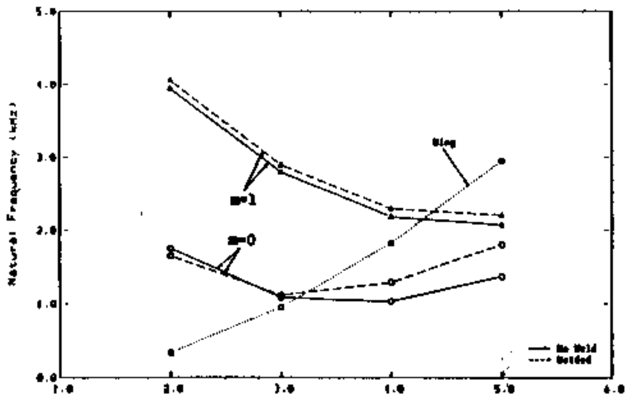


Figure 5 *Effect of the welded seam located in the middle of compressor shell model A on its natural frequencies*

Design Guidelines for Improved Dynamics

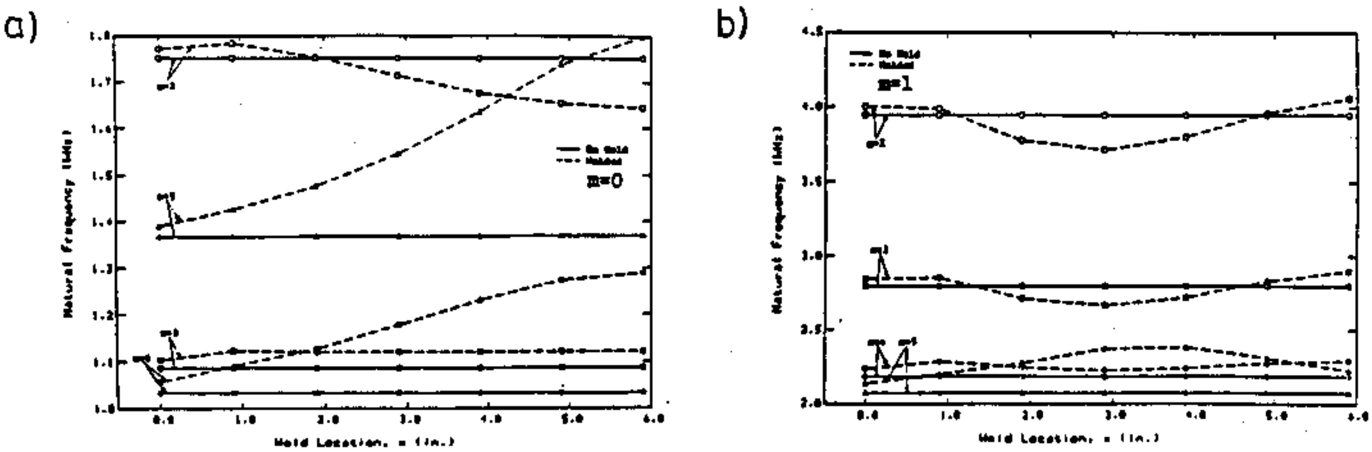
The various studies discussed in previous sections are essential in establishing practical design guidelines for the type of compressor shell that is of interest here. Within the scope of this work, the "stiffest" design for the compressor shell seems to be a hermetic can, similar to the compressor shell Model B, with a positively curved cylindrical body. The circular end plates of Model B, however, have low natural frequencies themselves. Hence, a further design improvement is to stiffen the end plates. An easy way to do this is to increase the end plate thickness. Using stiffening ribs may prove to be a more effective option in increasing the frequencies of the end plates.

As previously mentioned, the optimum position for the welded seam appears to be at a location where the targeted mode is likely to have an antinode. Since the lowest modes (m = 0) usually have an antinode in the middle of the cylindrical body, the welded seam should also be located in the middle of the cylinder.

Finally, knowledge of the interaction between the end caps and the cylinder is critical in determining which part of the compressor shell should be stiffened the most in order to raise a targeted natural frequency. The mode shapes of compressor shell Model E are presented in Figure 8 as an example. Knowledge of these mode shapes is also helpful in deciding on the suspension locations for the pump assembly inside the compressor shell. It is obviously best to keep these locations away from the primary vibrating substructure and close to the nodal points.

CONCLUDING REMARKS

A structural synthesis method based on state space mathematics was utilized to model the free vibrations of a refrigeration compressor shell. Five models that progressively approximate the compressor shell geometry were studied. It was concluded that traditionally popular simplistic models such as a shear diaphragmed cylinder fail to approximate the compressor shell reasonably, especially at lower natural frequencies. Comparison with measured data showed that the best estimation of com-



a) m = 0 Modes

b) m = 1 Modes

Figure 6 *Effect of the location of the welded seam on the natural frequencies of compressor shell model A*

TABLE 3
Effect of Positive Curvature S on Natural Frequencies (Hz) of Unwelded Compressor Shell Model A

n	S/L = 0.000	S/L = 0.025, Δ%		S/L = 0.050, Δ%		S/L = 0.100, Δ%	
2	1752	2059	, 18	2328	, 33	2818	, 61
3	1087	1464	, 35	1832	, 68	2530	, 133
4	1034	1310	, 27	1676	, 62	2429	, 135
5	1368	1468	, 7	1757	, 28	2451	, 79

$\Delta\% = 100X\,(f_{S/L} - f_0)/f_0$

TABLE 4
Natural Frequencies (Hz) for Unwelded Compressor Shell Model A with Positive Curvature: Comparison of SSM with Soedel (1981)

	S/L = 0.025			S/L = 0.050			S/L = 0.100		
n	SSM	Ref.[11]	Δ%	SSM	Ref.[11]	Δ%	SSM	Ref.[11]	Δ%
2	2059	2173	5.2	2328	2578	9.7	2818	3334	15
3	1464	1532	4.4	1832	1979	7.4	2530	2837	11
4	1310	1390	5.7	1676	1806	7.2	2429	2667	24
5	1468	1599	8.2	1757	1935	9.2	2451	2725	10

Δ% = 100 X (Soedel (1981) − SSM)/Soedel (1981)

pressor shell natural frequencies was generated by the model that approximated the compressor shell geometry the closest (Model E).

The state space method (SSM) was then employed to study the dependence of the compressor shell's natural frequencies on variables such as the location of the welded seam, the shape of the end caps, and the introduction of curvature in the cylindrical body. Several design improvements were suggested in this study.

Finally, the interaction between the medium inside the shell and the structure itself (fluid-structure coupling) is an issue that concerns the dynamics of hermetic compressor shell structures. In Figure 9, the measured frequency spectra for the compressor shell when the inside medium is air are compared to similar spectra when the inside medium is a refrigerant at different pressures. It is observed that as the refrigerant pressure increases, a larger number of resonant frequencies appear in the spectra, making it nearly impossible to distinguish the pure structural natural frequencies from the acoustic and/or combination natural frequencies. This should be investigated further, and the SSM may prove to be a suitable method for analyzing the structure-fluid coupling problem.

NOMENCLATURE

$[C]$	= substructure coupling matrix
D	= bending stiffness = $EH^3/12(1\text{-}\nu^2)$
E	= Young's Modulus of elasticity
f	= natural frequency in Hz
H	= thickness
h	= numerical discritization step size for space-variant substructures
m	= axial mode index indicating the number of nodal points along x excluding boundaries
M_x	= normalized bending moment at x boundary
N_x	= normalized force along x at x boundary
n	= circumferential mode index indicating the number of sine waves around the circumference

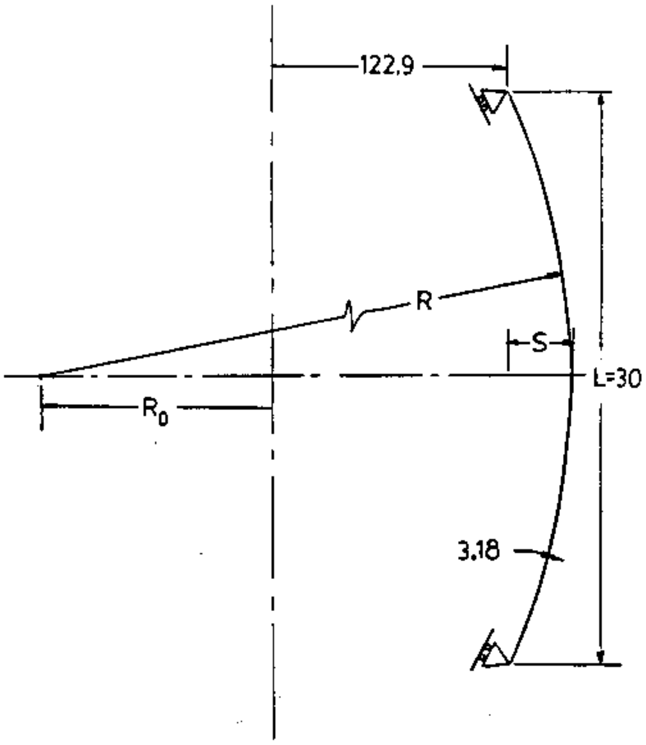


Figure 7 *Shear diaphragmed cylinder with positive curvature — (all dimensions in mm)*

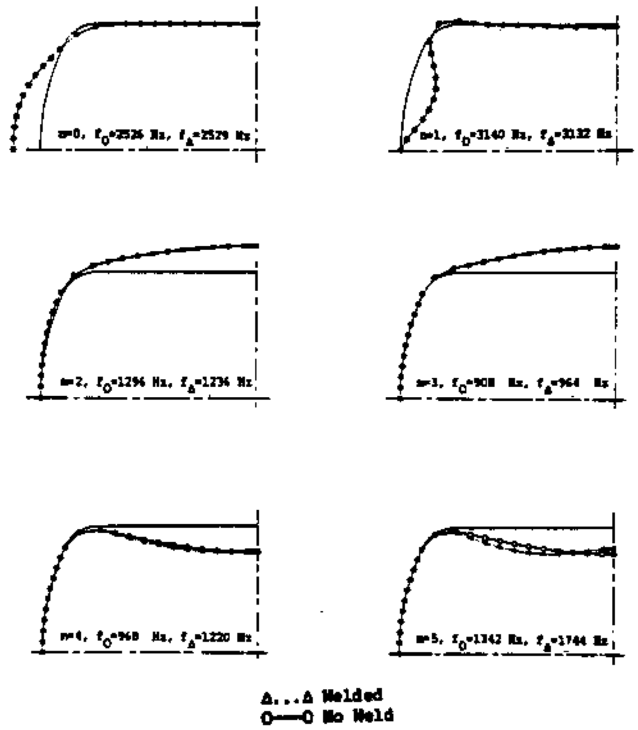


Figure 8 *Mode shapes of the refrigeration compressor shell (Model E)*

$S_{x\theta}$ = normalized in-plane Kirchhoff shear force at x boundary
SSM = state space method
$T[(x_0,x)]$ = transfer matrix relating state vectors at x_0 and x
$[U]$ = normalized substructure coefficient matrix
V_x = normalized transverse Kirchhoff shear force at x boundary

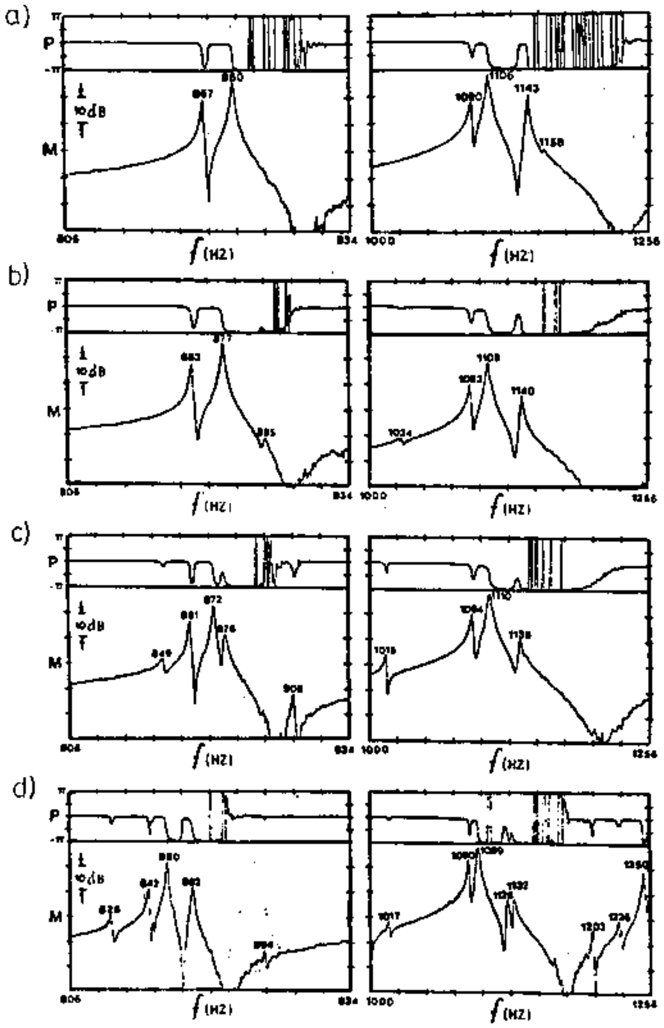


Figure 9 *Effect of the inside medium on the measured frequency spectra of the compressor shell: a) Air at 101 kPa, b) Refrigerant at 101 kPa, c) Refrigerant at 239 kPa, d) Refrigerant at 652 kPa.*

W_1 = normalized displacement along x
W_2 = normalized displacement along θ
W_3 = normalized transverse displacement along z
x = normalized substructure coordinate spanning from boundary to boundary
x_g = normalized global coordinate spanning from boundary to boundary
x_{0i}, x_{1i} = normalized boundary coordinates of ith substructure
x_{0g}, x_{1g} = normalized global boundary coordinates
z = normalized substructure coordinate normal to the surface
z_g = normalized global coordinate normal to the surface
β_x = normalized slope along x
θ = substructure circumferential coordinate
θ_g = global circumferential coordinate
ν = Poisson's ratio
ϱ = mass density
$\{\phi(x)\}$ = spatial state vector at x boundary
ω = circular frequency in rad/sec
$()'$ = d()/dx
$\{\}^T$ = vector transpose

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