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DYNAMICS OF AN OSCILLATOR WITH BOTH CLEARANCE AND CONTINUOUS NON-LINEARITIES

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1. FORMULATION

Consider a one-dimensional oscillator, under a preload F_m and a harmonic excitation $F_a \sin \omega t$, with both clearance and continuous stiffness non-linearities. In the dimensionless form, the equation of motion is as follows, where n denotes the type of the continuous non-linearity; i.e., $n=3$ is cubic [1–3], $n=2$ is quadratic [1], and non-integer n refers to the Hertzian contact problem [4, 5]:

$$\ddot{x}+2\zeta\dot{x}+f(x)=F_m+F_a\sin(\omega t), \tag{1a}$$

$$f(x)=\begin{cases}(x-1)+\alpha(x-1)^n, & x>1,\\ 0, & -1\leqslant x\leqslant 1,\\ (x+1)+\alpha(x+1)^n, & x<-1.\end{cases} \tag{1b}$$

Here, a clearance or backlash exists between $x=-1$ and $x=1$, and α defines the type of the spring in the contact regime, i.e., piecewise-linear for $\alpha=0$, softening for $\alpha<0$ and hardening for $\alpha>0$. While both clearance [6–11] and continuous [1–5] non-linearities have been examined separately, to the best of our knowledge these have not been considered simultaneously. Accordingly, the intent of this communication is to discuss the dynamic behavior of such a system encountered in practice [6].

Equation (1) is solved numerically by using a fifth–sixth order Runge–Kutta numerical integration routine [12], since none of the perturbation techniques may be applicable; see references [7, 10] for methodology. Three different continuous non-linearity examples are considered here: cubic, $n=3$; quadratic, $n=2$; and non-integer, $n=1·5$. The steady state solutions of equation (1) are searched at 441 points within a domain of initial conditions defined by $0\leqslant x(0)\leqslant 2$ and $-1\leqslant\dot{x}(0)\leqslant 1$.

2. RESULTS FOR A HEAVILY LOADED OSCILLATOR

First, we consider a heavily loaded oscillator with a small alternating to mean load ratio $\hat{F}=F_a/F_m$. When $\alpha=0$ (piecewise-linear oscillator), only period-1, harmonic (or almost harmonic) solutions are known to exist for a relatively small $\hat{F}$ [7, 10]. In Figure 1, the frequency response of the oscillator with $n=3$ is shown for $F_m=0·4$, $F_a=0·1$ ($\hat{F}=0·25$) and $\zeta=0·05$, for three different values of α. The alternating and mean components of the frequency response, $x_a(\omega)$ and $x_m(\omega)$, corresponding to hardening ($\alpha=0·1$), piecewise-linear ($\alpha=0$), and softening ($\alpha=-0·1$) springs are quite similar to each other, as is evident from Figure 1. The overall characteristics of the frequency response seem to be dictated by the clearance non-linearity. Regardless of the value of α, the frequency response has a

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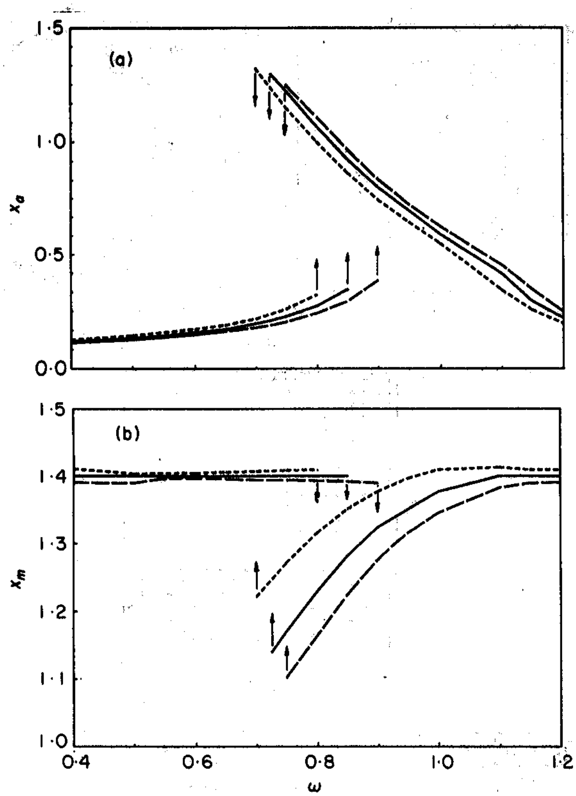

Figure 1. (a) $x_a(\omega)$ and (b) $x_m(\omega)$ spectra for a cubic non-linearity ($n=3$); $F_m=0{\cdot}4$, $F_a=0{\cdot}1$ ($\hat{F}=0{\cdot}25$) and $\zeta=0{\cdot}05$. ——, $\alpha=0$; – – –, $\alpha=-0{\cdot}1$; – - –, $\alpha=0{\cdot}1$.

dual-valued region which is bounded by the jump-up and jump-down frequencies. These transition frequencies are shifted to the left for $\alpha=-0{\cdot}1$ and to the right for $\alpha=0{\cdot}1$, but the width of the dual-solution region almost remains the same. The phase plane plots of the dual solutions at $\omega=0{\cdot}775$ are shown in Figures 2(a) and (b). In Figure 2(a), the system oscillates harmonically without losing contact at the loaded side, resulting in a no-impact (NI) motion ($x_{min}>1$). The other solution, as shown in Figure 2(b), is a single-sided-impact (SSI) motion with $-1<x_{min}<1$. Although the differences in x_a or x_m for different values of α are not very significant, the domains of attraction of each solution are affected considerably when α is varied, as shown in Figure 2(c–e). Similar results are obtained for $n=2$ and $n=1{\cdot}5$.

Furthermore, frequency responses shown in Figure 3 for a larger $\hat{F}$ ($F_m=0{\cdot}4$, $F_a=0{\cdot}2$, $\hat{F}=0{\cdot}5$ and $\zeta=0{\cdot}05$) do not differ significantly for $n=3$, 2 and $1{\cdot}5$. In Figure 3, double-sided-impact (DSI) motions ($x_{min}<-1$) exist in addition to NI and SSI motions, since $\hat{F}$ is large enough for such solutions to exist [7]. It is seen that, for a heavily loaded system, the frequency response is clearly dictated by the clearance non-linearity, and is not influenced significantly by the existence of the continuous non-linearity (cubic, quadratic or non-integer). However, the initial condition maps within the multi-solution regimes may be considerably different, as shown in Figure 4 for $\omega=0{\cdot}775$, at which NI, SSI and DSI solutions co-exist. For instance, for $n=3$, the initial conditions, $x(0)=1{\cdot}4$ and $\dot{x}(0)=1{\cdot}5$, correspond to the NI solution, whereas the same initial conditions are governed by the SSI and DSI solutions for $n=2$ and $n=1{\cdot}5$, respectively

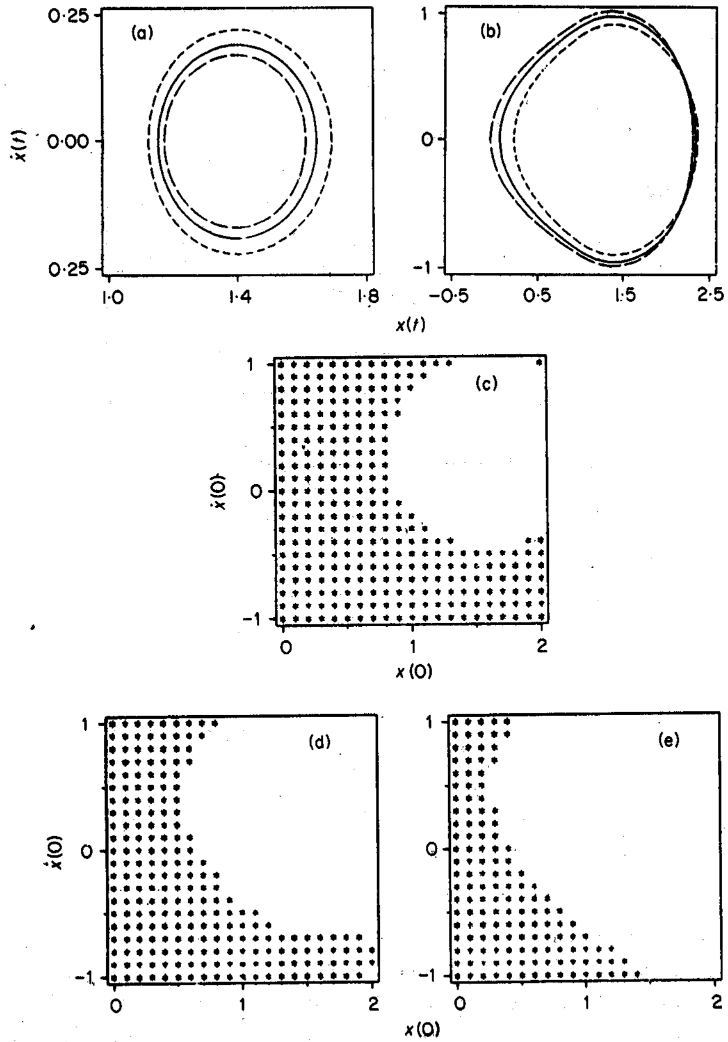

Figure 2. (a, b) Steady state solutions at $\omega = 0{\cdot}775$ with $F_m = 0{\cdot}4$, $F_a = 0{\cdot}1$ and $\zeta = 0{\cdot}05$; key as Figure 1. (c–e) Initial condition maps for different α; here an asterisk denotes the initial conditions governed by the SSI motion: (c) $\alpha = 0$; (d) $\alpha = -0{\cdot}1$; (e) $\alpha = 0{\cdot}1$.

3. RESULTS FOR A LIGHTLY LOADED OSCILLATOR

Next, we consider a lightly loaded oscillator with $F_m = 0{\cdot}5$, $F_a = 0{\cdot}6$ ($\hat{F} = F_a/F_m = 1{\cdot}2$) and $\zeta = 0{\cdot}05$. We have shown previously that chaotic and period-n subharmonic solutions are more likely to exist for the piecewise-linear system ($\alpha = 0$) when $\hat{F}$ is large [7, 10]. In Figure 5, the effect of α on $x(t)$ is shown for $n = 3$ and $\omega = 0{\cdot}5$. Two different period-2 solutions exist for $\alpha = 0{\cdot}05$, as shown in Figure 5(a) and (b). When α is reduced to $0{\cdot}02$, one of the period-2 solutions is replaced by a period-1 non-harmonic solution, while the other period-2 solution remains almost the same, as shown in Figure 5(c) and (d). Further reducing α to zero, we obtain a chaotic solution coexisting with a period-2 subharmonic motion, as evident from the strange attractor and the phase plane plot of Figure 5(e) and (f). Finally, for a slightly softening spring, $\alpha = 0{\cdot}02$, all of the solutions become chaotic, as shown in Figure 5(g). The steady state solutions are also found to be very sensitive to

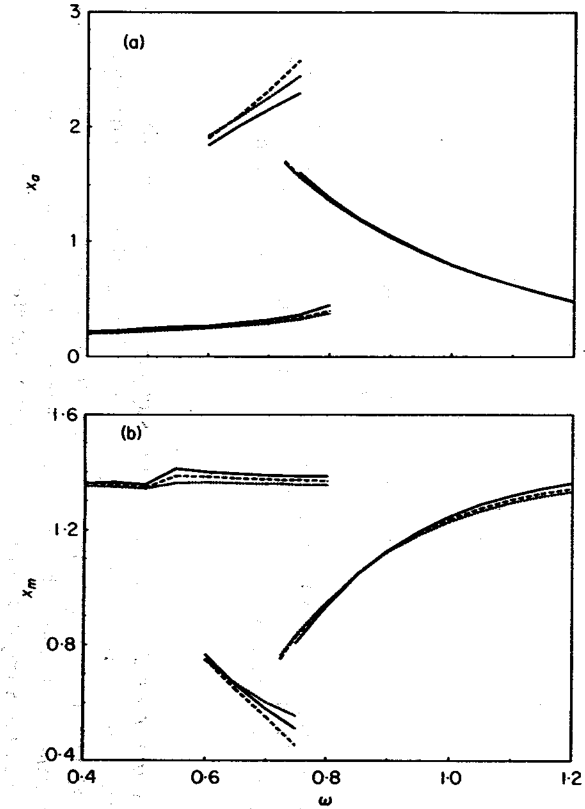

Figure 3. (a) $x_a(\omega)$ and (b) $x_m(\omega)$ spectra for $F_m=0{\cdot}4$, $F_a=0{\cdot}2$ ($\hat{F}=0{\cdot}5$), $\zeta=0{\cdot}5$ and $\alpha=0{\cdot}2$. ——, $n=3$; – – –, $n=2$; · · · · ·, $n=1{\cdot}5$.

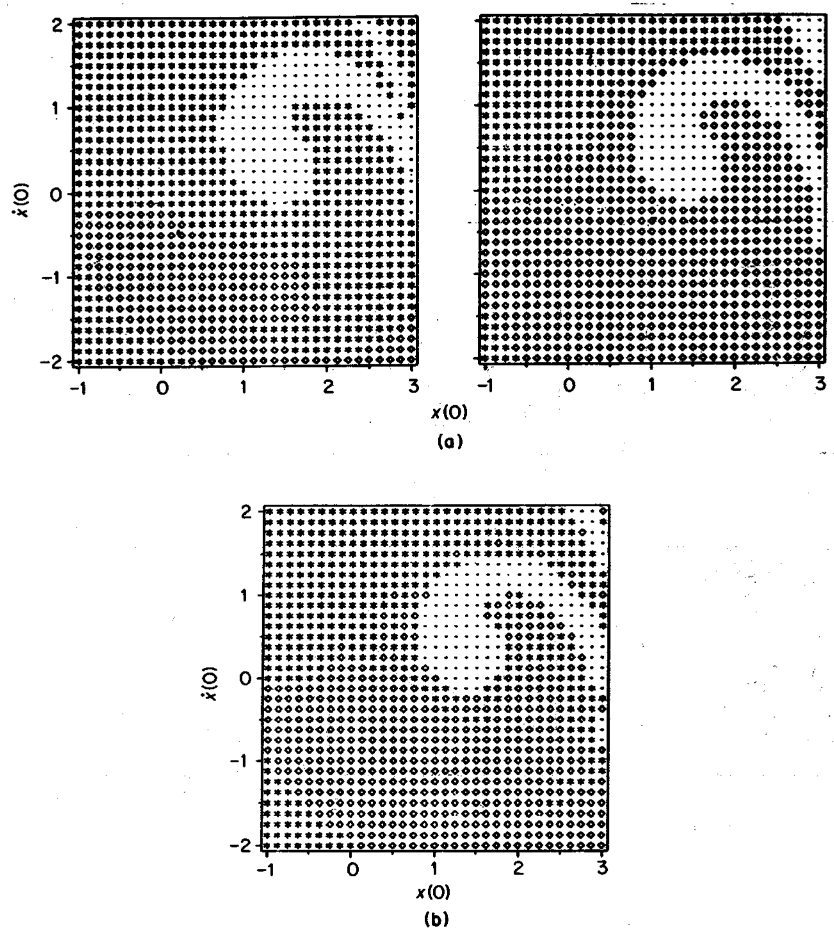

Figure 4. Initial condition maps for $\omega=0{\cdot}775$ in Figure 3: (a) $n=3$; (*b*) $n=2$; (c) $n=1{\cdot}5$. ●, NI motion; ◇, SSI motion; ∗, DSI motion.

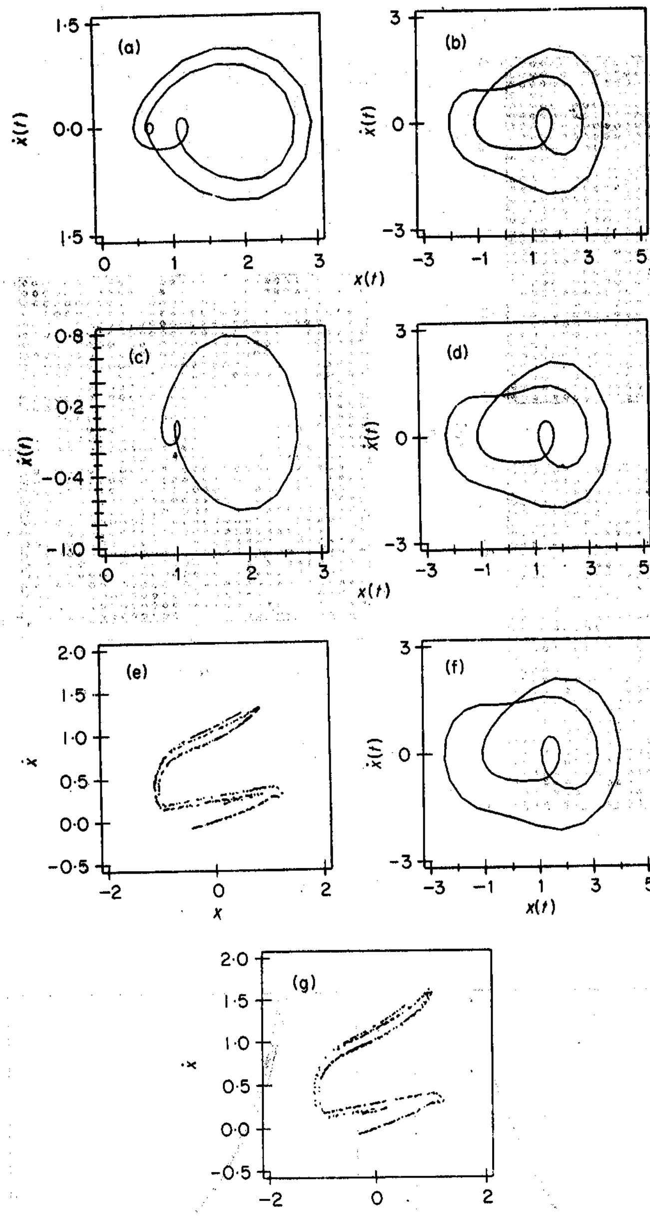

Figure 5. Response of a lightly loaded system with cubic non-linearity, or different values of α; $F_m=0{\cdot}5$, $F_a=0{\cdot}6$ ($F=1{\cdot}2$), $\omega=0{\cdot}5$ and $\zeta=0{\cdot}05$. (a, b) $\alpha=0{\cdot}05$, two period-2 solutions, (c, d) $\alpha=0{\cdot}02$, one period-1 and one period-2 solution, (e, f) $\alpha=0$, one period-2 solution and chaos, and (g) $\alpha=-0{\cdot}02$, chaos only.

the value of α for $n=2$ and $n=1{\cdot}5$. This suggests that, for a lightly loaded system, a minor variation in α may change the type of the solution(s) and the corresponding initial conditions. Hence, neglecting continuous non-linearities in a system with a clearance, no matter how small α is, could lead to incomplete or incorrect predictions.

Further research is required in order to fully understand the dynamic behavior of such systems.

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