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AN ACOUSTIC TRANSFER MATRIX MODEL FOR A REFRIGERATION CONDENSER

1. INTRODUCTION

Several previous investigators [1–3] have used acoustic transfer-matrix theory to model pressure pulsations in refrigeration systems. In each of these studies, the condenser and evaporator were treated as anechoïc ducts. Although this approach is mathematically convenient, it may not be adequate for predicting the behavior of real refrigeration systems. This is particularly true when one is interested in predicting the performance of reactive mufflers. Munjal [4], Munjal and Sreenath [5], and Crocker [6] have demonstrated that for internal combustion engines muffler performance is strongly influenced by the acoustic impedances of the source and load between which it is placed. Similar results might be expected for refrigeration systems with reciprocating compressors.

In this communication, the technique developed in references [1–3] is extended. A more realistic acoustic model for a refrigeration condenser which accounts for the effects of thermal conditions is developed using the transfer matrix method. To do this, solutions for the steady-state heat transfer in the condenser are obtained and used to construct a transfer matrix acoustic model of variable cross-sectional area. This permits study of the acoustic interaction between the compressor and condenser.

2. CONDENSER THERMAL MODEL

The condenser is considered to be a concentric tube heat exchanger, as shown in Figure 1, with refrigerant flowing through the inner tube of diameter d and area S, and cooling water flowing through the annulus. The outer tube is assumed to be well insulated, and the overall heat transfer coefficient between the fluids, U, incorporates convection resistances on both sides of the center tube and the conduction resistance of the tube wall. The refrigerant temperature, T_r, the cooling water temperature, T_w, and the quality of the refrigerant two-phase mixture, X, are assumed to vary only with the axial co-ordinate, x.

As Figure 1 indicates, the condenser may be considered as having three distinct regions. In the regions near the inlet and outlet, the refrigerant exists entirely as vapor and liquid, respectively. In the center region, it is a two-phase mixture. In either of the single-phase

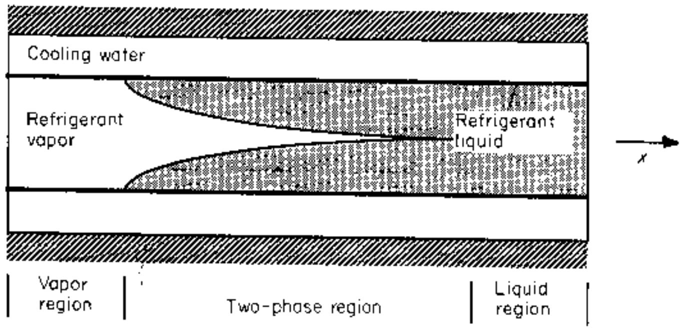

Figure 1. Schematic of water-cooled, concentric tube heat exchanger.

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regions, T_r and T_w are related by [7]

$$dT_r/dx + \Lambda_r(T_r - T_w) = 0, \qquad dT_w/dx + \Lambda_w(T_w - T_r) = 0, \tag{1a, b}$$

$$\Lambda_r = \pi dU/m_rC_r, \qquad \Lambda_w = \pi dU/m_wC_w. \tag{1c, d}$$

Here m_j and C_j are the mass flow rate and specific heat, respectively, of fluid j. Note that U and C_r take on different values in the vapor and liquid single phase regions. The axial co-ordinate x is a local co-ordinate with origin at the beginning of each region. The system (1) is solved subject to initial conditions $T_r(0) = T_{ir}$ and $T_w(0) = T_{iw}$. Solutions are

$$T_r(x) = (T_{ir} - \xi)\, e^{-(\Lambda_r + \Lambda_w)x} + \xi, \qquad T_w(x) = (T_{iw} - \xi)\, e^{-(\Lambda_r + \Lambda_w)x} + \xi \tag{2a, b}$$

$$\xi = (T_{ir}\Lambda_r + T_{iw}\Lambda_w)/(\Lambda_r + \Lambda_w). \tag{2c}$$

The temperature distribution given by equations (2) is valid to the point where the refrigerant vapor begins to condense. At this point, where T_r reaches the saturation temperature, T_{sat}, the two-phase region begins. Note that T_{sat} is determined by the condenser mean pressure. In this second region, $T_r = T_{sat}$ throughout while the refrigerant quality, X, satisfies [7]

$$dX/dx + \Lambda_{fg}(T_{sat} - T_w) = 0, \qquad \Lambda_{fg} = \pi dU/m_r h_{fg}, \tag{3a, b}$$

where h_{fg} is the refrigerant enthalpy of vaporization.

The cooling water temperature T_w again satisfies equation (1b) in the two-phase region [7]. In equations (1b) and (3a), however, U is evaluated to account for the new heat transfer conditions in the two-phase region, and the value for Λ_w is recalculated. Equations (3a) and (1b) are solved subject to initial values $X(0) = 1$ and $T_w(0) = T^*_{iw}$, where T^*_{iw} is the cooling water temperature given by equation (2b) at the end of the vapor region. Solutions in the two-phase region are

$$X(x) = (\Lambda_{fg}/\Lambda_w)(T_{sat} - T^*_{iw})(e^{-\Lambda_w x} - 1) + 1, \qquad T_w(x) = (T^*_{iw} - T_{sat})\, e^{-\Lambda_w x} + T_{sat}. \tag{4a, b}$$

Equation (4a) indicates that for $(\Lambda_{fg}/\Delta_w)(T_{sat} - T^*_{iw}) \geqslant 1$, X will eventually become zero. If the co-ordinate x where this occurs lies within the finite specified length of the condenser, the two-phase region must end, since negative values of X are meaningless. Beyond this point, the refrigerant exists as a liquid. The fluid temperatures in this region again satisfy (1), with appropriate values of Λ_r and Λ_w applied. Solutions given by equations (2) are again valid, but with T_{ir} and T_{iw} taken as the temperatures at the end of the two-phase region.

Zivi [8] found that for film-type condensation X is related to the part of the tube cross sectional area occupied by vapor, S_g, by

$$\frac{S_g}{S} = \left[1 + \left(\frac{1-X}{X}\right)\left(\frac{\rho_g}{\rho_f}\right)^{2/3}\right]^{-1}, \tag{5}$$

where S is the total tube area and ρ_g and ρ_f are the densities of the refrigerant vapor and liquid, respectively.

3. CONDENSER ACOUSTIC MODEL

Consider a rigid duct of uniform cross-section S and length L which is free of damping. For such a duct, the acoustic pressure, p, and the volume velocity, Q, at the ends of the duct are related by [9]

$$\begin{Bmatrix} p_1(f) \\ Q_1(f) \end{Bmatrix} = \begin{bmatrix} \cos\dfrac{2\pi fL}{c} & j\dfrac{\rho c}{S}\sin\dfrac{2\pi fL}{c} \\ j\dfrac{S}{\rho c}\sin\dfrac{2\pi fL}{c} & \cos\dfrac{2\pi fL}{c} \end{bmatrix} \begin{Bmatrix} p_2(f) \\ Q_2(f) \end{Bmatrix}, \tag{6}$$

where f is the frequency, c is the speed of sound, and j is the imaginary unit. Equation (6) may be used to model the vapor region of the condenser as a duct of area S filled entirely with refrigerant vapor and of length determined by the value of x from equations (2) when $T_r = T_{sat}$.

Physical property data for refrigerant 12 (CCl_2F_2) indicate that $\rho_f \approx 10\,\rho_g$ [10] and $c_f \approx 3c_g$ [11]. Hence, the characteristic acoustic impedance of the liquid, $\rho_f c_f$, is approximately 30 times that of the vapor. Because of this, it is reasonable to approximate the liquid/vapor interface in the two-phase region as an acoustically rigid boundary. This implies that an acoustic wave incident upon the interface is principally reflected back into the vapor rather than propagating through the liquid as previous investigators have assumed [1-3].

These facts may be used to construct an approximate acoustical model for the condenser two-phase region. Although S_g, computed from equations (4a) and (5), varies continuously with x, it may be approximated by a series of segments of uniform cross-section. This results in a "stepped tube" model for the two-phase region as shown in Figure 2. The number of steps used and the area of each are specified by the analyst such that the approximation to $S_g(x)$ is reasonable. The length of each element is then determined from equations (4a) and (5). For example, the first element might be chosen to have area $S_g = 0{\cdot}95\,S$. Its length would be the total axial displacement where $1{\cdot}0 < S_g/S < 0{\cdot}9$. Equation (6) is then applied to each element. The end of the final element is treated as an acoustically rigid boundary, again based on relative properties of the liquid and the vapor.

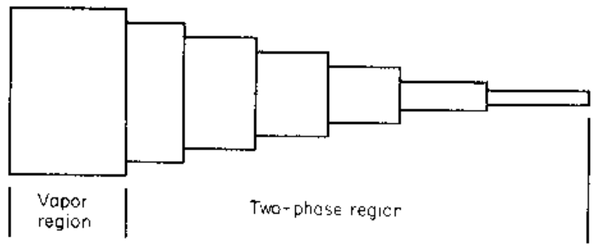

Figure 2. "Stepped tube" acoustic model for the condenser.

An acoustic model for the whole condenser is obtained by multiplying together the transfer matrices for the vapor region and each of the elements in the two-phase region. Such a model may be directly substituted for the anechoïc discharge line used in compressor and manifold models developed by Singh and Soedel [1]. This permits more realistic simulation of the performance of actual refrigeration systems.

4. CONCLUDING REMARKS

An approximate acoustic transfer matrix model for condensers in vapor compression refrigeration systems has been developed. This work is a first attempt to account for the effects of thermal variables on the acoustical performance of refrigeration systems. Although a simple and mathematically tractable heat exchanger geometry was considered, the method proposed for constructing a "stepped tube" acoustic model could be applied to real condensers with more complicated heat transfer. This method could also be applied to evaporators for analysis of suction pressure pulsations.

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