New Rating Indices for Gear Noise Based Upon Vibro-Acoustic Measurements*

Case Study

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This article presents a new method of computing subjective rating indices for gear whine (steady state noise at the gear meshing frequency) using frequency domain objective descriptors obtained from physical, narrow band, vibro-acoustic measurements such as radiated sound pressure and gearbox acceleration levels. The proposed method makes use of spatially, spectrally, and temporally weighted averages performed over a range of operating conditions to obtain several indices which are found to be compatible with human subjective response under certain conditions. Once established, the resulting gear whine index is free from the inconsistencies of human auditors and further, the noise rating criteria remain constant over time. Variations of this new rating method may be applicable to other types of quasi-steady state machinery noise problems which exhibit similar vibro-acoustic signatures.

Nomenclature

A	Number of spectral averages, integer
B	Analysis bandwidth (Hz)
E	Weighted mesh energy estimator
f	Frequency (Hz)
Δf	Frequency resolution (Hz)
g	Gear mesh frequency multiplier, integer
h	Sampling period (seconds)
i	Data record index, integer
k	Frequency index, integer
N	Number of points in discrete frequency analysis, integer
α	Scalar weighting parameter
ψ^2	Band-limited, weighted mean square value
σ_{rad}	Radiation efficiency of gearbox
Ω	Rotational speed (rpm)
q	Analysis half-bandwidth, integer
r	Mesh harmonic index, integer
$\mathbf{r}$	Position vector
ρ	Correlation coefficient
s	Number of transducer signals
T	Mean torque
t	time (seconds)
w	Weighting function
$\bar{w}$	Overall weighting function
W	One-sided, discrete, auto power spectrum
x	Measured gear noise signal
X	Discrete Fourier transform of x
Z	Number of gear teeth, integer

Measured Signal Levels

L_a	Gearbox housing vibration acceleration level, dB
L_{w}	Perceived gear whine level, dB
L_p	Sound pressure level, dB
L_v	Gearbox housing vibration velocity level, dB
L_W	Sound power level, dB

Subscripts

c	Center frequency
g	Gear mesh
r	Gear mesh harmonic index
s	Shaft
x	Transducer signal

Superscripts

* Spatially averaged

† Overall estimator

Introduction

Gear noise has always been a concern among the designers and manufacturers of various consumer products such as automobiles, power tools, and household appliances, since it has a significant influence on the perceived quality and performance of a product by its user. Accordingly, strategies for gear noise reduction usually focus on reducing perceived noise levels or annoyance rather than reducing absolute sound pressure or power levels. Typically, the gear noise generated by such a product is evaluated subjectively by appropriately trained auditors or selected juries representing typical product users or customers.[1-7]

Frequently, a single numerical index is assigned to rate product noise and its acceptance on some arbitrary, predetermined scale.[7] However, the effectiveness of using such subjective ratings as an engineering tool for diagnostic purposes or to assess various engineering changes on the reduction of objectionable gear noise is limited because of several reasons. First, the resolution of a given subjective rating is limited by the auditor's ability to detect small changes in the character of the gear noise. Many times, gear noise is masked by noises from other sources both internal and external to the product. Second, many products must operate over a range of speeds and loading conditions. Dynamic gear mesh force, which is the primary exciter of gear noise, is known to be highly dependent on the mean transmitted torque and system gear pair dynamics.[8] Further, system resonant behavior may significantly alter the amplitude and frequency content of the resulting gear noise. Product noise may be acceptable under certain operating conditions while unacceptable at others. In such cases, an overall rating index which accurately describes the acceptability of a particular product over its full range of operation is extremely difficult to obtain. Third, if a system containing multiple gear meshes is considered, the ability to distinguish subjectively between noise generated by individual gear meshes whose mesh frequencies and/or their harmonics coincide or nearly coincide at certain operating speeds may be impossible. Beating phenomenon, amplitude modulation and frequency modulation effects may also exist which add further complexity to the gear noise. Finally, the repeatability of a subjective rating and correlation between individual auditors is always of concern. Personal inconsistencies among auditors and the inability to maintain uniform reference indices are responsible for sometimes large variations in subjective rating criteria, especially during those gear design and noise reduction programs which continue by necessity over relatively long periods of time.

Nature of Gear Noise

Gear noise may be divided into two main categories: gear whine and gear rattle. Gear rattle is a highly nonlinear, impulsive phenomenon which generally occurs under lightly loaded conditions and consists of repeated gear tooth impacts through backlash due to torsional vibrations of the geared system.[9] Gear whine, which is the focus of this article, is a continuous, steady state sound which is primarily attributed to transmission error and elastic deformations of the gear teeth under load.[10] Gear whine normally manifests itself at gear meshing frequency f_g and its harmonics. Gear meshing frequency f_g corresponds to the frequency of gear tooth engagement and is given by

$$f_g = Z\,\Omega / 60 \qquad (1)$$

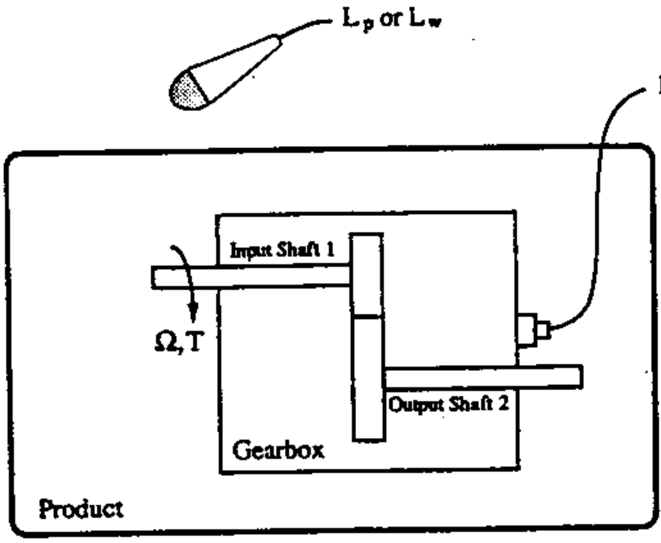
where f_g is in Hz, Ω is the shaft rotational speed in rpm and Z is the integer number of gear teeth on the respective gear. Gear whine tone is often modulated by both amplitude and frequency modulation effects due to mounting eccentricities, radial stiffness variations of the gear wheels, and other design and manufacturing factors.[11] These modulation effects give rise to sidebands about f_g and its harmonics as shown later. For any enclosed gear drive, the primary radiator is the gearbox housing and attached panels which are excited by gear mesh forces transmitted structurally through shafting, bearings and other internal components.

Problem Formulation

Several investigators have developed measurement and diagnostic techniques which may be used to quantify gear noise.[12] However, very few of these techniques provide indices specifically intended to correlate with human subjective response. Methods have been developed for gear rattle using housing acceleration measurements, but we are unaware of any prior efforts to develop a similar perception rating for gear whine.[5,6] This article presents a new method to compute subjective rating indices for gear whine based upon objective descriptors of physical vibro-acoustic measurements such as sound pressure level L_p, sound power level L_w, and gearbox acceleration L_a and velocity levels L_v. The method makes use of spatially, spectrally, and temporally weighted averages performed over a range of operating conditions to obtain several indices which have been shown to correlate with subjective response under certain conditions. This technique is intended to augment, not replace, traditional subjective rating methods during the product development phase to reduce objectionable gear noise. The system of Fig. 1, which consists of a generic product containing a single mesh gear drive, is used as an example throughout much of this document which focuses primarily on steady-state operation under constant loading conditions.

Development of a Rating Scheme

Vibro-Acoustic Measurements. Consider the geared system shown in Fig. 1 which is operating at shaft rotational speed Ω and mean transmitted torque T. Typical gear whine spectra



Gearbox Characteristics

	Input Side	System	Output Side
Number of Teeth	$Z_1 = 20$		$Z_2 = 50$
Shaft Frequency, Hz	$f_{s1} = \Omega/60$		$f_{s2} = 0.4\, f_{s1}$
Gear Mesh Frequency, Hz		$f_g = 20\, f_{s1}$	
Operating Speed, rpm		Ω	
Mean Torque, N-m		T	

Figure 1. Physical example: generic product with single mesh gear drive

for this system, as discussed earlier, are shown in Fig. 2. Gear noise radiated by a particular product is highly dependent upon the structureborne (dominant) and airborne paths by which the sound energy is transferred from the gear mesh source to a position, say r, near the operator's or user's ear. For a typical product, sound pressure level $L_p(t) = L_p[\Omega(t),r(t),T(t)]$ is a function of operating speed $\Omega(t)$, torque $T(t)$ and position vector $r(t)$, all of which may vary in time. Accordingly, a logical measurement to use as a basis for a rating index is L_p at position r near the listener's ear. For cases where r varies according to product use, it may be more prudent to measure sound power level L_w. Since the development of noise rating indices with products having variable operating conditions is normally very difficult, the current discussion is limited to the special case where Ω, T and r are nearly time-invariant for clarity. Under many circumstances, the measurement of L_p or L_w is not practical and other measurements, such as gearbox acceleration or velocity, must be made. This is valid since L_w radiated from the gearbox housing and attached panels is known to be proportional to the mean square vibrational velocity of the radiator L_v, and the radiation efficiency of the panel σ_{rad}.[19] Accordingly, L_v may also be used as a basis for a rating index which should correlate with rating indices formed from L_p under certain circumstances. Furthermore, if Ω is held constant, L_a, which is related to L_v by a factor equal to the frequency of interest, may be used to approximate L_v over a relatively small frequency bandwidth. Empirical evidence has shown that gearbox vibration measurements are usually superior to acoustical measurements for the implementation of such methods.[5-7] Vibration measurements are typically easier to perform, contain less extraneous information, and provide increased resolution and repeatability because of their direct proximity to the gear mesh source. Vibration measurements are clearly

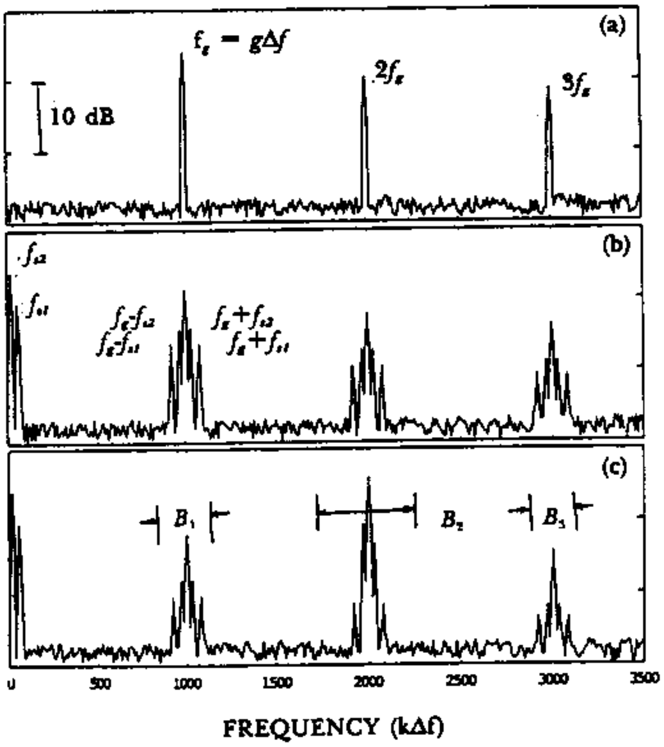


Figure 2. Typical gear noise auto power spectra for the system of Fig. 1 at operating speed $\Omega = 3000$ rpm ($f_g = 1000$ Hz): (a) classical gear whine spectrum; (b) classical gear whine spectrum with substantial modulation sidebands; (c) system resonance excited by second harmonic of gear mesh frequency, also with substantial modulation sidebands

desirable for in-situ product testing, such as production line or dynamometer testing, especially when ambient noise levels are high and isolation of gear whine from other masking noises is not possible.

Noise Perception Issues. Five factors which influence noise perception indices such as perceived noise level, PNL, are (1) intensity and frequency content, (2) concentration of spectral energy, (3) duration, (4) rise time and (5) impulsiveness.[14] Miller suggests that physical sound measurements may be weighted accordingly to obtain perceived noise levels which are consistent with subjective ratings.[15] However, there is much debate as to the significance of each of these factors. Most of the extensive research in psycho-acoustics has dealt with loudness perception, frequency selectivity, temporal resolution, pitch perception, spatial perception, pattern recognition, speech intelligibility and annoyance using synthesized audio stimuli presented to subjects under closely controlled laboratory conditions.[16] The development of subjective rating indices for machinery noise with complex signatures presents a formidable challenge. Nonetheless, several investigators have developed noise quality rating criteria with limited success, especially in the automotive field.[2-6]

Croker, *et. al.*, and Johnson and Hirami have developed rating indices for gear rattle based upon gearbox housing acceleration measurements.[5-6] In these studies, human subjects consisting of engineers and rattle experts were presented with several transient gear rattle audio stimuli and then required to express both preference and distance responses. Multi-dimensional scaling techniques were then employed to determine the dominate dimensions of the subjective response. Several time domain and frequency domain objective descriptors were calculated from the measured vibration data and correlated with each dominate dimension of the subjective response. A rattle index was then formed from a combination of *time-domain* objective descriptors, which correlated to an acceptable degree with the subjective response, using multivariate regression analysis.

The steady-state gear whine phenomenon is entirely different than the transient gear rattle phenomenon. Consequently, different *frequency-domain* objective descriptors were investigated in order to obtain an index which correlates to an acceptable degree with subjective rating of gear whine. The use of linear and A-weighted overall, 1/1 octave and 1/3 octave L_p and L_a measurements was investigated and found not to correlate well with noise perference ratings. This observation is consistent with the findings of other investigators.[2-7]

Frequency Domain Descriptors. In this development we consider mostly the content and energy concentration in auto power spectra on a narrow band basis. Consider a discrete, one-sided, narrow band, auto power spectrum $W_{xx}(k\Delta f)$ of a gear noise signal $x(t)$ which may represent sound pressure level, sound power level, gearbox acceleration, etc.[17]

$$W_{xx}(k\Delta f) = \frac{2}{AN^2h^2}\sum_{i=1}^{A}|X_i(k\Delta f)|^2 \quad k=0,\ 1,\ldots,N/2 \qquad (2a)$$

where $X_i(k\Delta f)$ is the discrete Fourier transform of the i *th* time record $x_i(nh)$ given by

$$X_i(k\Delta f) = h\sum_{n=0}^{N-1} x_i(nh)\exp\left(\frac{-2j\pi kn}{N}\right) \begin{matrix} i=1,\ 2,\ldots\ldots,A \\ k=0,1,2,\ldots,N\text{-}1\ . \end{matrix} \qquad (2b)$$

Here $j=\sqrt{-1}$ and $x_i(nh)$ represents successive digitized records of $x(t)$ sampled at N equally spaced points a distance h apart. The number of spectral averages is denoted by A and the frequency resolution is given by $\Delta f=1/Nh$. Frequency analysis parameters N and h should been chosen such that $f_g \cong g\ \Delta f$ as shown in Fig. 2 where g is an integer multiplier. Further, the analysis frequency range must include a sufficient number of gear mesh harmonics as discussed later. We further assume that the necessary low pass filtering and windowing operations have been performed on the measured data in order to avoid aliasing and leakage effects respectively.[17]

Now consider a frequency interval centered about the r *th* harmonic of f_g or g with bandwidth $B_r=2q_r\Delta f$ where q_r is an integer. We proceed on the premise that B_r is less than the critical bandwidth at center frequency $f_{cr}=r\,g\,\Delta f$.[18] Hence, the perceived sound energy within this band may be considered to be proportional to the weighted mean square value Ψ_r^2 computed over this same frequency band

$$\Psi_r^2 = \sum_{k=rg-q_r}^{rg+q_r} w_r(k-rg)\ W_{xx}(k\Delta f) \qquad (3a)$$

$$\text{where } w_r(k) = \begin{cases} w_r(k) & -q_r\leq k\leq q_r \\ 0 & \text{otherwise} \end{cases} \qquad (3b)$$

Note that the B_r may be chosen separately for each harmonic of f_g considered as illustrated in Fig. 2(c). Equation (3) assigns a weighting to each of the spectral components within the B_r via the weighting function $w_r(k)$. Practical experience has shown that a uniform weighting works well for most cases. However, at other times, it is desirable to weight sidebands differently from mesh harmonics; especially when the bandwidth of significant modulation sidebands about a particular harmonic of gear mesh frequency exceeds the respective critical bandwidth or if modulation effects are clearly perceptible by human subjects.

We now define an objective descriptor E_m which has been found to correlate with subjective gear whine ratings under certain circumstances. This weighted estimator E_m represents the perceived acoustical or vibrational energy associated with the gear mesh and is formed from a linear combination of the Ψ_r^2 computed about m harmonics of f_g

$$E_m = \sum_{r=1}^{m}\alpha_r\Psi_r^2 \qquad (4)$$

Here the α_r are scalar weighting parameters which are highly frequency dependent and may include such effects as human loudness perception at each center frequency f_{cr} as well as the transmission ratio between the band-limited energy of measured response signal $x(t)$ and the band-limited sound energy level at the listener's ear. Determination of the α_r will be discussed later. The analysis frequency range defined by $N\Delta f$ must be chosen sufficiently large to include the desired number of mesh harmonics m. Further, the upper and lower analysis bandwidths $B_1 = 2\, q_1 \Delta f$ and $B_m = 2\, q_m \Delta f$ must satisfy the conditions $g - q_1 \geq 1$ and $mg + q_m \leq N/2$. Once the B_r, w_r, α_r and m have been determined using the method outlined in the next section, they may be combined to form a single overall weighting function $\bar{w}(k)$ defined on the interval $k=[0,N/2]$ for efficient real time implementation using a programmable spectrum analyzer or equivalent digital data analysis system

$$E_m = \sum_{k=0}^{N/2} \bar{w}(k)\ W_m(k\Delta f) \qquad (5a)$$

$$\text{where } \bar{w}(k) = \begin{cases} \alpha_r w_r(k) & rg - q_r \leq k \leq rg + q_r \quad r = 1,2,\ldots,m \\ 0 & \text{otherwise} \end{cases} \qquad (5b)$$

We note that overlapping of realistic B_r is not likely for systems containing only a single gear pair due to manufacturing restrictions on the minimum number of gear teeth. We define the *perceived gear whine level* L_m by expressing the objective descriptor E_m in decibel (dB) units as follows where E_{ref} is an arbitrary reference value chosen to obtain convenient numerical values of L_m

$$L_m = 10 \log_{10} \left(\frac{E_m}{E_{ref}}\right) \qquad (6)$$

Determination of Weighting Parameters

The various weighting parameters B_r, w_r and α_r must be found empirically for each product considered using a process analogous to adjusting gains in an automatic control system. No analytical formulas exist for their determination. However, useful parameter values can be estimated given a few guidelines and the benefit of prior experience with similar product noise problems. The procedure is shown in the flow diagram of Fig. 3 and illustrated in two examples which follow.

Once vibro-acoustical data and corresponding subjective ratings have been collected from a sample of units, trial weighting functions w_r can be chosen and values of Ψ_r^2 computed for various B_r ranging from 2 to 15 percent of gear mesh frequency. The optimal widths of the B_r depend heavily upon the nature of modulation sidebands. A statistical regression analysis should then be performed to determine the m, α_r and B_r values which provide the best fit between E_m or L_m and subjective rating.[19] Statistical preconditioning of the subjective data may also be required prior to regression analysis. Normally uniform w_r are chosen as the first trial functions and m typically ranges from 1 to 5 harmonics. If a satisfactory correlation is not obtained, the entire process is repeated with new w_r. Once suitable weighting parameters are found,

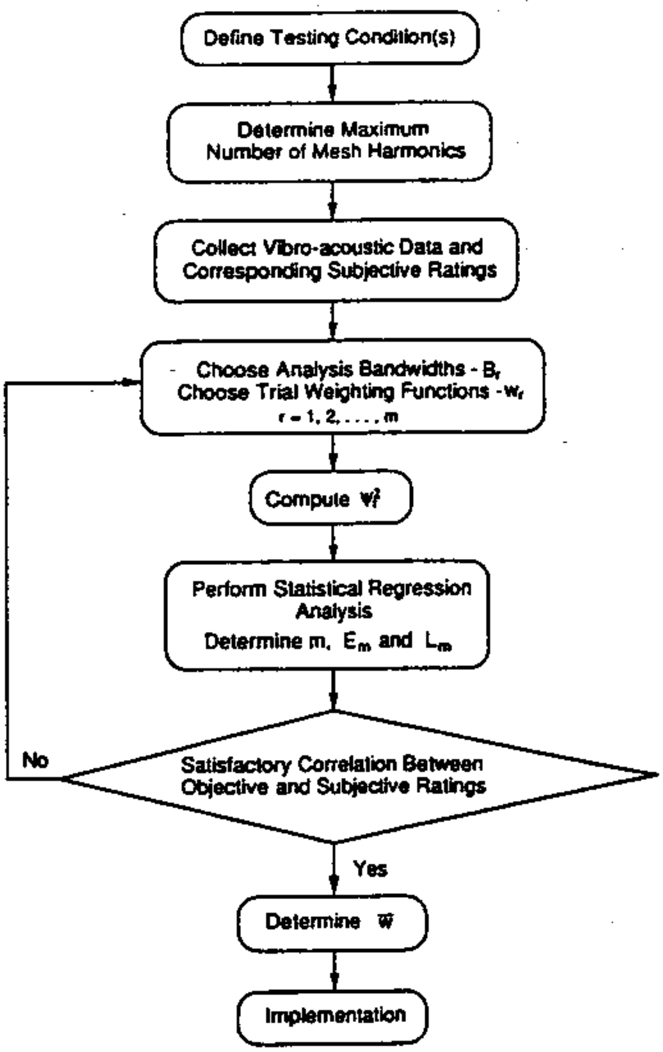


Figure 3. Flow diagram illustrating procedure to determine weighting parameters and weighting functions for a single mesh gear system operating at steady state condition

$\bar{w}(k)$ is determined from (5b) and future values of E_m and L_m are computed from (5a) and (6) respectively. We note that if uniform w_r are employed, the process is reduced to determining only the proper values of m, α_r and B_r. Furthermore, if N is initially chosen such that several gear mesh harmonics are included in $W_m(k\Delta f)$, then several Ψ_r^2 can be computed prior to the regression analysis using different values of B_r. A single regression analysis can then be conducted to determine the values of m, α_r and B_r which result in the best fit.

Simulation Example

Consider the three simulated gearbox vibration spectra shown in Fig. 2 each with $f_g = 1000$ Hz corresponding to input shaft speed $\Omega = 3000$ rpm. Each spectrum represents characteristics commonly found in measured gear noise spectra:

(2a) classical gear whine spectrum; (2b) classical gear whine spectrum with substantial modulation sidebands; and (2c) system resonance excited by second mesh harmonic, also with substantial modulation sidebands.[12] For illustration purposes, let $m=3$, $\alpha_r=1$ and consider unity valued $w_r(k)$. Values of Ψ_r^2 corresponding to $r=1$ through 3 and L_r in units of decibels are listed in Table 1 for cases 2a-2c using three analysis bandwidths $B_r=20$, 50, and 150 Hz. Here E_{ref} is chosen such that $L_r=0$ dB for case a when $B_r=20$ Hz. Although the character of each spectrum is quite different, it is possible to obtain almost any desired relationship between the various L_r values given in Table 1.

TABLE 1
COMPARISON OF BAND-LIMITED PERCEIVED GEAR MESH ENERGY VALUES (Ψ_r^2) AND PERCEIVED WHINE LEVEL (L_3) FOR THE EXAMPLE GEAR WHINE SPECTRA OF FIGS. 2A-2C GIVEN $m=3$, $\alpha_r=1$ AND UNITY VALUED w_r*

	Case 2a				Case 2b				Case 2c			
B_r	Ψ_1^2	Ψ_2^2	Ψ_3^2	L_r	Ψ_1^2	Ψ_2^2	Ψ_3^2	L_r	Ψ_1^2	Ψ_2^2	Ψ_3^2	L_r
20 Hz	-2.5	-5.9	-7.6	0.0	-5.5	-8.7	-9.8	-2.8	-7.8	-1.4	-8.7	0.1
50	-2.4	-5.8	-7.4	0.1	-2.5	-5.8	-7.5	0.0	-6.3	1.0	-7.8	2.2
150	-2.3	-5.5	-7.0	0.2	-0.7	-2.1	-4.7	2.6	-5.2	1.3	-7.7	2.6

*All values are expressed in decibels relative to L_r for Case 2a with $B_r=20$ Hz.

The proper choice of the B_r depends upon the subjective rating assigned to each case by auditors. For a product exhibiting wide spectral variations, it may be possible to develop several E_m descriptors using distinct weighting parameters in order to distinguish between gear whines of different spectral character which are perceived to be of comparable annoyance.

Comparison between various Ψ_r^2 values computed using relatively narrow and wide B_r also provides a useful diagnostic procedure to identify certain types of gear errors such as excessive mounting eccentricities or runout by comparing the energy in the modulation sidebands about f_g. For example, consider the values of Ψ_1^2 when $B_r=20$ and 150 Hz corresponding to the spectra of cases a and b respectively. The value of Ψ_1^2 increases by only 0.2 dB as B_r is increased from 20 to 150 Hz for case a while an increase of 4.8 dB is observed for case *b* which exhibits substantial modulation sidebands. By using the proper non-uniform W_r it may also be possible to identify certain types of errors associated with individual gears. Such information can be very useful to the gear designer or analyst.

Single Mesh Gear Drive Example

Objective and Subjective Descriptors. The viability of the proposed rating scheme is demonstrated through an application example which considers a single mesh gear drive similar to the generic product of Fig. 1. A rating index based upon objective descriptors of measured data was developed which correlated with an existing noise quality rating scale. Eleven units were evaluated independently by three noise quality auditors, each of whom assigned a single numerical index to rate the acceptability of product noise on a predetermined scale from 1 to 10. A value of 1 (noisy) corresponds to intolerable annoyance while a value of 10 (quiet) is assigned to units which exhibit no perceptible gear whine. This subjective evaluation procedure may not be optimum since unavoidable biases are introduced by the predetermined scale and further, no preference response testing methods were employed. However, variations of such subjective evaluation procedures pervade the field of machinery noise quality rating where more sophisticated methods are not practical or cost effective.[7] Any objective descriptor or index must conform to the established noise quality rating methods employed by a particular product design or manufacturing group in order to be adopted by industry.

Measured vibration data were collected from three transducers: a single microphone located at a fixed distance and orientation from each unit (L_p); an accelerometer mounted on the gearbox housing directly over a shaft-bearing interface (L_{a1}); and an accelerometer mounted on a structural support which is attached directly to the gearbox housing (L_{a2}). The accelerometer locations were determined such that the localized dynamic response of the system was relatively insensitive to normal manufacturing errors and transducer mounting variations while being sufficiently close to the gear mesh source. Each unit was operated under identical speed and loading conditions corresponding to input shaft speed $\Omega=1350$ rpm, mesh frequency $f_g=450$ Hz and shaft frequencies $f_{s1}=22.5$ Hz and $f_{s2}=9$ Hz, respectively. This test condition was established by the auditors and corresponds to the operating condition where the potential for objectionable gear whine was greatest. Typical auto power spectra $W_m(k\Delta f)$ of each transducer signal are shown in Fig. 4 for a single unit. The analysis frequency range is from 0 to 2000 Hz with $\Delta f=5$ Hz and $A=32$. Hence $m\leq 4$ harmonics may be used in the construction of E_m and L_m.

An examination of the measured spectra for each of the eleven units revealed that none exhibited radically different noise spectra. It is therefore reasonable to expect that a single index can be determined which correlates with the gear whine subjective rating. Note that the critical bandwidth for each harmonic of mesh frequency exceeds 100 Hz. Unity valued weighting functions $w_r(k)=1$ were chosen. Weighting parameters α_r were determined from a regression analysis to give the best straight line fit between L_m and subjective rating in the least square sense for values of m ranging from 1 to 4 and B_r ranging from 10 to 100 Hz. While there is no particular reason to expect a linear relationship between L_m and subjective rating, the data analysis which follows suggests that a linear model is reasonable.

Calibration. Typical calibration curves obtained using each of the transducer signals are shown in Figs. 5(a-c) corresponding to $m=3$ and $B_r=70$ Hz. The average subjective rating is plotted against L_m and error bars indicate the range of the subjective rating for each unit. In each case, the value E_{mf} was chosen such that $L_m = 0$ dB corresponds to the minimum acceptable subjective rating, which in this particular example case was 6. Correlation coefficient $0 \leq |\rho| \leq 1$ is also shown in each figure. The best correlation between L_m and subjective rating was obtained from accelerometer signal L_{a1} with $m = 3$ and $B_r = 70$ Hz. For this case, correlation coefficient $|\rho| = 0.970$. Note however that respectable correlations $|\rho| > 0.8$ were obtained using objective descriptors from each of the three transducer signals for these particular values of m and B_r. The fact that a better fit was achieved between the descriptors obtained from accelerometer measurements, rather than from the descriptor obtained from microphone measurements, may be due to the high ambient noise levels in the measured sound pressure signal and the presence of other acoustic sources which are evident in Fig. 4(a). Further note that the relative amplitudes of the mesh harmonic components of L_p and L_{a1} shown in Figs. 4(a) and 4(b), respectively are substantially different from the relative mesh harmonic amplitudes of L_{a2} in Fig. 4(c). This is due to a localized resonance in the support structure to which accelerometer L_{a2} was attached that was excited by the second harmonic of f_g at this particular operating speed. Weighting parameters α_r must be determined independently for each transducer signal considered. Furthermore, it is impossible to accurately estimate values of the α_r for any specific product prior to the regression analysis. Consequently, this method should not be used to compare the noise quality of machines which may have similar functions or operating characteristics, but which are significantly different in design or configuration unless a calibration is first performed by using a sufficiently large sample of each different machine design.

Calibration curves and corresponding $|\rho|$ values obtained from L_{a1} data are shown in Fig. 6 for four cases: (a) $m = 1$, $B_r = 10$ Hz with $|\rho| = 0.579$; (b) $m = 1$, $B_r = 70$ Hz with $|\rho| = 0.636$; (c) $m = 3$, $B_r = 10$ Hz with $|\rho| = 0.892$; and (d) $m = 3$, $B_r = 100$ Hz with $|\rho| = 0.954$. An examination of these curves reveals that it is necessary to include higher harmonics $m \geq 1$ in the computation of L_m and further, the energy contained within the modulation sidebands must also be included. Figures 5(b), 6(c) and 6(d) suggest that there may be an optimal bandwidth $10 \leq B_r \leq 100$ Hz. Any optimal values of individual B_r may be different for each mesh harmonic.

Spatial Averaging. In order to obtain an even better objective gear whine descriptor E_m^s, a spatial average was formed from Ψ_{xr}^2 values determined from each transducer signal which is denoted here by subscript x

$$E_m^s = \sum_{r=1}^{m} \sum_{x=1}^{s} \alpha_{xr} \Psi_{xr}^2 \qquad (7)$$

Here s represents the number of transducer signals considered in the spatial average, and for this example case $s = 3$. A multivariate regression analysis was performed to determine the values of B_{xr} and α_{xr} which gave the best straight line fit between L_m^s and subjective rating in the least square sense for $m = 3$. The resulting calibration curve L_m^s versus average

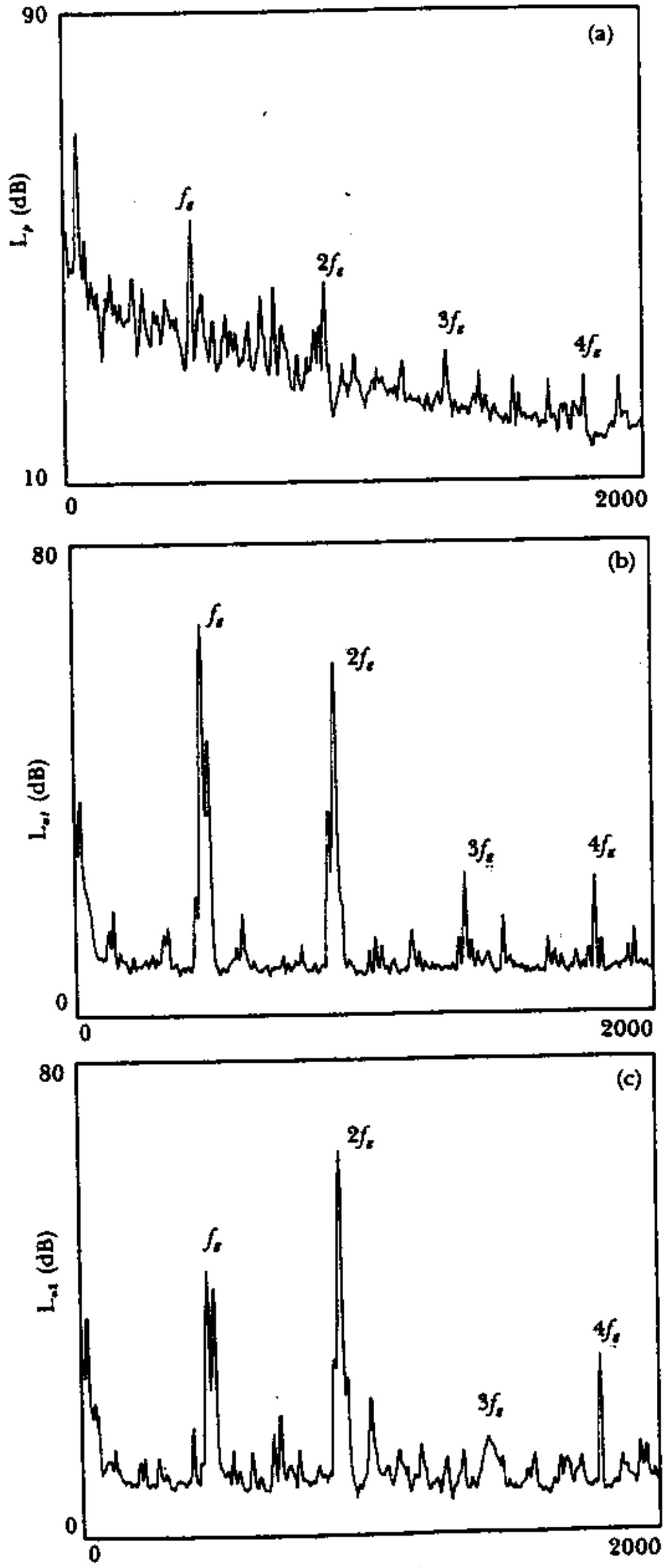


Figure 4. Typical auto power spectra of gear whine: (a) microphone signal (L_p); (b) signal from an accelerometer mounted on the gearbox housing directly over a bearing-shaft interface (L_{a1}); (c) signal from an accelerometer mounted on a structural support which is attached directly to the gearbox housing (L_{a2})

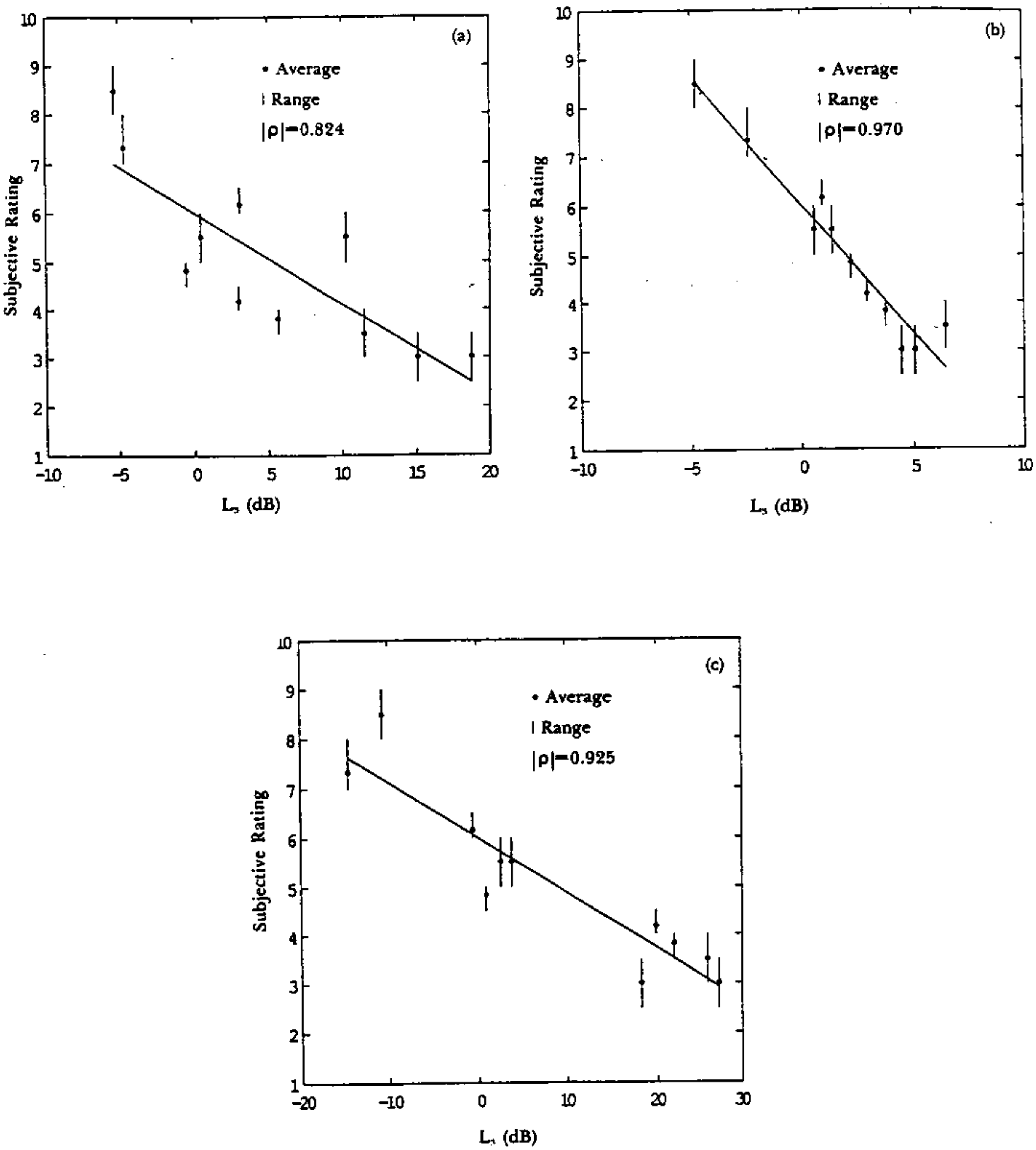


Figure 5. Calibration curves of average subjective rating versus perceived gear whine level L_w in decibels corresponding to 11 units, $m=3$ and $B_r=70$ Hz: (a) microphone (L_p) with $|\rho|=0.824$; (b) accelerometer (L_{a1}) with $|\rho|=0.970$; (c) accelerometer (L_{a2}) with $|\rho|=0.925$

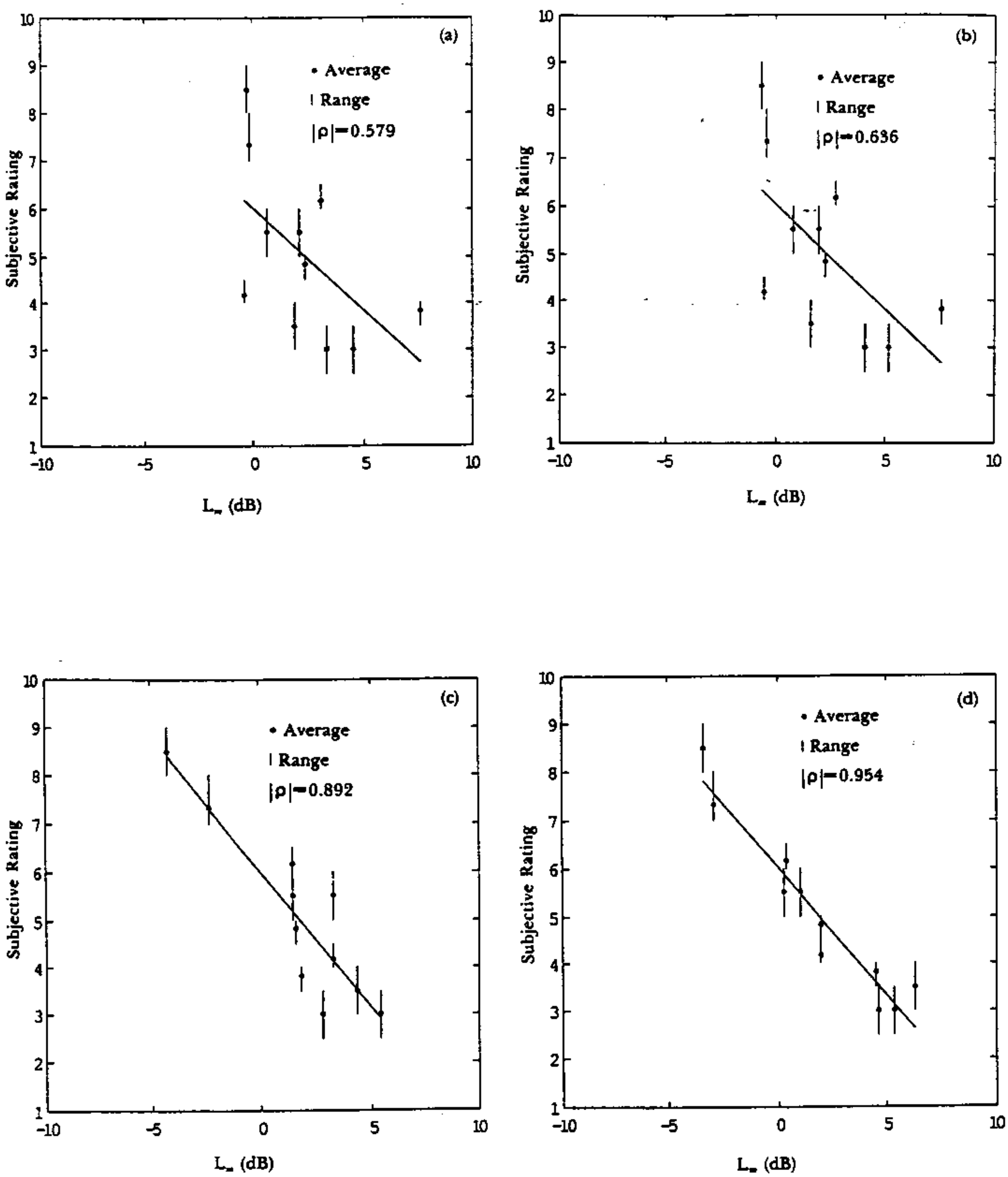


Figure 6. Calibration curve of average subjective rating versus perceived gear whine level L_w in decibels obtained from accelerometer L_{a2} data: (a) $m=1$, $B_r=10$ Hz and $|\rho|=0.579$; (b) $m=1$, $B_r=70$ Hz and $|\rho|=0.636$; (c) $m=3$, $B_r=10$ Hz and $|\rho|=0.892$; (d) $m=3$, $B_r=100$ Hz and $|\rho|=0.954$

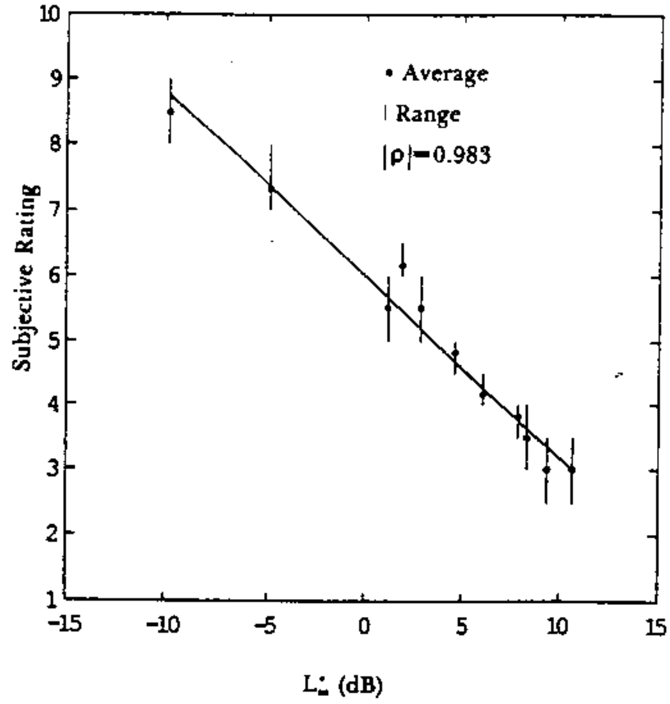


Figure 7. Calibration curve of average subjective rating versus perceived gear whine level L_m in decibels obtained from spatially averaged microphone (L_p) and accelerometer signals (L_{a1} and L_{a2}) with $m = 3$, B_m ranging from 10 to 70 Hz and $|\rho| = 0.983$

subjective rating is shown in Fig. 7 where $|\rho| = 0.983$. The above example further suggests that the correlation between L_m and subjective rating must be viewed within a confidence interval of subjective rating ±1 with respect to the 1 to 10 subjective rating scale employed in this study. This is consistent with the observed resolution and repeatability (estimated ±1) of the subjective auditor ratings.

We note that the use of weighted spatial averaging of two or more transducer signals is very often required in order to compensate for random variations in testing conditions, transducer mountings, dynamic characteristics of individual units, and the like. If the product is excited at or near a system resonance, temporal averaging over a range of operating conditions may be required to compensate for random shifts in resonant frequencies due to manufacturing variations. If individual units having identical subjective ratings are found to exhibit radically different noise spectra, then independent E_m^* must be determined for each case. These objective descriptors can then be combined to form an overall rating index as illustrated in the next example.

Extended Application: Automotive Transmission

The gear whine rating scheme discussed above can also be extended to gear systems which operate over a range of speed and loading conditions and to systems containing several gear meshes, although the procedure is obviously much more complicated. This can be illustrated through an example case which considers gear whine associated with an automotive transmission.

Figure 8 shows a typical spectral map of transmission housing vibration acceleration level $L_a = L_a[\Omega, T, f]$, for an automobile transmission during an in-vehicle test consisting of light acceleration (drive) from 40 to 60 mph and unbraked deceleration (coast) back to 35 mph. The system rotational speed Ω is controlled directly while system torque T is determined by vehicle dynamics and driving conditions. The response L_a is dominated by two gear meshes whose respective mesh frequencies are well separated. Amplitude varies significantly with operating condition, particularly with drive and coast conditions. For this application, the proposed rating technique was extended to include weighted temporal averaging over three vehicle speed ranges and spatial averaging of two accelerometer signals. The accelerometers were each mounted on the transmission casing directly over two respective shaft-bearing interfaces in close proximity to each gear mesh of interest. The method was further modified by employing B_m equal to constant percentages of each gear mesh harmonic center frequency f_{cr} (Ω), rather than B_m of constant frequency. This is readily accomplished by performing the analysis in the spectral order domain rather than the frequency domain. For this case, B_m corresponds to the number of spectral orders. Order domain analysis requires the addition of a tachometer signal to measure Ω, but has the added advantage of reducing smearing effects which are inherent to the frequency analysis.[17] The weighting functions w_m and weigting parameters α_m now must also vary with Ω. Unlike systems which contain only a single gear pair, overlap of the B_m is possible in a system which contains multiple gear meshes creating a potentially difficult situation. In such a case, judicious selection of the B_m and testing conditions is required. Spatial averaging of several transducer signals must be employed in some cases to distinguish between individual gear meshes.

Several distinct E_m^* were determined in accordance with (7) to obtain several distinct L_m^* levels corresponding to each gear pair for both drive and coast conditions. A single overall estimator Lt_m^* was also developed which was fit to an overall subjective rating index. A typical calibration curve for 52 units with correlation coefficient $|\rho| = 0.841$ is shown in Fig. 9 where Lt_m^* is plotted against average subjective rating. The subjective evaluation procedure employed in this study was identical to that described in the previous example. Ideally, a sample of transmissions uniformly distributed over the full subjective range would have been considered. However, this was not possible due to the actual distribution of production transmissions as well as cost and time constraints. The degree of correlation between any objective descriptor or noise quality index and corresponding subjective rating is always limited due to the variances associated with human perception and the inability of any objective index to account for all possible factors influencing human response. Accordingly, given the resolution and repeatability of subjective ratings, the com-

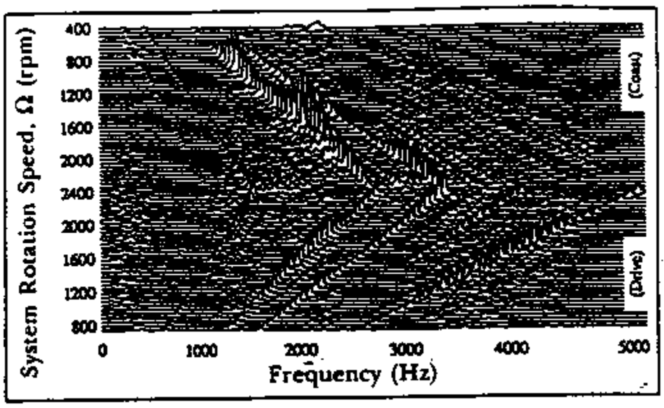


Figure 8. Typical spectral map $L_s = L_s[\Omega(t), T(t), f]$ of an automobile transmission in-situ test

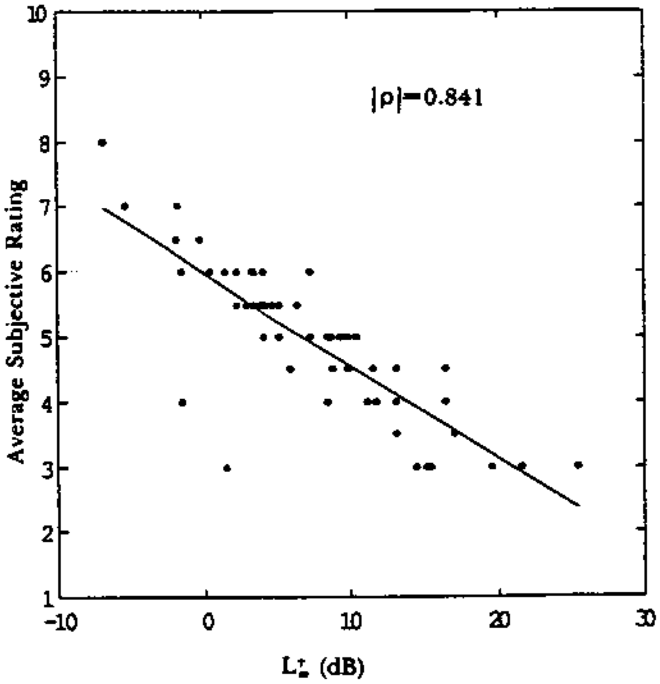


Figure 9. Calibration curve of average subjective rating versus overall perceived gear whine level L^*_m in decibels for automotive transmission example of Fig. 8 corresponding to 52 units with $|\rho| = 0.841$

plexity of the product involved and the nature of the in-situ test conditions, the best fit curve in Fig. 9 was deemed to provide a satisfactory correlation with subjective response.

Concluding Remarks

A new method for computing noise quality rating indices for gear whine using frequency-domain objective descriptors obtained from physical, vibro-acoustic measurements has been developed. The proposed technique differs from conventional PNL methods by focusing on energy in narrow band frequency spectra. Implementation of this rating scheme requires several weighting functions and parameters which must be determined specifically for each product or machine considered. Once these weighting functions and parameters are found, the method provides a very useful and cost effective noise quality rating tool which can be used to assess various engineering changes on the reduction of objectionable gear whine during the product development phase. The resulting noise quality index is free from the inconsistencies of human auditors and the noise rating criteria remain constant over time. Further refinements in the proposed method are possible based on extensive applications to specific gear noise problems. Variations of this new rating method may be applicable to other types of quasi-steady state machinery noise problems which exhibit similar noise signatures.

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