Journal of Sound and Vibration (1994) **169**(4), 547–553

LETTERS TO THE EDITOR

VIBRATION TRANSMISSION THROUGH ROLLING ELEMENT BEARINGS, PART V: EFFECT OF DISTRIBUTED CONTACT LOAD ON ROLLER BEARING STIFFNESS MATRIX

T. C. LIM

SDRC, Structural Dynamics Research Corporation, Engineering Services Division, Inc., 2000 *Eastman Drive, Milford, Ohio* 45150-2789, *U.S.A.*

AND

R. SINGH

Acoustics and Dynamics Laboratory, Department of Mechanical Engineering, The Ohio State University, Columbus, Ohio 43210-1107, *U.S.A.*

(*Received* 11 *January* 1993)

1. INTRODUCTION

In a series of recent papers [1–4], we presented a new rolling element bearing stiffness matrix theory, along with its computational scheme and application to vibratory systems. The objective was to model the dynamic coupling phenomenon between shaft bending motion and casing plate flexural motion in a rotating mechanical system. It has been brought to our attention that our bearing load–deflection formulation as defined in Part 1 [1] is somewhat *similar* to the 1960 work of Jones [5]. A careful comparison of references [1] and [5] shows that these two distinct analyses lead to virtually identical results for ball bearings, even though Jones did not define a bearing stiffness matrix. However, our theory [1] for roller bearings is quite different, and more *advanced* than Jones' formulation. While our theory [1] was pursued *without* the knowledge of Jones' prior work [5], other researchers may wish to use both, especially for ball bearings. The rest of this communication is concerned with roller bearings.

For roller bearings, our mathematical model was derived by assuming a discrete normal load vector at each loaded rolling element. Consequently, the load distribution, either uniform or non-uniform, along the roller line of contact was not modelled explicitly. Recently, while applying this roller bearing model to a certain class of machine vibration problems, we have observed that this assumption is invalid for a few cases such as when the taper angle is very small or zero. Accordingly, our model cannot predict angular stiffness coefficients and bearing moment transmissibility of a mechanical system with misaligned radial needle bearings; these effects are primarily due to an uneven load distribution along the roller effective length. Therefore, it is the purpose of this communication to address such unique cases by formulating a new roller bearing stiffness matrix model.

The contact load distribution in roller bearings was not discussed in the 1960 paper by Jones [5]. Its significance was presented in 1964 by Dareing and Radzimovsky [6], who developed a linear semi-empirical relationship between the distribution of roller normal load per unit length and angular displacement to predict life of a misaligned roller bearing. In 1966, Harris [7] presented a linear formulation, based on the Hertzian theory, to compute the bearing moment caused by angular misalignment. As a result, he indirectly formulated an angular stiffness coefficient $k_{b\theta\theta}$ in terms of the constant roller effective length L:

$$k_{b\theta\theta} = (L^3/6K_n) \sum_{\psi_j = 0}^{\psi_j = \pm\pi} \cos^2 \psi_j. \tag{1}$$

0022–460X/94/040547 + 07 \$08.00/0

Here, K_n is the well known roller load–deflection stiffness constant [7, 8]. Other analyses, such as the discrete spring-gap element proposed by Demerling *et al.* [9], are primarily based on a simple concept of using a finite number of distributed unidirectional springs along the roller line of contact. This type of bearing model typically results in a model with a large dimension, since each roller is represented explicitly. Furthermore, the selection of such spring elements is often arbitrary. None of these models has ever been used to study the effect of distributed contact load and angular stiffnesses of a roller bearing in a vibratory system. This communication attempts to fill this void in the literature.

2. THEORY

The basic assumptions of the original bearing stiffness matrix theory and nomenclature of Part [1] are adopted here unless indicated otherwise. Consider a generic roller bearing with a taper angle α subjected to generalized mean bearing displacements $\{q\}_{bm}$ and loads $\{f\}_{bm}$, as shown in Figure 1. The resultant elastic deformation pattern along the jth roller effective length, which is characterized by a dimensionless parameter ζ, is

$$\delta_{Rj}(\psi_j, \zeta) = \begin{Bmatrix} V(\psi_j) + \zeta L W(\psi_j), & \delta_{Rj} > 0 \\ 0, & \delta_{Rj} \leqslant 0 \end{Bmatrix}, \qquad -0{\cdot}5 \leqslant \zeta \leqslant 0{\cdot}5, \tag{2a}$$

$$V(\psi_j) = \delta_{xm} \cos\alpha \cos\psi_j + \delta_{ym} \cos\alpha \sin\psi_j + \delta_{zm} \sin\alpha - r_c$$
$$+ \beta_{xm} r_j \sin\alpha \sin\psi_j - \beta_{ym} r_j \sin\alpha \cos\psi_j - r_L \cos\alpha, \tag{2b}$$

$$W(\psi_j) = -\beta_{xm} \sin\psi_j + \beta_{ym} \cos\psi_j, \tag{2c}$$

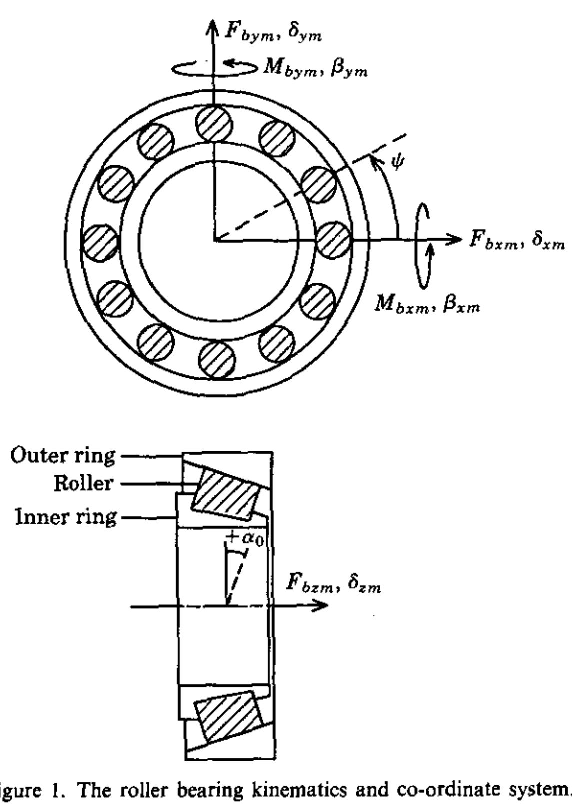

Figure 1. The roller bearing kinematics and co-ordinate system.

where r_j, r_L and r_c are the mean rolling radius, radial clearance and crown drop, respectively. Note that the expression $\zeta LW(\psi_j)$ in equation (2) represents the variation of the jth roller deflection due to bearing angular misalignments $\{\beta_{xm}, \beta_{ym}\}$, which was not accounted for in our original theory [1]. According to Harris [7], the Hertzian load–deflection equation is valid for distributed normal load. Using this assumption, the total normal load Q_j for the jth roller can be obtained by integrating along the contact line:

$$Q_j = K_n \int_{\zeta_1}^{\zeta_2} \{V(\psi_j) + \zeta LW(\psi_j)\}^n \,\mathrm{d}\zeta. \tag{3}$$

Equation (3) is non-zero only if the roller is either fully or partially loaded, which implies that $\delta_{Rj} > 0$ for a subset of $-0{\cdot}5 \leqslant \zeta \leqslant 0{\cdot}5$. This subset is bounded by the following limits of integration (ζ_1, ζ_2) where the operators max $[A, B]$ and min $[A, B]$ give the larger or smaller of the two values, A or B, respectively:

$$\zeta_1 = \begin{cases} \max[-V(\psi_j)/LW(\psi_j), -0{\cdot}5], & LW(\psi_j) > 0 \\ -0{\cdot}5, & LW(\psi_j) \leqslant 0 \end{cases}, \tag{4a}$$

$$\zeta_2 = \begin{cases} 0{\cdot}5, & LW(\psi_j) \geqslant 0 \\ \min[-V(\psi_j)/LW(\psi_j), 0{\cdot}5], & LW(\psi_j) < 0 \end{cases}. \tag{4b}$$

The mean bearing forces $\{F_{bxm}, F_{bym}, F_{bzm}\}$ and moments $\{M_{bxm}, M_{bym}\}$ (note that $M_{bzm} = 0$) are obtained by applying a vectorial sum to the roller loads:

$$\begin{Bmatrix} F_{bxm} \\ F_{bym} \\ F_{bzm} \\ M_{bxm} \\ M_{bym} \end{Bmatrix} = \sum_j^Z Q_j \begin{Bmatrix} \cos\alpha \cos\psi_j \\ \cos\alpha \sin\psi_j \\ \sin\alpha \\ (r_j \sin\alpha + e_j)\sin\psi_j \\ -(r_j \sin\alpha + e_j)\cos\psi_j \end{Bmatrix}, \tag{5}$$

where the load eccentricity e_j is the distance between the resultant load vector line of action and roller mid-point:

$$e_j = \left[L \int_{\zeta_1}^{\zeta_2} \zeta \{V(\psi_j, \zeta) + \zeta LW(\psi_j, \zeta)\}^n \,\mathrm{d}\zeta \right] \Big/ \left[\int_{\zeta_1}^{\zeta_2} \{V(\psi_j, \zeta) + \zeta LW(\psi_j, \zeta)\}^n \,\mathrm{d}\zeta \right]. \tag{6}$$

The resultant load vector line of action moves from its ideal mid-point location in proportion to the degree of bearing angular misalignments. Therefore, e_j provides a qualitative measure of the effect of angular misalignments on the bearing stiffness matrix $[K]_{bm}$.

The $[K]_{bm}$ matrix is formulated next by computing the partial derivatives of the roller load vectors in equation (5) with respect to their displacements. This approach is basically similar to our original scheme [1]. The $[K]_{bm}$ matrix is symmetric and its coefficients are given by

$$k_{bxx} = nK_n \cos^2\alpha \sum_j^Z I_0 \cos^2\psi_j, \qquad k_{bxy} = nK_n \cos^2\alpha \sum_j^Z I_0 \cos\psi_j \sin\psi_j, \tag{7a, b}$$

$$k_{bxz} = nK_n \cos\alpha \sin\alpha \sum_j^Z I_0 \cos\psi_j \tag{7c}$$

$$k_{bx\theta_x} = nK_n \cos\alpha \sum_j^Z (I_0 r_j \sin\alpha - I_1)\cos\psi_j \sin\psi_j, \tag{7d}$$

$$k_{bx\theta_y} = nK_n \cos\alpha \sum_j^Z (I_1 - I_0 r_j \sin\alpha)\cos^2\psi_j, \tag{7e}$$

$$k_{byy} = nK_n \cos^2\alpha \sum_j^Z I_0 \sin^2\psi_j, \qquad k_{byz} = nK_n \cos\alpha \sin\alpha \sum_j^Z I_0 \sin\psi_j, \tag{7f, g}$$

$$k_{by\theta_x} = nK_n \cos\alpha \sum_j^Z (I_0 r_j \sin\alpha - I_1)\sin^2\psi_j, \qquad k_{by\theta_y} = -k_{bx\theta_x}, \tag{7h, i}$$

$$k_{bzz} = nK_n \sin^2\alpha \sum_j^Z I_0, \qquad k_{bz\theta_x} = nK_n \sin\alpha \sum_j^Z (I_0 r_j \sin\alpha - I_1)\sin\psi_j, \tag{7j, k}$$

$$k_{bz\theta_y} = nK_n \sin\alpha \sum_j^Z (I_1 - I_0 r_j \sin\alpha)\cos\psi_j, \tag{7l}$$

$$k_{b\theta_x\theta_x} = nK_n \sum_j^Z (I_0 r_j^2 \sin^2\alpha - 2I_1 r_j \sin\alpha + I_2)\sin^2\psi_j, \tag{7m}$$

$$k_{b\theta_x\theta_y} = nK_n \sum_j^Z (2I_1 r_j \sin\alpha - I_0 r_j^2 \sin^2\alpha - I_2)\sin\psi_j \cos\psi_j, \tag{7n}$$

$$k_{b\theta_y\theta_y} = nK_n \sum_j^Z (I_0 r_j^2 \sin^2\alpha - 2I_1 r_j \sin\alpha + I_2)\cos^2\psi_j, \tag{7o}$$

$$k_{bx\theta_z} = k_{by\theta_z} = k_{bz\theta_z} = k_{b\theta_x\theta_z} = k_{b\theta_y\theta_z} = k_{b\theta_z\theta_z} = 0, \tag{7p}$$

where Z is the number of rolling elements, and

$$I_p = T(p, \xi_2) - T(p, \zeta_1), \qquad p = 0, 1 \text{ or } 2, \tag{8}$$

in which

$$T(p,\zeta) = \left\{ \begin{array}{ll} \dfrac{(V+\zeta LW)^n}{nW}\left[(L\zeta)^p - \dfrac{p(L\zeta)^{p-1}(V+\zeta LW)^1}{(n+1)W} + \dfrac{p(p-1)(V+\zeta LW)^2}{(n+1)(n+2)W^2}\right], & W \neq 0 \\ \dfrac{(L\zeta)^{p+1}V^{n-1}}{p+1}, & W = 0 \end{array} \right\}. \tag{9}$$

Although the form of new stiffness coefficients k_{bij} is considerably more complex than the original one presented in Part 1 [1], the computational scheme remains essentially the same. To review briefly, k_{bij} can be evaluated directly given $\{q\}_{bm}$ by employing equations (7)–(9). If the bearing load vector $\{f\}_{bm}$ is known instead, an additional non-linear numerical solution step is performed using equation (5) to obtain $\{q\}_{bm}$ prior to applying equations (7)–(9).

3. RESULTS AND DISCUSSION

The present $[K]_{bm}$ matrix formulation developed here is applied to a typical roller bearing characterized by the following design data: load–deflection exponent $n = 10/9$, $K_n = 3{\cdot}0 \times 10^8$ N/m^n, $Z = 14$, $r_L = 0{\cdot}00175$ mm, $r_c = 0{\cdot}0$ and $r_j = 20{\cdot}0$ mm. First, consider the obvious case of taper angle $\alpha = 0°$ subjected to only angular misalignment $-0{\cdot}01 \leqslant \beta_{ym} \leqslant 0{\cdot}01$, which we know must be analyzed by using a distributed contact load

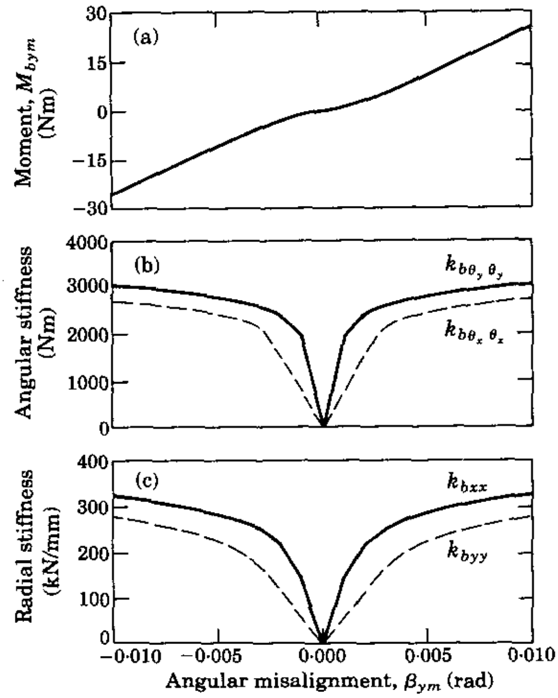

Figure 2. Straight roller bearing ($\alpha = 0°$) moment and stiffness coefficients caused by an angular misalignment β_{ym}; (a) moment; (b) angular stiffness; (c) radial stiffness.

bearing model. Also, let the roller length be $L = 10{\cdot}0$ mm. Equations (5) and (7) predict that M_{bym}, k_{bxx}, k_{byy}, $k_{b\theta_x\theta_x}$ and $k_{b\theta_y\theta_y}$ are the only non-zero load and stiffness coefficients, as shown in Figure 2, compared to our original formulation [1] in which all loads and stiffness coefficients were identically zero.

Second, we vary α between 0° and 90° to examine the extent of the influence of the distributed contact load on $[K]_{bm}$. The roller bearing is now preloaded such that its mean displacements are $\delta_{xm} = \delta_{zm} = 0{\cdot}01$ mm and $\beta_{ym} = 0{\cdot}01$ radian. The non-zero stiffness coefficients are k_{bxx}, k_{bxz}, k_{byy}, k_{bzz}, $k_{bx\theta_y}$, $k_{by\theta_x}$, $k_{bz\theta_y}$, $k_{b\theta_x\theta_x}$ and $k_{b\theta_y\theta_y}$. In this case, most of these coefficients are slightly affected by the distributed contact load, except for the angular stiffness coefficients $k_{b\theta_x\theta_x}$ and $k_{b\theta_y\theta_y}$, as shown in Figure 3. It is observed that our new and original bearing models differ considerably, for a smaller α, for the following reason. For

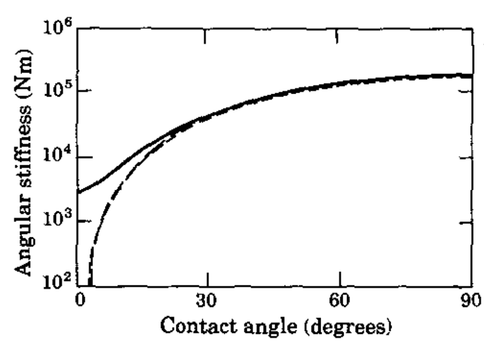

Figure 3. The effect of the taper angle on the stiffness coefficients, $k_{b\theta_x\theta_x}$ and $k_{b\theta_y\theta_y}$. ——, New formulation; - - -, original formulation. Note that $k_{b\theta_x\theta_x} \leqslant k_{b\theta_y\theta_y}$.

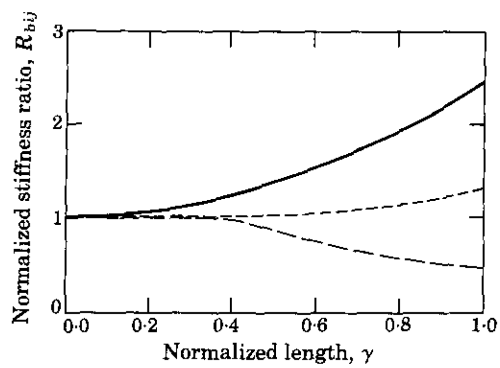

Figure 4. The effect of the roller normalized length γ on the normalized stiffness coefficients. Each new stiffness coefficient is normalized by the original calculation. [1]. ——, $R_{b\theta_y\theta_y}$, $R_{b\theta_x\theta_x}$; - - - -, $R_{bz\theta_y}$; — — —, R_{bxz}.

a large value of α, the effect of β_{ym} is insignificant. As α becomes smaller, the angular stiffness coefficients are increasingly governed by the uneven contact load or load eccentricity e_j due to β_{ym}.

Finally, we keep $\alpha = 15°$ and vary $\gamma = L/r_j$ from 0 to 1. The mean displacements are kept the same, like the second parametric study. Each new stiffness coefficient is normalized with respect to the original one [1]. This new variable is called here the stiffness coefficient ratio R_{bij}; a value of one implies that new and original results are identical. Our analysis indicates that R_{bxz}, $R_{bz\theta y}$, $R_{b\theta_x\theta_x}$ and $R_{b\theta_y\theta_y}$ deviate significantly from one, as shown in Figure 4. Discrepancies between the two formulations are most noticeable for a wide roller bearing whose axial and radial dimensions are comparable. Also, the abrupt transition observed at $\gamma = 0{\cdot}4$ is caused by a sudden change in the loading pattern as a result of its wider dimension.

Our results, although limited, suggest that the effect of distributed contact load on roller bearing stiffness matrix is most significant in either small taper angle or wide dimension bearings. Therefore, it is imperative that new stiffness coefficients be used to model such bearings, especially when dynamic moment transmissibility and the effect of angular stiffnesses are of primary interest. For most other types of roller bearing applications, our original formulation is still valid.

ACKNOWLEDGMENTS

We wish to thank Dr Chungda Lee for his assistance in this quantitative comparison of our original theory and Jones' formulation. The second author acknowledges the support from the U.S. Army Research Office.

REFERENCES

1. T. C. Lim and R. Singh 1990 *Journal of Sound and Vibration* **139**, 179–199. Vibration transmission through rolling element bearings, part I: bearing stiffness formulation.
2. T. C. Lim and R. Singh 1990 *Journal of Sound and Vibration* **139**, 201–225. Vibration transmission through rolling element bearings, part II: system studies.
3. T. C. Lim and R. Singh 1991 *Journal of Sound and Vibration* **151**, 31–54. Vibration transmission through rolling element bearings, part III: geared rotor system studies.
4. T. C. Lim and R. Singh 1992 *Journal of Sound and Vibration* **153**, 37–50. Vibration transmission through rolling element bearings, part IV: statistical energy analysis.

5. A. B. JONES 1960 *Transactions of the American Society of Mechanical Engineers, Journal of Basic Engineering* **82**, 309–320. A general theory for elastically constrained ball and radial roller bearings under arbitrary load and speed conditions.
6. D. W. DAREING and E. I. RADZIMOVSKY 1964 *Machine Design* **36**, 175–179. Misaligned roller bearings.
7. T. A. HARRIS 1966 *Rolling Bearing Analysis*. New York: John Wiley.
8. B. J. HAMROCK and W. J. ANDERSON 1983 *NASA Reference Publication* 1105. Rolling-element bearings.
9. D. J. DEMERLING, B. J. DORMAN and R. E. SOUTHAM 1984 *SAE Earthmoving Conference Technical Paper* 840773. Modeling of tapered roller bearings in structural analyses.