A Linear Time Varying Model for On-Off Valve Controlled Pneumatic Actuators

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A new linear time varing (LTV) model has been developed for open loop, on-off valve controlled pneumatic actuation systems. This formulation is based on a periodic profile description for variable operating points and directional control valve flow variations. The dynamic behavior of the example case, a single acting cylinder controlled by a two way-two port rotary valve, under the cyclic pressure loading is obtained using the proposed LTV model. Experimental evidence and digital simulation predictions based on the nonlinear mathematical equations validate the analytical formulation. The proposed LTV model is found to be better and more applicable than linear time invariant (LTI) models used previously by many investigators.

1 Introduction

Pneumatic power control is becoming increasingly more popular as it offers an attractive alternative to hydraulic and electromechanical actuation for low to medium power requirements (Thayer, 1988). Pneumatic drives are used in new applications including small robots and manipulators because of low cost and ease of maintenance. Such actuation systems commonly use on-off valve control because of its simplicity of design compared with proportional flow control. It is also less sensitive to leakage eliminating the need for close tolerances. Related studies, for instance by Burrows and Webb (1968, 1970), have shown that on-off flow control is indeed promising in the development of low cost pneumatic servomechanisms. Hence, there is much to be gained by focusing research efforts in the area of on-off air flow control as evident from the recent publications by Morita et al. (1985), Noritsugu (1985 and 1987), Liu and Bobrow (1987), and Lai et al. (1989). This study addresses the problem of predicting dynamic response of such a system, which is inherently nonlinear.

2 Literature Review

Early studies on proportional and on-off controlled pneumatic actuators employed analog computer techniques (Shearer 1956; Blackburn et al. 1960; Burrows and Ballard 1972). Recently digital simulation methods have been adopted. For example, Hundal (1978) and Wang et al. (1984) analyzed pneumatic shock-absorbing cylinders, and Lai et al. (1989) have used similar methods to examine the position accuracy of an actuator under on-off control. Such an approach usually takes substantial computer time especially when a series of parametric studies is conducted. Also, difficulties associated with numerical integration may arise sometimes due to the strong nonlinearities exhibited by pneumatic systems (Wang and Singh, 1986 and 1987). Perturbation methods have been applied by a few investigators to represent pneumatic system nonlinearities analytically. Araki (1984) has used the describing function method to analyze the frequency response of a pneumatic cylinder under proportional flow control. Wang and Singh (1986, 1987) have used the method of harmonic balance as well as quasi-linearization and digital simulation techniques to study the frequency response of open and closed pneumatic chambers including compressibility, orifice flow, and dry friction nonlinearities. A similar study by Chen (1977) was based on the Krylov-Bogoliubov method of variation of parameters.

Pneumatic actuators have also been analyzed using the linear system theory (Araki 1967; Botting et al. 1970; McCloy and Martin 1980). For example, Doebelin (1980) has analyzed the stability of an air cushion device using a third order linearized model. Sweet et al. (1975) have used a seventh order linearized model to analyze the dynamics of an air suspension system including the effects of pipeline dynamics. However, such a linearized model is suitable only for the actuator midstroke position. All of these analyses have been limited to linear time invariant (LTI) or constant coefficient differential equation formulation. Such a model is valid only for small and symmetric fluctutations about a mean operating point. Recently, Scavanda et al. (1987) and Liu et al. (1987) have utilized the state space approach to extend the linearized model over a series of invariant operating points. However, no reference on the variable operating point analysis could be found.

3 Problem Formulation

The primary objective of this paper is to develop a new analytical method to predict the dynamic behavior of open

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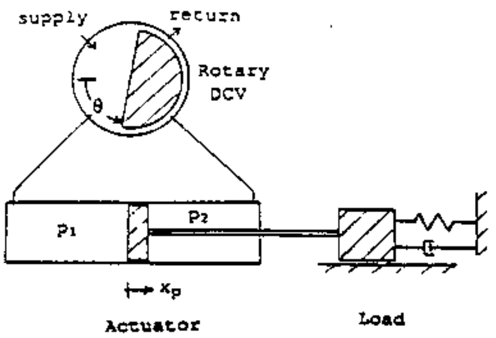

Fig. 1 Generic on-off valve controlled pneumatic actuator in the open loop mode. Here a rotary directional control valve (DCV) is used which is comparable to the commercially available spool or poppet valves.

loop, on-off valve controlled pneumatic actuators. It should yield accurate predictions under the general loading condition, not limited to the vicinity of an operating point. Accordingly, formulation of a linear time varying (LTV) model is undertaken in an effort to describe "large" fluctuations in a system response variable which can not be predicted with the currently available LTI models.

For this purpose, a generic but simple pneumatic system as shown schematically in Fig. 1 is considered. It comprises an on-off directional control valve (DCV) and a cylinder actuator with an inertia load. A rotary DCV is used instead of the conventional sliding spool or poppet control valve. Flow switching is accomplished by means of a rotary spool, hence periodic excitation is obtained by having the rotary valve (RV) operate at a constant rotational speed $\dot{\theta}(t)=\Omega_p$. RV's have the advantage of running at high speeds without the adverse inertial effects associated with sliding spool type DCV's, as described by Kunt (1988). In order to ensure that the proposed pneumatic system is simple but representative, a two-way RV prototype with two ports has been designed carefully and fabricated; further details will be given in Section 4. Of interest here is the determination of controlled motion $x_p(t)$ along with the chamber pressures $p_1(t)$ and $p_2(t)$ in response to command DCV spool displacement $\theta(t)$. Periodic excitation with constant Ω_p, which is the case in most industrial applications, will be considered.

Specially the objectives of this study are as follows: (i) develop an LTV formulation for pneumatic system based on a periodic staircase type profile for variable operating points and DCV flow area variations, (ii) apply the proposed model to a two-way rotary valve controlled actuator and predict dynamic response under the cyclic pressure excitation, (iii) validate the new LTV model by comparing its predictions, with the results obtained by LTI models, digital simulation technique based on nonlinear mathematical formulation and an analogous experimental study, and (iv) discuss various modeling issues including the validity range of LTV model.

4 Rotary Valve Analysis

The RV is modeled as a general DCV with N_w flow paths or ways, and N_0 operating positions. The number of its flow ports $N_p \geq N_w$, since a flow path may be connected to more than one flow port. In anticipation of the lumped parameter analysis that will be resorted to, the internal volume of the RV is discretized into N_v control volumes. The minimum flow area between the ith flow port and the kth control volume is denoted by a_{ik} and varies with spool position θ. The flow area A_{ik} between the ith flow path and the kth control volume can be determined from

$$A_{ik}(\theta)=\sum_{m=L_i}^{M_i} a_{mk}(\theta)\ ,\ i=1,2,\ldots,N_w;\ k=1,2,\ldots,N_v. \tag{1}$$

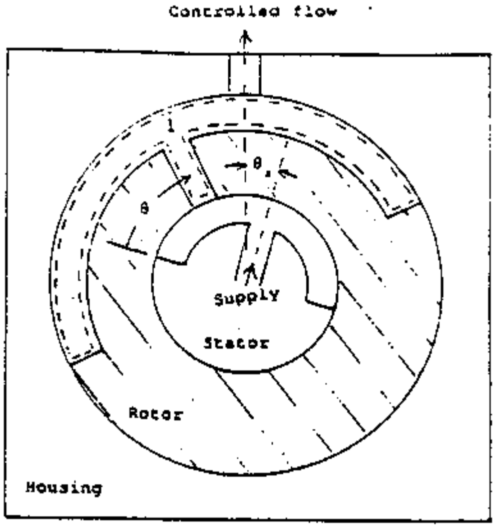

Fig. 2(a)

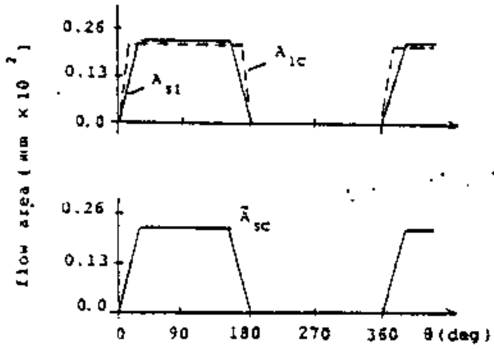

Fig. 2(b)

Fig. 2 Two-way rotary valve (RV) with 2 positions (on and off). (a) Internal details. (b) Flow area variations.

where flow ports L_i through M_i are assumed to be connected to the ith flow path. Under certain conditions, control volume dynamics need not be included into the analysis, in which case flow paths are in effect directly coupled with each other. It is then necessary to determine the resulting flow area variation $\bar{A}_{ik}$ between the ith and the kth flow paths. Since mass flow rate calculations are based on the minimum cross sectional area of a flow conduit, it follows that

$$\bar{A}_{ik}(\theta)=\sum_{m=1}^{N_v} \min\{A_{im}(\theta)\ ,\ A_{mk}(\theta)\},\ i,k=1,2,\ldots,N_w. \tag{2}$$

where each term in the summation represents the flow area contributed as a result of collapsing the mth control volume to a point. Equations (1) and (2) represent general RV kinematics, and can be extended to include the kinematics of the actuation process by increasing the number of control volumes.

Figure 2 schematically shows the internal details of the RV prototype specially designed for this study. For this prototype $N_p=N_w=2$, the two flow ports being supply and controlled flow. Flow area variations result from the relative motion θ of the rotor with respect to the housing and the stator. In Fig. 2(a), RV internal space is discretized into a single volume as indicated by dashed lines and hence $N_v=1$. Flow area variations for this RV are shown in Fig. 2(b); subscripts 1, s, and c refer to RV control volume and supply and controlled flow ports respectively. Flow area profiles show that this prototype is basically an on-off valve ($N_0=2$). RV on-time can be adjusted by varying the offset angle θ_s between the stator and the housing shown in Fig. 2(a). Flow area variations of Fig. 2(b) correspond to $\theta_s = 0$ deg and have an on-time of 180 deg or 50 percent.

5 Nonlinear Mathematical Model

Thermofluid processes for the system shown in Fig. 1 are modeled through constitutive and continuity equations based on control volumes defined for the actuator chambers and the RV internal volume. For the kth control volume, the ideal gas and polytropic equations are

$$p_k = R_a m_k T_k / V_k, \quad T_k(t) = T_k(0)[p_k(t)/p_k(0)]^{(n_k - 1)/n_k} \quad (3,4)$$

where t denotes time, R_a is the gas contant, and p_k, m_k, T_k, V_k, and n_k are the fluid pressure, mass, temperature, volume, and polytropic constant for the kth control volume. Actuator chamber volumes V_1 and V_2 are functions of the piston displacement x_p. Mass flow rate is modeled as one dimensional. A compressible mass flow rate function $M_f(i,k)$ between the ith flow path and the kth control volume is introduced for this purpose based on isentropic relations (Andersen, 1967)

$$M_f(i, k) = \begin{cases} p_i \sqrt{\dfrac{2\gamma}{R_a T_i(\gamma - 1)}\left[\left(\dfrac{p_k}{p_i}\right)^{2/\gamma} - \left(\dfrac{p_k}{p_i}\right)^{(\gamma+1)/\gamma}\right]}, & 1 \geq \dfrac{p_k}{p_i} \geq r_c \\ p_i \sqrt{\dfrac{\gamma}{R_a T_i}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}, & \dfrac{p_k}{p_i} \leq r_c \end{cases} \quad (5)$$

where $r_c = [2/(\gamma+1)]^{\gamma/(\gamma-1)}$ is the critical pressure ratio, and γ is the ratio of specific heats. This formulation neglects fluid inertia effects. It also assumes that total and static properties of the fluid within a control volume are equal. Conservation of mass for the kth control volume requires

$$\dot{m}_k = \sum_{i=1}^{N_w} \begin{cases} C_{ik} A_{ik} M_f(i, k), & p_i \geq p_k \\ -C_{ik} A_{ik} M_f(k, i), & p_i < p_k \end{cases} \quad (6)$$

where C_{ik} is a discharge coefficient incorporating flow losses due to passage discontinuity, friction, and leakage. For the system under study $N_w = 4$; the four flow paths being actuator left and right chambers ($i = 1, 2$), and supply and return ports ($i = 3, 4$).

Equation of motion for the single-degree-of-freedom actuator-load system is written as

$$\ddot{x}_p(t) = [F_e(t) - F_f(t)]/M_t \quad (7a)$$

where M_t is the total moving mass. The external force F_e and the total friction force F_f are given by

$$F_e(t) = (A_{p1} - C_p)p_1(t) + (A_{p2} + C_p)p_2(t) - K_s x_p(t) \quad (7b)$$

$$F_f(t) = \begin{cases} F_d \text{sign}[\dot{x}_p(t)] + C_v \dot{x}_p(t), & \dot{x}_p(t) \neq 0 \\ F_e(t), & \dot{x}_p(t) = 0,\ F_e(t) \leq F_s \\ F_s \text{sign}[F_e(t)], & \dot{x}_p(t) = 0,\ F_e(t) > F_s \end{cases} \quad (7c)$$

where A_{p1} and A_{p2} are the areas of the piston right and left side pressure faces, respectively. Stiction force is denoted by F_s and dry friction by F_d. C_p, K_s, and C_v denote the sliding friction coefficient, spring stiffness, and viscous damping coefficient, respectively.

6 LTV Formulation

Parametric studies using the nonlinear model have shown that the thermodynamic processes involved are much closer to isothermal ($n_k = 1.0$) than to adiabatic ($n_k = \gamma$); see Kunt (1988) for further details. Accordingly we assume $n_k = 1.0$ in order to simplify the algebra involved with the LTV formulation. In addition, since RV internal volume is usually much smaller than that of the actuator, RV charging and discharging processes are modeled as zeroth order. This assumption has also been verified through parametric studies conducted using the nonlinear model. Operating points are defined for the remaining system variables p_1, p_2, and x_p for periodic excitation, i.e., $A_{ik}(t + T_p) = A_{ik}(t)$ where $T_p = 1/\Omega_p$. Equations (3)–(7) are linearized yielding

$$\dot{\mathbf{z}}(t) = \mathbf{G}(t)\mathbf{z}(t) + \mathbf{f}(t) \quad (8a)$$

with

$$z(t) = \begin{Bmatrix} x_p(t) \\ \dot{x}_p(t) \\ p_1(t) \\ p_2(t) \end{Bmatrix}, \quad G(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_n^2 & -2\xi\omega_n & K_{p1} & -K_{p2} \\ 0 & -K_{v1} & -1/\tau_1(t) & 0 \\ 0 & -K_{v2} & 0 & -1/\tau_2(t) \end{bmatrix}, \quad f(t) = \begin{Bmatrix} 0 \\ 0 \\ f_{p1}(t) \\ f_{p2}(t) \end{Bmatrix} \quad (8b\text{–}d)$$

The system parameters are given by

$$\tau_1(t) = \frac{1}{\dfrac{R_a T_a}{V_{01} + A_{p1}\bar{x}_p} \displaystyle\sum_{i=3,4} C_{i1}\bar{A}_{i1}(t)\beta_{i1}},$$

$$\tau_2(t) = \frac{1}{\dfrac{R_a T_a}{V_{02} - A_{p2}\bar{x}_p} \displaystyle\sum_{i=3,4} C_{i2}\bar{A}_{i2}(t)\beta_{i2}}$$

$$f_{p1}(t) = \frac{R_a T_a}{V_{01} + A_{p1}\bar{x}_p} \sum_{i=3,4} [C_{i1}\bar{A}_{i1}(t)(\alpha_{i1} + \beta_{i1}\bar{p}_1)],$$

$$K_{p1} = (A_{p1} - C_p)/M_t$$

$$f_{p2}(t) = \frac{R_a T_a}{V_{02} - A_{p2}\bar{x}_p} \sum_{i=3,4} [C_{i2}\bar{A}_{i2}(t)(\alpha_{i2} + \beta_{i2}\bar{p}_2)],$$

$$K_{p2} = (A_{p2} + C_p)/M_t$$

$$K_{v1} = \frac{\bar{p}_1}{V_{01} + A_{p1}\bar{x}_p}, \quad K_{v2} = \frac{\bar{p}_2}{V_{02} - A_{p2}\bar{x}_p}, \quad \omega_n = \sqrt{\frac{K_s}{M_t}},$$

$$\xi = \frac{C_v}{2\sqrt{K_s M_t}} \quad (8e\text{–}j)$$

where overhead bars indicate operating points. Actuator left and right chamber volumes corresponding to zero piston displacement are denoted by V_{01} and V_{02}, respectively. Temperature is assumed to be uniform throughout at T_a. Mass flow rate linearization constants are denoted by α_{ik} and β_{ik}, $k=1, 2$; $i=3, 4$. For $\bar{p}_k \leq p_i$

$$\alpha_{ik} = M_f(i,k)\,|_{\bar{p}_k} = \begin{cases} p_i\sqrt{\dfrac{2\gamma}{R_aT_a(\gamma-1)}\left[\left(\dfrac{\bar{p}_k}{p_i}\right)^{2/\gamma} - \left(\dfrac{\bar{p}_k}{p_i}\right)^{(\gamma+1)/\gamma}\right]}, & 1 \geq \dfrac{\bar{p}_k}{p_i} \geq r_c \\ p_i\sqrt{\dfrac{\gamma}{R_aT_a}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}, & \dfrac{\bar{p}_k}{p_i} \leq r_c \end{cases}$$

$$\beta_{ik} = -\frac{\partial M_f(i,k)}{\partial p_k}\bigg|_{\bar{p}_k} = \begin{cases} \dfrac{\dfrac{2}{\gamma}\left(\dfrac{\bar{p}_k}{p_i}\right)^{(2-\gamma)/\gamma} - \dfrac{\gamma+1}{\gamma}\left(\dfrac{\bar{p}_k}{p_i}\right)^{1/\gamma}}{\sqrt{\dfrac{2R_aT_a(\gamma-1)}{\gamma}\left[\left(\dfrac{\bar{p}_k}{p_i}\right)^{2/\gamma} - \left(\dfrac{\bar{p}_k}{p_i}\right)^{(\gamma+1)/\gamma}\right]}}, & 1 \geq \dfrac{\bar{p}_k}{p_i} > r_c \\ \sqrt{\dfrac{\gamma}{R_aT_a}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}, & \dfrac{\bar{p}_k}{p_i} \leq r_c \end{cases}$$

and for $\bar{p}_k > p_i$

$$\alpha_{ik} = -M_f(k,i)\,|_{\bar{p}_k} = \begin{cases} \bar{p}_k\sqrt{\dfrac{2\gamma}{R_aT_a(\gamma-1)}\left[\left(\dfrac{p_i}{\bar{p}_k}\right)^{2/\gamma} - \left(\dfrac{p_i}{\bar{p}_k}\right)^{(\gamma+1)/\gamma}\right]}, & 1 \geq \dfrac{p_i}{\bar{p}_k} \geq r_c \\ \bar{p}_k\sqrt{\dfrac{\gamma}{R_aT_a}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}, & \dfrac{p_i}{\bar{p}_k} \leq r_c \end{cases}$$

$$\beta_{ik} = \frac{\partial M_f(k,i)}{\partial p_k}\bigg|_{\bar{p}_k} = \begin{cases} \dfrac{2\left(\dfrac{p_i}{\bar{p}_k}\right)^{2/\gamma} - \left(\dfrac{p_i}{\bar{p}_k}\right)^{(\gamma+1)/\gamma}}{\sqrt{\dfrac{2R_aT_a\gamma}{\gamma-1}\left[\left(\dfrac{p_i}{\bar{p}_k}\right)^{2/\gamma} - \left(\dfrac{p_i}{\bar{p}_k}\right)^{(\gamma+1)/\gamma}\right]}}, & 1 \geq \dfrac{p_i}{\bar{p}_k} > r_c \\ \sqrt{\dfrac{\gamma}{R_aT_a}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}, & \dfrac{p_i}{\bar{p}_k} \leq r_c \end{cases} \qquad (8k\text{–}n)$$

Equation (8) constitutes a periodic LTV model due to the presence of variable coefficients $\tau_1(t)$ and $\tau_2(t)$ of period T_p. The corresponding block diagram is shown in Fig. 3. For fixed operating points time varying blocks are those of actuator chamber dynamics. However, this model can accommodate variable operating points, in which case the gains K_{v1} and K_{v2} will also be time varying.

Periodic LTV system analysis has been extensively applied for the study of electrical networks and circuits (Richards, 1983). In mechanical systems, chief applications up-to-date involve the analysis of rotating structures such as helicopter blades (Friedmann and Hammond, 1977; Dugundji and Wendell, 1983), and railway wheel sets (Lieh and Haque, 1988). No reference could be found on the LTV formulation of pneumatic or hydraulic actuation systems.

The general solution to equation (8) with initial conditions at $t=0$ can be written as (DeRusso, 1965)

$$\mathbf{z}(t) = \mathbf{W}(t)\mathbf{W}^{-1}(0)\mathbf{z}(0) + \int_0^t \mathbf{W}(t)\mathbf{W}^{-1}(\tau)\mathbf{f}(\tau)d\tau \qquad (9)$$

where $\mathbf{W}(t)$ is the Wronskian matrix, whose columns are the basis solutions to the system homogeneous equation. Unlike the LTI system, it is not in general possible to determine all the basis solutions for the periodic LTV system and construct an analytical solution. On-off flow area variations and the resulting coefficient profiles can be approximated using a staircase (piecewise constant) waveform as illustrated in Fig. 4. This leads to the following characterization of the system matrix and the forcing vector:

$$\mathbf{G}(t) = \mathbf{G}_m \text{ and } \mathbf{f}(t) = \mathbf{f}_m,\ t\in\Delta_m,\ m=1, 2, \ldots, N_s \qquad (10a)$$

with

$$\Delta_m = \left\{t; \frac{k+\theta_{m-1}/2\pi}{\Omega_p} \leq t < \frac{k+\theta_m/2\pi}{\Omega_p},\ k=0, 1, 2, \ldots\right\} \qquad (10b)$$

Here $\mathbf{G}_m$ and $\mathbf{f}_m$ denote constant matrices and vectors, respectively. The number of steps in the staircase profiles $N_s \geq N_0$. The angle at which the spool moves from position m to $m+1$ is denoted by θ_m. Letting $\theta_0 = \theta(t=0) = 0$, the duration T_m of step m is given by

$$T_m = \frac{\theta_m - \theta_{m-1}}{2\pi\Omega_p} \quad \text{and} \quad \sum_{m=1}^{N_p} T_m = T_p. \tag{10c}$$

Then, the response during step m can be found from

$$\mathbf{z}_m(t_m) = \mathbf{z}_{\infty,m} + \exp(\mathbf{G}_m t_m)[\mathbf{z}_m(0) - \mathbf{z}_{\infty,m}] \tag{11}$$

where $t_m \geq 0$ is a local time variable and

$$\mathbf{z}_{\infty,m} = \lim_{t_m \to \infty} \mathbf{z}_m(t_m) = \mathbf{G}_m^{-1}\mathbf{f}_m \tag{12}$$

assuming the system to be asymptotically stable within each step, i.e., $\exp(\mathbf{G}_m t_m) \to 0$ as $t_m \to \infty$. The initial conditions of step m are equal to the terminal conditions of step $m-1$

$$\mathbf{z}_m(0) = \mathbf{z}_{\infty,m-1} + \exp(\mathbf{G}_{m-1} T_{m-1})[\mathbf{z}_{m-1}(0) - \mathbf{z}_{\infty,m-1}] \tag{13}$$

The complete response can be calculated using equations (10)–(13). If the system is asymptotically stable, a periodic steady state solution exists, because the frequency of coefficient variations is equal to that of the forcing function (Richards, 1983). Denoting the identity matrix by $\mathbf{I}$, the initial conditions $\mathbf{z}_m(0)$ for the steady-state solution can be found from

$$\begin{bmatrix} -\exp(\mathbf{G}_1 T_1) & \mathbf{I} & \mathbf{0} & \cdot\ \cdot & \mathbf{0} \\ \mathbf{0} & -\exp(\mathbf{G}_2 T_2) & \mathbf{I} & \cdot\ \cdot & \mathbf{0} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \mathbf{0} & \cdot\ \cdot\ \cdot & \mathbf{0} & & \mathbf{I} \\ \mathbf{I} & \mathbf{0}\ \cdot\ \cdot & \mathbf{0} & & -\exp(\mathbf{G}_{N_p} T_{N_p}) \end{bmatrix} \begin{Bmatrix} \mathbf{z}_1(0) \\ \mathbf{z}_2(0) \\ \cdot \\ \cdot \\ \mathbf{z}_{N_p}(0) \end{Bmatrix} = \begin{Bmatrix} [\mathbf{I} - \exp(\mathbf{G}_1 T_1)]\mathbf{z}_{\infty,1} \\ [\mathbf{I} - \exp(\mathbf{G}_2 T_2)]\mathbf{z}_{\infty,2} \\ \cdot \\ \cdot \\ [\mathbf{I} - \exp(\mathbf{G}_{N_p} T_{N_p})]\mathbf{z}_{\infty,N_p} \end{Bmatrix}. \tag{14}$$

7 Example Case

The dynamic response of a single acting cylinder controlled by a two-way two-position RV is determined experimentally as well as by the nonlinear, LTV, and LTI models. The experimental set up is shown schematically in Fig. 5. A standard air unit is used with a maximum supply pressure of 0.60 MPa (87 psi). The RV is operated by a fractional horsepower DC motor. Instrumentation is included for the measurement of RV operating speed Ω_p, actuator pressure $p_1(t)$ and piston displacement $x_p(t)$.

The response of the nonlinear model is determined numerically using a fourth order Runge-Kutta integration scheme with fixed time increment. Table 1 lists the numerical values of system parameters used in the computer simulation. Note that two main assumptions, namely the isothermal process and the zeroth order charging or discharging valve dynamics, involved in the development of LTV model are included in the nonlinear model as well. Hence both models can now be compared directly.

The LTI model is obtained from the LTV model by using time averaged values for the variable coefficients. Steady state operating points are also found in a similar manner. For example, the steady state operating point for the actuator pressure is defined as

$$\bar{p}_1 = \frac{1}{T_p}\int_0^{T_p} p_1(t)dt \quad \text{or} \quad \bar{p}_1 = \frac{1}{2\pi}\int_0^{2\pi} p_1(\theta)d\theta. \tag{15}$$

Since p_1 is not known beforehand, it is approximated by a quasi-static pressure

$$p_{s1} = \lim_{\Omega_p \to 0} p_1(\theta), \tag{16}$$

which can be found by setting the time derivatives in equation (6) of the nonlinear model equal to zero and solving the resulting set of nonlinear algebraic equations. A quasi static displacement x_{sp} is defined in a similar manner.

Variable operating points are used with the LTV model to enhance accuracy. These are periodic with a staircase profile: $\bar{x}_p(t) = \bar{x}_{pm}$, $\bar{p}_1(t) = \bar{p}_{1m}$, $t \in \Delta m$. The constants $\bar{x}_{pm}$ and $\bar{p}_{1m}$ are found from:

$$\bar{x}_{pm} = \frac{1}{\theta_m - \theta_{m-1}}\int_{\theta_{m-1}}^{\theta_m} x_p(\theta)d\theta,$$

$$\bar{p}_{1m} = \frac{1}{\theta_m - \theta_{m-1}}\int_{\theta_{m-1}}^{\theta_m} p_1(\theta)d\theta \tag{17}$$

Here x_p and p_1 are based on the predictions of the LTI model, since using quasi-static considerations in this case would introduce wrong phase relationships. Caution should be exercised while using variable operating points as the accuracy of the LTV model may be adversely affected depending on the error content of the LTI results.

8 Results and Discussion

Experimental and theoretical rsults for the example case are compared for different operating speeds of the RV. Figures 6–8 display steady-state time histories for piston displacement x_p and actuator pressure p_1 for $\Omega_p = 1$, 4, and 10 Hz, respectively. Good agreement is obtained between the experimental and nonlinear model results especially at low speeds (say $\Omega_p < 8$ Hz) with a discrepancy of less than 10 percent based on the experimental results. The nonlinear model results can be further improved by a more accurate determination of the friction coefficients and the effective flow areas.

The LTV model with $N_s = 2$ is found to perform considerably better than the LTI model, which suffers especially at low speeds due to the large changes in the variable operating points. LTV model predictions are in satisfactory agreement with the experimental and nonlinear model results over the

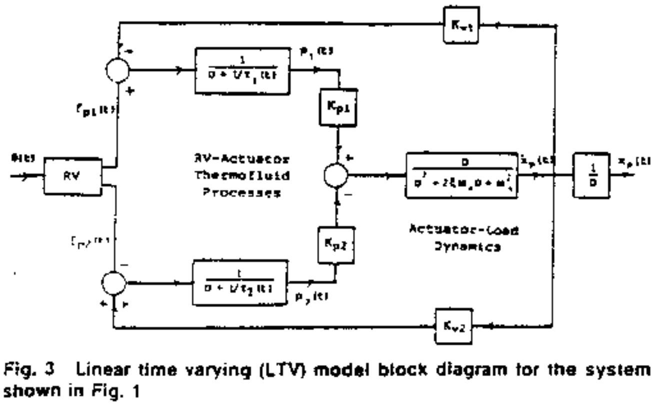

Fig. 3 Linear time varying (LTV) model block diagram for the system shown in Fig. 1

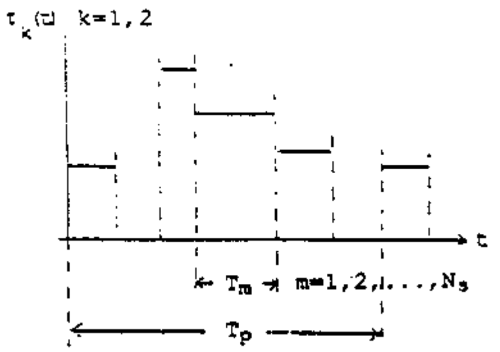

Fig. 4 Assumed staircase profile of the on-off valve flow area coefficients used in the LTV model

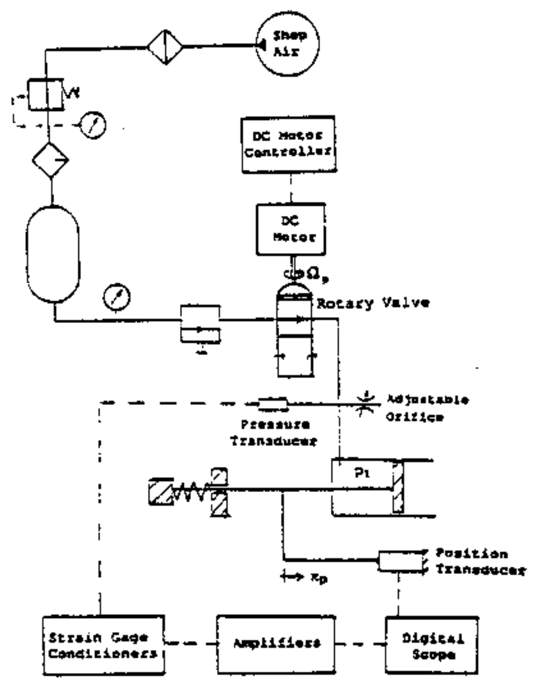

Fig. 5 Experimental schematic and instrumentation system for the example case (single acting cylinder controlled by a two-way rotary valve)

Table 1 Numerical values for the system of Fig. 5 with $N_v = 1$. Subscripts 1,2,3, and 4 refer to actuator chamber, RV control volume, supply port and adjustable orifice (return port), respectively

$A_{12} = 0.16 \times 10^{-4}$ m^2 (0.024 in.^{2}),
$A_{14} = 0.97 \times 10^{-5}$ m^2 (0.015 in.^{2})
$A_{23} = 0.32 \times 10^{-4}$ m^2 (0.049 in.^{2})
$C_{12} = 0.4$, $C_{14} = 0.4$, $C_{23} = 0.65$
$V_{01} = 0.82 \times 10^{-5}$ m^3 (0.5 in.^{3}), $V_2 = 0.41 \times 10^{-5}$ m^3 (0.25 in.^{3})
$\theta_t = 180$ deg
$n_1 = n_2 = 1.0$
$p_3 = 0.41$ MPa (60 psi), $p_4 = 0.10$ MPa (14.7 psi)
$T_3 = T_4 = 20$ C (68°F)
$A_{p1} = 0.59 \times 10^{-3}$ m^2 (0.92 in.^{2}),
$A_{p2} = 0.64 \times 10^{-3}$ m^2 (0.99 in.^{2})
$M_t = 1.75$ kg (0.01 lb-s^2/in.)
$K_s = 2.1$ kN/m (12 lb/in.)
$F_s = 26.6$ N (6 lb), $F_d = 13.3$ N (3 lb)
$C_v = 43.7$ N-s/m (0.25 lb-s/in.)
$C_p = -0.32 \times 10^{-4}$ m^2 (−0.05 in.^{2})

frequency range of interest ($\Omega_p \leq 10$ Hz). The variable operating point for the actuator pressure has been found to be the most important factor in improving the accuracy of the LTV model over the LTI model. It should be noted that the LTV model with a constant operating point for piston displacement is found to give more accurate results. This is

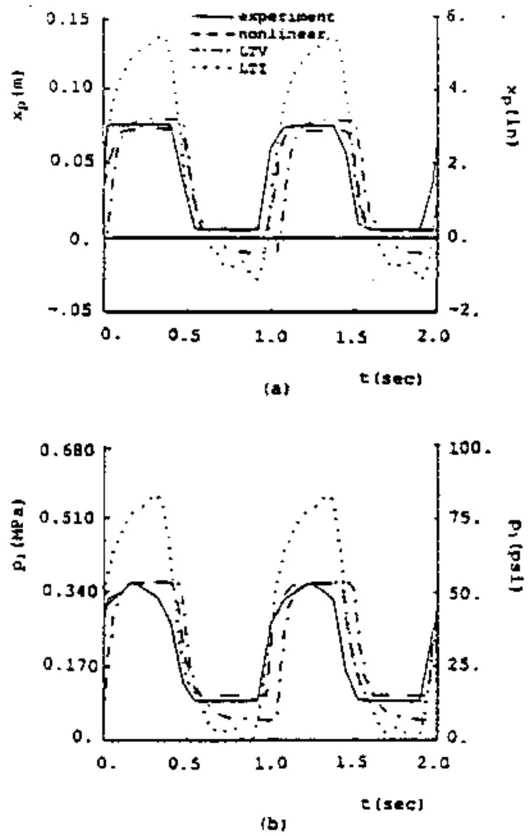

Fig. 6 Cyclic time histories for $\Omega_p = 1$ Hz. Here x_p = piston displacement, p_1 = actuator chamber pressure and Ω_p = rotary valve running speed.

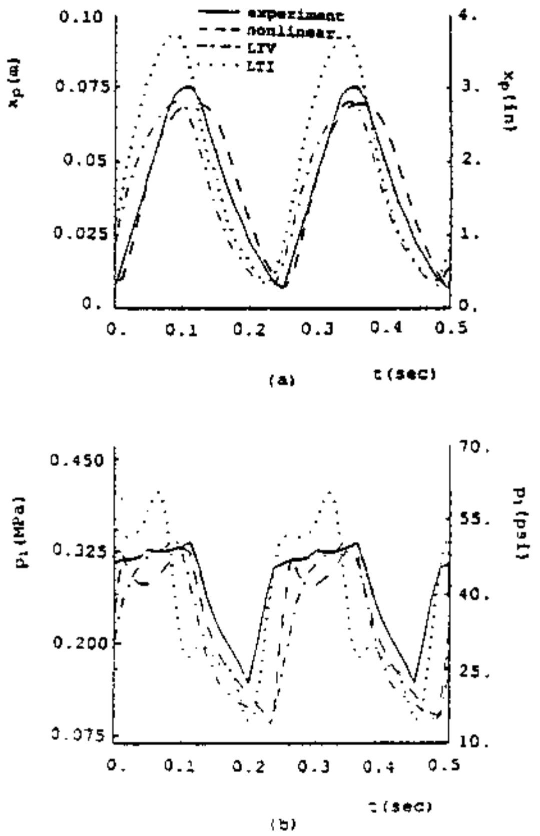

Fig. 7 Cyclic time histories for $\Omega_p = 4$ Hz

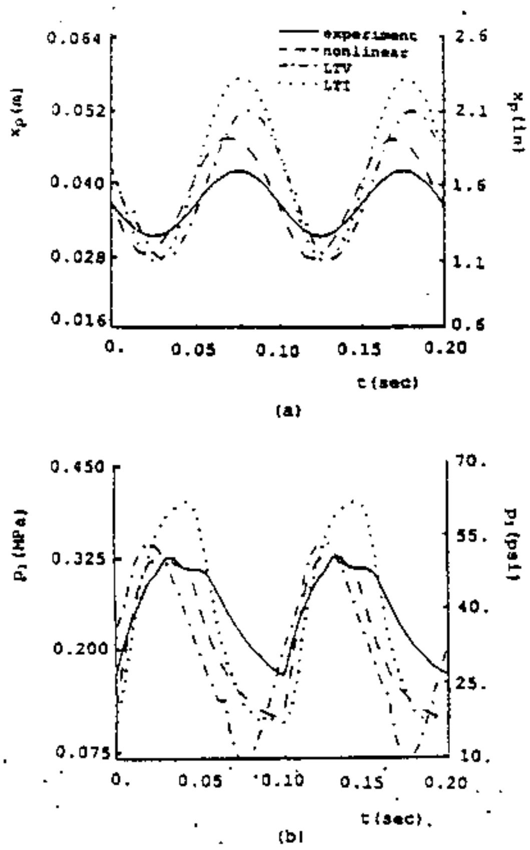

Fig. 8 Cyclic time histories for $\Omega_p = 10$ Hz

due to the singularity associated with the piston displacement as can be seen in equation (3). This singularity renders the LTV model more sensitive to the error content of the LTI results and degrades accuracy especially at low speeds.

For the frequency response of the single acting cylinder system, a steady-state amplitude is defined for the piston displacement as

$$X_p = \lim_{t \to \infty} [\max\{x_p(t)\} - \min\{x_p(t)\}]/2. \qquad (18)$$

This is calculated using the nonlinear and linearized models for $\Omega_p \leq 10 \cdot$ Hz. Resulting amplitude frequency response curves are shown in Fig. 9. Once again, the nonlinear and LTV models are in close agreement. Conversely, the LTI model results are acceptable only at higher speeds. The amplitude response is dominated by spring effects at very low speeds and frequency independent as accurately predicted by the LTV model.

9 Conclusion

A new LTV model has been formulated for on-off valve controlled pneumatic systems. It has a periodic staircase profile for flow variable operating points and control valve area variations. It has been successfully applied to a single acting cylinder. The proposed model has been found to be in good agreement with the experimental study, and its predictions are comparable to those yielded by the digital simulation based on nonlinear model. The superiority of the proposed formulation over the currently available LTI models is evident from the following: (i) LTV model predictions are more accurate than those of the LTI model, (ii) LTV model provides a transition from the LTI analysis to the nonlinear model by incorporating interactions between the dynamic system and excitation, and (iii) unlike the LTI model, LTV formulation can be applied to a wider range of problems. Also, the LTV model can be used

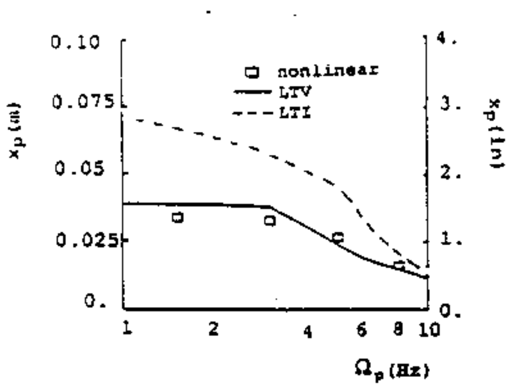

Fig. 9 Amplitude frequency response characteristics for the piston displacement

to seek an improved understanding of the dynamic behavior and to conduct design parametric studies in an efficient manner. Accordingly, the proposed formulation can be treated as a suitable alternative to the nonlinear digital simulation technique. Perhaps of more fundamental importance is the fact that the principle of superposition is valid for the LTV model which obviously can not be employed for the nonlinear model. This should be very useful in designing motion and force control strategies (Lai et al. 1989).

The accuracy of the LTV model can be further improved by incorporating dry friction effects, which are neglected in the present formulation. This requires defining a variable operating point for the piston velocity. Also, refinement of the coefficient profiles by increasing N_s to 4–6 should enhance accuracy. Further, real gases can be modeled by replacing the ideal gas law given by equation (3) with the appropriate equation of state. Any polytropic process with known n_k can also be included in the formulation. The mass flow rate expression of equation (5) needs to be modified accordingly. Such changes can be incorporated in the LTV model through the forcing functions f_{p1} and f_{p2}, variable time constants τ_1 and τ_2, and mass flow rate linearization constants α_{ik} and β_{ik}. It should be, however, noted that the LTV model will still be efficient computationally.

The LTV model developed here can be modified for proportional valve controlled actuation systems by introducing piecewise linear profiles for the coefficient variations. The analysis of hydraulic actuation systems can also benefit from the proposed LTV formulation. Since it is not as highly nonlinear as the pneumatic system, the LTV model should yield even better results for the hydraulic system.

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References

Andersen, B. W., 1967, *The Analysis and Design of Pneumatic Systems*, Wiley, New York.

Araki, K., 1986, "Frequency Response of a Pneumatic Valve-Controlled Cylinder with an Uneven-Underlap Four-Way Valve, Parts I and II," *J. Fluid Control*, Vol. 16, No. 2, pp. 47–94.

Blackburn, J. F., Reethof, G., and Shearer, J. L., 1960, *Fluid Power Control*, MIT Press.

Botting, L. R., Eynon, G. T., and Foster, K., 1970, "The Response of a High-pressure Penumatic Servomechanism to Step and Sinewave Inputs," *Proc. Instn. Mech. Engrs.*, Vol. 184, No. 1, pp. 993–1012.

Bowns, D. E., and Ballard, R. L., 1972, "Digital Computation for the Analysis of Pneumatic Actuator Systems," *Institution of Mechanical Engineers Proc.*, Vol. 186, pp. 881–889.

Burrows, C. R., and Webb, C. R., 1968, "Simulation of an On-Off

Pneumatic Servomechanism," *Institution of Mechanical Enginers Proc.*, Vol. 182, pp. 631–642.
Burrows, C. R., and Webb, C. R., 1970, "Further Study of a Low-Pressure On-Off Pneumatic Servomechanism," *Institution of Mechanical Engineers Proc.*, Vol. 184, pp. 849–858.
Chen, F. Y., 1977, "Dynamic Response of a Cam-actuated Mechanism with Pneumatic Coupling," *ASME Journal of Engineering for Industry*, Vol. 99, No. 3, pp. 598–603.
DeRusso, P. M. et al., 1965, *State Variables for Engineers*, Wiley, New York.
Doebelin, E. O., 1980, *System Modeling and Response: Theoretical and Experimental Approaches*, Wiley, New York.
Dugundji, J., and Wendell, J. H., 1983, "Some Analysis Methods for Rotating Systems with Periodic Coefficients," *AIAA J.*, Vol. 21, No. 6, pp. 890–897.
Eun, T., Cho, H. S., and Lee, C. W., 1984, "On the Development of a Modified On-off Controller for Pneumatic Servo Mechanisms," *Proc. American Control Conference*, pp. 468–473.
Friedmann, P., and Hammond, C. E., 1977, "Efficient Numerical Treatment of Periodic Systems with Application to Stability Problems," *Int. J. Numerical Methods in Engineering*, Vol. 11, pp. 1117–1136.
Lai, J. Y., Menq, C. H., and Singh, R., 1989, "Accurate Position Control of a Pneumatic Actuator," *Proc. American Control Conference*.
Lieh, J., and Haque, I., 1988, "Parametrically Excited Behavior of a Railway Wheelset," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 110, pp. 8–17.
Liu, S., and Bobrow, J. E., 1987, "An Analysis of a Pneumatic Servo System and its Application to a Computer-Controlled Robot," *Proc. ASME Winter Mtg.*, Vol. 6, pp. 385–392.
McCloy, D., and Martin, H. R., 1980, *The Control of Fluid Power: Analysis and Design*, Ellis Horwood, New York.
Morita, Y. S., Shimizu, M., and Kagawa, T., 1985, "An Analysis on Pneumatic PWM and its Application to a Manipulator," *Proc. of International Symposium on Fluid Control and Measurement*, Tokyo, pp. 3–8.
Noritsugu, T., 1985, "Pulse-Width Modulated Feedback Force Control of a Pneumatically Powered Robot Hand," *Proc. of International Symposium on Fluid Control and Measurement*, Tokyo, pp. 47–52.
Noritsugu, T., 1987, "Development of PWM Mode Electro-pneumatic Servomechanism Part II: Position Control of a Pneumatic Cylinder," *J. Fluid Control*, Vol. 66, pp. 65–80.
Richards, J. A., 1983, *Analysis of Periodically Time-Varying Systems*, Springer-Verlag, Berlin.
Scavarda, S. et al., 1987, "Linearized Models for an Electropneumatic Cylinder Servovalve System," *Proc. 3rd. Int. Conference in Advanced Robotics*, pp. 149–160.
Shearer, J. L., 1956, "Nonlinear Analog Study of a High Pressure Pneumatic Servomechanism," *Trans. ASME*, Paper No. 56-IRD-1, pp. 465–472.
Thayer, W. J., 1988, "Electropneumatic Servoactuation—An Alternative to Hydraulics for Some Low Power Applications," *Moog Technical Bulletin*, No. 151.
Wang, Y. T., and Singh, R., 1986, "Pneumatic Chamber Nonlinearities," ASME *Journal of Applied Mechanics*, Vol. 53, pp. 956–58.
Wang, Y. T., and Singh, R., 1987, "Frequency Response of a Nonlinear Pneumatic System," ASME *Journal of Applied Mechanics*, Vol. 54, pp. 209–214.
Wang, Y. T. et al., 1984, "Computer Simulation of a Shock-Absorbing Pneumatic Cylinder," *J. Sound and Vibration*, Vol. 93, pp. 353–364.