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ERROR ASSOCIATED WITH A REDUCED ORDER LINEAR MODEL OF A SPUR GEAR PAIR

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1. FORMULATION

An appropriate dynamic model of a spur gear pair must include the coupling between the transverse and torsional motions at the gear mesh [1–7]. Although detailed numerical models with large degrees of freedom based on the transfer matrix method [6] or the finite element method [5, 7] are available, reduced order analytical models are preferred for design calculations or for non-linear analyses [1–4]. In this communication, we propose such a model, and determine the associated error in the undamped eigensolution by a comparison with a benchmark finite element model [5].

The example case, as shown in Figure 1(a), consists of two identical spur gears of mass M, moment of inertia J and base circle diameter D, two identical flexible shafts of mass m and diameter d, and four identical rolling element bearings of radial stiffness K. The gear mesh constraint is represented by a linear, time-invariant (average) mesh stiffness k_m. When the shafts and bearings are assumed to be rigid, a two-degree-of-freedom,

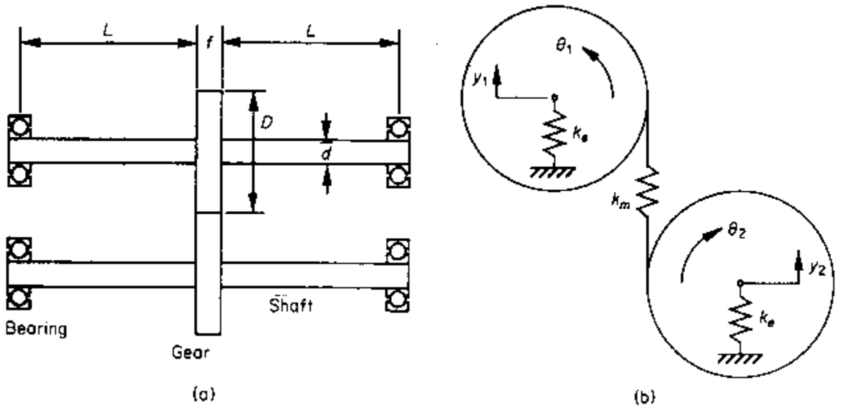

Figure 1. (a) Physical system and dimensions; (b) proposed model.

semi-definite, torsional model can be developed. Defining $q(t) = D(\theta_1(t) - \theta_2(t))/2$ and $m_c = 2J/D^2$, this model is reduced to a single-degree-of-freedom, torsional system with natural frequency $\hat{\omega}_1 = \sqrt{k_m/m_c}$. However, in practical geared systems, shafts and/or bearings are typically compliant. Therefore, $\hat{\omega}_1$, as predicted by the single-degree-of-freedom torsional model, may involve a large amount of error. Accordingly, a three-degree-of-freedom torsional-transverse model is proposed in Figure 1(b). Here, for a single-geared shaft, the fundamental mode shape is assumed to be given by the static deflection curve,

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and the corresponding equivalent transverse stiffness k of a shaft with simple supports at both ends and with a gear in the middle is $k=(3\pi Ed^4)/(32L^3)$, where E is the modulus of elasticity. The combined shaft-bearing stiffness k_e is found by considering K and k in series as $k_e=(2kK)/(2K+k)$. The governing undamped equations of motion and the natural frequencies ω_i $(i=1,2,3)$ resulting from the corresponding eigensolution are given as follows:

$$\begin{bmatrix} m_c & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_m & k_m & -k_m \\ k_m & (k_m+k_e) & -k_m \\ -k_m & -k_m & (k_m+k_e) \end{bmatrix} \begin{Bmatrix} q \\ y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{1}$$

$$\omega_{1,3}^2 = \frac{(k_mM+(2k_m+k_e)m_c) \pm \sqrt{(k_mM+(2k_m+k_e)m_c)^2 - 4Mm_ck_mk_e}}{2Mm_c}, \tag{2a}$$

$$\omega_2 = \sqrt{\frac{k_e}{M}}\,. \tag{2b}$$

2. RESULTS

The natural modes Φ_i corresponding to equations (1) and (2) are illustrated in Figure 2 and described below; these are the only ones excited by the kinematic transmission error excitation at the gear mesh point [5]. (a) First coupled transverse-torsional mode Φ_1: the gear centers translate and rotate in the opposite directions; consequently transverse and torsional motions cancel each other to yield a small relative displacement $q(t)$ at the gear mesh, as illustrated in Figure 2(a). (b) Purely transverse mode Φ_2: there is no torsional vibration in this mode as both gear centers translate in phase along the pressure line direction y without any rotation, as shown in Figure 2(b), and consequently $q(t)=0$ since the gear ratio is one. (c) Second coupled transverse-torsional mode Φ_3: as evident from Figure 2(c), both transverse and torsional motions are additive at the gear mesh. The calculated modes Φ_i are close to the predictions Φ_i^* of the benchmark finite element model [5], as is evident from Table 1.

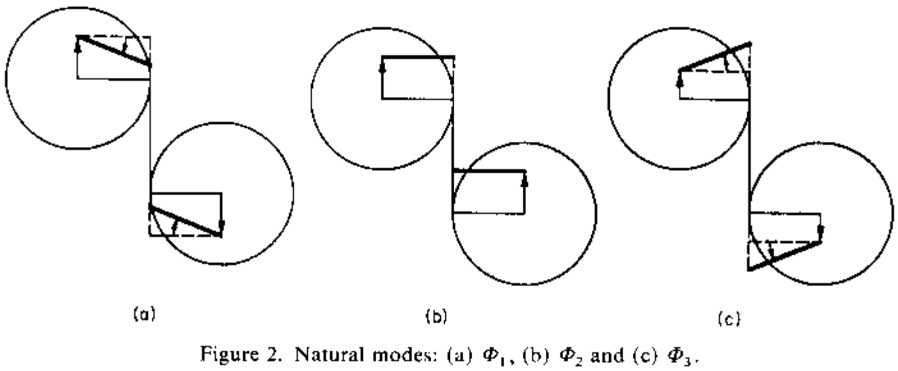

Figure 2. Natural modes: (a) Φ_1, (b) Φ_2 and (c) Φ_3.

The calculated natural frequencies ω_i of the proposed model differ from the natural frequencies ω_i^* predicted by the benchmark finite element model [5], since the proposed reduced order model does not account (i) the masses and inertias of the shafts, and (ii) the rotor-dynamic effects of the continuous shaft, especially at higher frequencies. The error is defined as $e_i=(\omega_i-\omega_i^*)/\omega_i$. We define two dimensionless parameters, L/D and $M/m=fD^2/2Ld^2$ for error analysis. In Figure 3 is shown, on a log-log scale, a set of

TABLE 1

Comparison of modal vectors

	Proposed model			Finite element model [5]		
	Φ_1	Φ_2	Φ_3	Φ_1^*	Φ_2^*	Φ_3^*
q	1	0	1	1	0	1
y_1	−0·441	1	0·190	−0·439	1	0·283
y_2	0·441	1	−0·190	0·439	1	−0·283

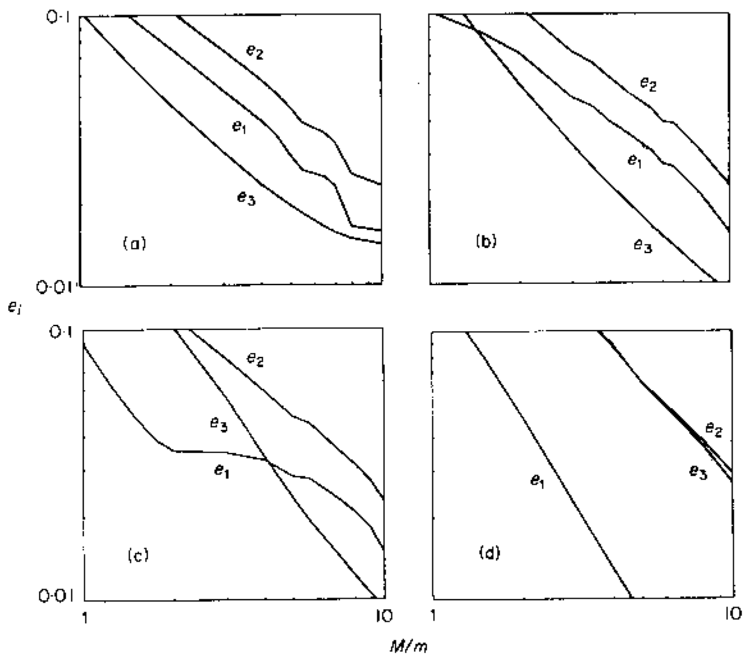

Figure 3. Errors in natural frequency predictions for $K/k_m = 1000$ and $L/D =$ (a) 1·0, (b) 0·75, (c) 0·5 and (d) 0·25.

error curves as a function of M/m for a given ratio of L/D. Here, the bearings are assumed to be very stiff, say $K/k_m = 1000$, so that $k_e \to k$. It is suggested in Figure 3 that the proposed model is better for a highly compliant shaft. Error bounds as a function of M/m for several bearing stiffness values are given in Figure 4. Note that e_1 and e_2 become smaller when the bearings are softer, but e_3 remains almost the same.

As evident from Figures 3 and 4, $e_i < 0{\cdot}1$. This suggests that the proposed model, which has already been used for the non-linear studies [1, 2, 4], is indeed reasonable. One can even improve its predictions using the error curves of Figures 3 and 4. For example, consider $k_m = 1 \times 10^8$ N/m, $D = 0{\cdot}2$ m, $f = 0{\cdot}04$ m, $d = 0{\cdot}0283$ m, $L = 0{\cdot}2$ m and $K/k_m = 1000$, which yield $k_e = 4{\cdot}877 \times 10^6$ N/m and $m_e = 2{\cdot}45$ kg. Then, equation (2) predicts $\omega_{1,2,3}$ as 91·5, 107 and 1247 Hz. Errors for corresponding dimensionless parameters $M/m = 5$ and $L/D = 1$ are estimated from Figure 3 as 0·03, 0·045 and 0·0195. Hence, adjusted values of $\omega_{1,2,3} = 89$, 107 and 1222 Hz should be used.

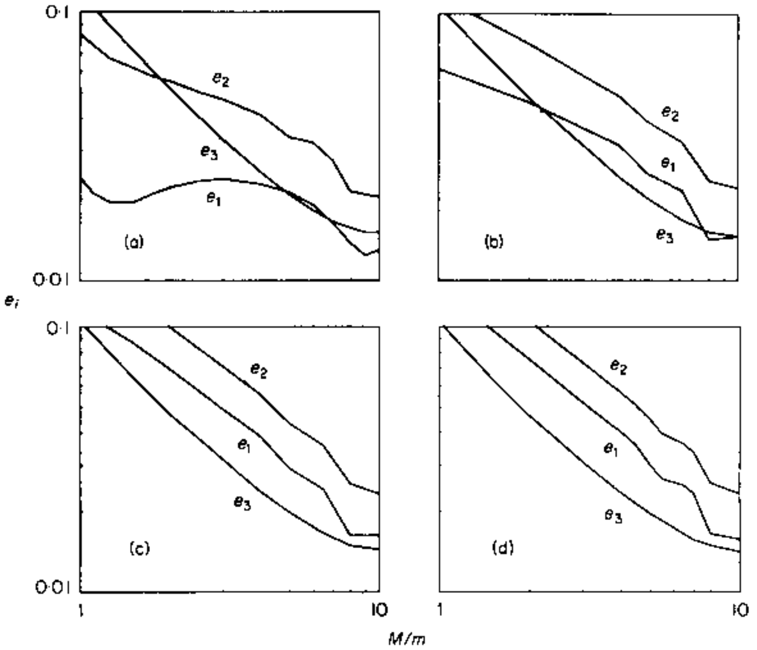

Figure 4. Errors in natural frequency predictions for $L/D = 1{\cdot}0$ and K/k_m = (a) 1, (b) 2, (c) 10 and (d) 1000.

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