A STUDY OF PASSIVE AND ADAPTIVE HYDRAULIC ENGINE MOUNT SYSTEMS WITH EMPHASIS ON NON-LINEAR CHARACTERISTICS

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Passive hydraulic mounts exhibit excitation frequency variant and deflection amplitude sensitive stiffness and damping properties. Such non-linear dynamic characteristics are examined by using analytical and experimental methods, both at the device level and within the context of a simplified vehicle model. A new lumped parameter non-linear mathematical model of the hydraulic mount is developed by simulating its decoupler switching mechanism and inertia track dynamics. The low frequency performance features and limitations of several passive mounts are made clear through the non-linear vehicle model simulation and comparable laboratory vibration tests. The high frequency performance problems of the passive hydraulic mount are identified by applying the quasi-linear analysis method. Based on these results, a new adaptive mount system is developed which exhibits broad bandwidth performance features up to 250 Hz. It implements an on–off damping control mode by using engine intake manifold vacuum and a microprocessor based solenoid valve controller. A laboratory bench set-up has already demonstrated its operational feasibility. Through analytical methods, it is observed that our adaptive mount provides superior dynamic performance to passive engine mounts and comparable performance to a small scale active mount over a wide frequency range, given the engine mounting resonance control, shock absorption and vibration isolation performance requirements. Although technical prospects of the proposed adaptive system appear promising, the in situ performance needs to be evaluated.

1. INTRODUCTION

Vehicular engine mounting vibration control is generally achieved by either using passive mounts or implementing a semi-active/active vibration control strategy [1–7]. Passive hydraulic mounts (as shown in Figure 1(a) and described in section 2) exhibit improved dynamic properties over rubber–metal mounts through their inherent excitation frequency variant and deflection amplitude-sensitive stiffness and damping properties. In our previous paper [8], linear system theory had been employed to estimate the dynamic characteristics of hydraulic mounts. Recently, a non-linear mathematical model has been developed for a generic hydraulic mount with only an inertia track (Figure 1(b)), by formulating fluid system equations and measuring non-linear system parameters [9]. It has been shown that this model can simulate the frequency variant non-linear dynamic properties of the inertia track mount reasonably well up to 50 Hz. However, switching decoupler dynamics must be formulated in order to model the deflection amplitude sensitive non-linear dynamic properties of the regular hydraulic mount. In addition, passive hydraulic mounts are known to exhibit a few performance problems at higher frequencies, say beyond 150 Hz, which may deteriorate vibration isolation properties and
acoustic behavior of an engine mounting system. However such high frequency problems are yet to be examined in a systematic and analytical manner.

The basic idea of a semi-active control scheme is to dissipate the vibratory energy by changing the mount damping properties with a low speed, low power actuator at a minimal additional cost. Conversely, in the active control mode, an active energy source is needed, but it may be cumbersome and cost-prohibitive for automotive applications. Therefore, from the practical viewpoint, semi-active vibration control looks more promising, as it may be sufficient to provide an improved ride quality for vehicle occupants. The adaptive hydraulic mount is one example of semi-active vibration control; typically, a passive fluid damping mode is utilized to dissipate the engine-mounting vibratory energy. On-off adaptive mounts employ an electrically operated rotary solenoid valve [1], a vacuum operated rotary valve, or an electrodynamic decoupling system; all of these can be controlled electronically by an on-board microprocessor. On the other hand, continuous adaptive mounts are based on the following methods: to employ electronic orifice valves and a servohydraulic injection system [2]; to apply vacuum in adjusting the fixed decoupler compliance [3]; to use electro-rheological fluid with a high voltage power supply [4]; and to employ an electrodynamic actuator in the hydraulic working chamber for the vibration compensation system [5]. One typical example of the active control strategy is to utilize an electromechanical actuator as an active mass damper [6]. This active system requires
a power amplifier to generate the dynamic force which counteracts the disturbance force during engine idling. Frequency shaped active control has been employed with a servo-hydraulic system in order to isolate the steady state narrow-band engine disturbance forces from the aircraft frame [7]. Almost all of the adaptive or active engine mounts, as reported in the literature, have been designed essentially to control the low frequency noise and vibration problems, especially during engine idling. Conversely, the adaptive hydraulic mount incorporating an electromagnetic decoupling system in the mount top element was aimed specifically at influencing the high frequency acoustic behavior [5].

The overall objective of this study is to understand the non-linear dynamic performance characteristics of the hydraulic mount both analytically and experimentally, and to develop a new adaptive hydraulic mount system with broad bandwidth performance features, i.e., up to 250 Hz. This paper is essentially an extension of our prior articles [8, 9], with an emphasis placed on automotive applications.

2. PROBLEM FORMULATION

2.1. PASSIVE HYDRAULIC MOUNT

The construction details of the regular hydraulic mount, equipped with an inertia track and a decoupler, are shown in Figure 1(a). The hydraulic mount is connected to the engine and chassis through the mounting studs (1) and (2). The top element (3), made up of rubber material (duro 51), supports the static engine weight. The upper chamber (4) and lower chamber (5) are filled with the glycol fluid mixture of antifreeze and distilled water. A cyclic engine motion causes oscillating fluid flow between two chambers. A fraction of the displaced fluid is accommodated by the decoupler (6) motion and the remaining portion is forced to flow through the inertia track (7). The decoupler gap \( D_d \) is the available space between the upper plate (8) and the lower plate (9) within which the decoupler may move freely. The decoupler is typically produced from duro 70 rubber. The compliant thin rubber bellow (10) comprising the lower chamber is produced from duro 51 rubber. The air breather (11) enables the rubber bellow to move freely without any air compression effect. The canister (12) contains the inside parts mentioned above. Provided that the inertia track flow is not desired up to a relative engine displacement of \( \pm X_d \), \( D_d \) is chosen at the design stage, as stated below, without considering any dynamic effect:

\[
D_d = 2X_dA_p/A_d, \tag{1}
\]

where \( A_p \) is the equivalent piston area of the top element and \( A_d \) is the decoupler area. In the automotive industry, \( X_d \approx 0.2 \text{ mm} \) generally. For the regular mount, \( D_d = 0.7 \text{ mm} \) normally. The decoupler enables the hydraulic mount to be an amplitude sensitive device, and the inertia track renders it to be a frequency variant device.

The regular mount has been modified into various configurations for this study. The inertia track mount which is not equipped with the decoupler is shown in Figure 1(b). The simple orifice mount, where the upper and lower chambers are separated only by the upper plate having a center hole, is illustrated in Figure 1(c). Furthermore, in Figure 1(d) is shown the low damping dry rubber mount, the dynamic properties of which coincide with the top element, since the glycol fluid is drained out completely by removing the rubber bellow.

2.2. VEHICLE MODEL

In Figure 2(a) is shown a conceptual non-linear vehicle model which includes a generic engine mount and a suspension system, where \( H_x(\omega, X, T) \) and \( H_s(\omega, X, T) \) represent the
Figure 2. Vehicle models with engine disturbance force: (a) generic quarter-car model; (b) simplified model.

frequency ($\omega$), deflection amplitude ($X$) and temperature ($T$) dependent non-linear spring rates of these components. In addition, $m_e$ and $m_s$ are the engine and sprung masses, $F(t)$ is the unbalanced or disturbance engine force, and $x_e(t)$ and $x_s(t)$ are the dynamic displacements of $m_e$ and $m_s$, respectively. The road excitation is ignored here. Next, neglecting the variation of the engine mount spring rate with $T$, and considering a linear suspension system since the main focus of this study is on the engine mount, we obtain the following complex spring rates:

$$K_e(\omega, X, T) = K_e(\omega, X), \quad K_s(\omega, X, T) = K_s(\omega). \quad (2a, b)$$

In Figure 2(b) is shown the simplified two-degree-of-freedom (DOF) vehicle model including $K_e(\omega, X)$ and $K_s(\omega)$, i.e., the non-linear engine mount and the linear suspension system with a constant spring stiffness $k_s$ and a linear viscous damping coefficient $b_s$. In the case of hydraulic mount, $k_r(\omega)$ and $b_r(\omega)$ are the frequency variant elastic stiffness and damping coefficients of its top element in the shear mode, and $u(\omega, X)$ is the time varying non-linear hydraulic reaction force. The effect of $X$ on $k_r$ and $b_r$ is assumed to be negligible. The masses of the top element (\(\approx 0.3 \text{ kg}\)) and canister (\(\approx 0.56 \text{ kg}\)) are incorporated directly into $m_e$ and $m_s$ since they are insignificant in the context of the vehicle model. Note that $u = 0$ for a conventional rubber–metal mount, and $u(\omega, X)$ represents the actuation force in the case of active mount.

2.3. SCENARIOS AND OBJECTIVES

First, a “black box” approach is undertaken to examine the dynamic behavior of the hydraulic mount. The high frequency dynamic characteristics are examined specifically in the frequency domain by employing the simple orifice mount of Figure 1(c). A quasi-linear analysis method, neglecting the mechanical impedance loading effect of mounting components, is developed to identify the performance features and limitations of the regular mount (Figure 1(a)) in comparison with the low damping rubber mount (Figure 1(d)), in the context of the vehicle model (Figure 2(a)), especially at higher frequencies over 50–250 Hz.

Second, a lumped parameter non-linear mathematical model is developed to evaluate the dynamic performance of hydraulic mounts at lower frequencies over 1–50 Hz. The decoupler switching dynamics are formulated and the non-linear model is verified by comparing predictions with measured data in the frequency domain. Incorporating the mount mathematical model into the vehicle model as in Figure 2(b), both harmonic (3–20 Hz) and impulse responses are examined rigorously for the regular mount, inertia track mount and rubber mount of Figures 1(a), (b) and (d), respectively. Vibration tests of an engine mounting system are performed with a large scale electrodynamic shaker by
employing harmonic and shock excitations. Such an experiment is necessary because controlled excitation conditions are virtually impossible to achieve in a vehicle test. Experimental results are used to qualitatively support the vehicle model simulation results.

Third, a new adaptive hydraulic mount system is developed to resolve the low frequency performance limitations and circumvent the high frequency performance problems of the passive mount. The engine intake manifold vacuum of gasoline engines is utilized as the actuation power source in implementing the on–off damping control mode by means of solenoid valves, and a microcontroller system is employed as the electronic control module (ECM). During the preliminary phase of this study, the dynamic performance of the proposed adaptive system is evaluated analytically and compared with those of the passive mount and a small scale active mount in terms of harmonic and impulse responses.

3. DYNAMIC CHARACTERISTICS OF PASSIVE HYDRAULIC MOUNT

We take a “black box” approach to examine the dynamic behavior of the hydraulic mount and to evaluate its performance characteristics in a vehicle model. Therefore, the internal configuration of the mount is not of any concern in this section.

3.1. MEASUREMENT OF ENGINE MOUNT DYNAMIC PROPERTIES

In general, the frequency, strain–amplitude and temperature dependent dynamic properties of a mount/isolator are represented in terms of dynamic stiffness spectra. In Figure 3(a) it is shown how to determine the cross-point dynamic stiffness of a non-linear mounting component conceptually. Since an engine mount generally exhibits non-linear dynamic characteristics, the mean value $F_{trans}$ of the transmitted force $F_r(t)$ is not necessarily

![Figure 3](image-url)
zero and $F_T$ contains higher harmonics of $\omega_0$ in addition to the fundamental harmonic, even though the excitation $x(t)$ is sinusoidal:

$$x(t) = x_c(t) - x_s(t) = X \sin \omega_0 t,$$

$$F_T(t) = F_{m0} + \sum_{n=1}^{\infty} \tilde{F}_{Tm} \sin (m\omega_0 t + \phi_m) = \sum_{n=1}^{\infty} \tilde{F}_{Tm} e^{j m \omega_0 t},$$

where $\tilde{F}_{Tm}$ is the amplitude of each harmonic, and $\phi_m$ is the phase lead of $F_{Tm}(t)$ with respect to $x(t)$. Note that $x(t)$ is equivalent to the relative engine displacement with respect to the vehicle chassis or sprung mass. The complex-valued cross-point dynamic stiffness $\mathcal{K}$ for the given $X$ and $T$ may be defined as

$$\mathcal{K}(\omega) = \mathcal{F}[F_T(t)]/\mathcal{F}[x(t)],$$

where $\mathcal{F}$ represents the Fourier transformation. Suppose that the fundamental harmonic $F_{T1}(t)$ of the non-sinusoidal $F_T(t)$ is much greater than its higher harmonics and $F_{m0}$:

$$F_T(t) \approx F_{T1}(t) = \tilde{F}_{T1} \sin (\omega_0 t + \phi_1).$$

Note that

$$\mathcal{F}[x(t)] = j\pi X[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)],$$

$$\mathcal{F}[F_{T1}(t)] = j\pi \tilde{F}_{T1}[e^{-j\phi_1} \delta(\omega + \omega_0) - e^{j\phi_1} \delta(\omega - \omega_0)],$$

where $\delta(\cdot)$ is the impulse function. We may define the cross-point dynamic stiffness $K^*(j\omega)$ in terms of the fundamental harmonic as

$$K^*(j\omega) = \mathcal{F}[F_{T1}(t)]/\mathcal{F}[x(t)], \quad \text{at } \omega = \omega_0$$

$$= (\tilde{F}_{T1}/X)e^{j\phi_1} = K(\omega) e^{j\phi(\omega)} = K(\omega) \cos \phi_k(\omega) + j K(\omega) \sin \phi_k(\omega),$$

where $K$ is the dynamic stiffness modulus and $\phi_k$ is the loss angle representing the phase lead of $F_{T1}(t)$ with respect to $x(t)$. In what follows, an asterisk denotes that the response is approximated only at its fundamental harmonic. We obtain, from equations (9b) and (9c),

$$K(\omega) = \tilde{F}_{T1}(\omega)/X, \quad \phi_k(\omega) = \phi_1(\omega).$$

In practice, an electrohydraulic material testing system is employed to measure $K(\omega)$ and $\phi_k(\omega)$ of an engine mount. A compressive preload corresponding to a given engine mass is applied to the mount top element through the hydraulic actuator. The actuator is then excited with a sinusoidal stroke $x(t)$ under closed loop control. A typical industrial practice is to apply $X = 1\cdot0$ mm in the frequency range of 1–50 Hz and $X = 0\cdot1$ mm over 50–250 Hz. The Fourier series analysis or Fourier filter algorithm [10] is employed to extract $F_{T1}(t)$; note that $K(\omega)$ and $\phi_k(\omega)$ as given by equation (10) can be found directly from equations (3) and (6).

The typical $K(\omega)$ and $\phi_k(\omega)$, in the frequency range 1–250 Hz, of the low damping rubber mount are shown in Figure 3(b). Note that $K$ and $\phi_k$ are slightly frequency dependent. The typical $K(\omega)$ and $\phi_k(\omega)$ of the hydraulic mount ($A_i = 0\cdot7$ mm) at lower frequencies are shown in Figure 3(c). Its dynamic properties are highly frequency dependent. The peak loss angle, which is a measure of the maximum attainable damping coefficient at a given frequency, reaches 50–60° and decays thereafter. The typical dynamic stiffness spectra of the hydraulic mount at higher frequencies are shown in Figure 3(d). Observe the differences from the spectra of the rubber mount. The hydraulic mount exhibits a reduction in $K$ with a minimum occurring at around 120 Hz, followed by a steep rise thereafter. The level of $\phi_k(\omega)$ is relatively much higher in the entire frequency range.
In addition, $\phi_k > 90^\circ$ over a certain frequency range. These spectral data signify that the high frequency dynamic behavior of the hydraulic mount cannot be described by a single-DOF system linear or non-linear model.

3.2. NON-LINEAR DYNAMIC CHARACTERISTICS AT LOWER FREQUENCIES

Carrying out a series of component tests with various excitations, the non-linear dynamic characteristics of the hydraulic mount ($d_a = 2.24 \text{ mm}$) are illustrated. Its $K(\omega)$ and $\phi_k(\omega)$, measured with $X = 0.1, 1.0, 2.0$ and $3.0 \text{ mm}$ are shown in Figure 4. We observe the deflection amplitude dependent non-linearity; the levels of $K$ and $\phi_k$ become higher initially as $X$ increases from 0.1 to 1.0 mm, while these levels are reduced as $X$ increases further.

3.3. HIGH FREQUENCY DYNAMIC CHARACTERISTICS

In order to understand the high frequency fluid dynamics processes, a controlled experiment is carried out on the simple-orifice mount of Figure 1(c). In Figure 5 are shown the variations of $K(\omega)$ and $\phi_k(\omega)$ with the orifice diameter $d_o$, where $X = 0.1 \text{ mm}$. In the plots of $K(\omega)$, we observe a consistent pattern. As $d_o$ increases, the frequency $f_{K_{\min}}$ at which the minimum $K$ is exhibited shifts to a higher value and the level of $K(\omega)$ is decreased. It seems that $f_{K_{\min}}$ is a natural frequency related to the chamber fluid dynamics. Examining the plots of $\phi_k(\omega)$ at $f_{K_{\min}}$ for all $d_o$, we first note that $\phi_k = 90^\circ$ at $f_{K_{\min}}$; this signifies that there is a resonance. Second, $\phi_k$ shifts abruptly from about $0^\circ$ toward $180^\circ$, like the phase response of an undamped second order dynamic system. This indicates that the resonance is not dominated by the fluid flow resistance but rather by the fluid inertial effect.

Returning to the plots of $K(\omega)$, we observe that $f_{K_{\min}}$ takes a higher value as $d_o$ increases.
This observation looks reasonable in view of linear system analysis. It shows symbolically that, in this fluid system, natural frequencies are proportional to \( I^{-1/2} o \), which in turn is proportional to \( \phi_e \), where \( I \) is fluid inertance. Therefore, the resonant fluid mass may be related to \( m_1 \) rather than \( m_2 \), with respect to Figure 1(c). Precise mathematical modelling is not pursued here, since the fluid motion is highly turbulent at higher frequencies. We may examine the high frequency response in more detail by plotting the spectra of \( 20 \log_{10} (K/k) \) and \( f_K \), where the nominal static stiffness of the hydraulic mount \( k = 2 \cdot 2 \times 10^5 \) N/m. For instance, in case that \( \phi_e = 30 \) mm, four distinct natural frequencies are observed at about 100, 150, 190 and 250 Hz, with \( \phi_e = 90^\circ \) at each natural frequency. It appears that such high frequency fluid dynamic behavior may not be represented with a one-dimensional lumped parameter model.

3.4. QUASI-LINEAR ANALYSIS OF A NON-LINEAR VEHICLE MODEL

It has been shown in the preceding sections that the hydraulic mount exhibits non-linear dynamic characteristics with deflection amplitude, and the highly turbulent fluid dynamics processes at higher frequencies are not amenable to precise mathematical modelling. With that reasoning, a quasi-linear analysis method is employed in order to evaluate the dynamic performance of the generic engine mount in a simple but reasonable way, without developing its rigorous mathematical model. The main feature of this method is that it incorporates the measured cross-point dynamic stiffness spectra, obtained from a component test, directly into the non-linear vehicle dynamics. It is, however, assumed that \( X \) is uniform over the frequency range of concern, as employed in the component test, or the mechanical impedance loading effect [10] is neglected.

The equations of motion for the vehicle model of Figure 2(a) may be expressed in matrix form as

\[
\begin{bmatrix}
    m_e & 0 \\
    0 & m_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_e \\
    \ddot{x}_s
\end{bmatrix}
+ \begin{bmatrix}
    -K_e & -K_e \\
    -K_e & -K_e + K_s
\end{bmatrix}
\begin{bmatrix}
    x_e \\
    x_s
\end{bmatrix}
= \begin{bmatrix}
    F_e \\
    0
\end{bmatrix}.
\]  

(11)

Assuming that \( F(t) \) is sinusoidal, and presuming that the fundamental harmonics of the non-sinusoidal responses \( x_e(t) \) and \( x_s(t) \) are dominant over the higher harmonics, we may write the following for the steady state response:

\[
F(t) = F_e \cos(\omega t), \quad x_e(t) = X_e \cos(\omega t), \quad x_s(t) = X_s \cos(\omega t), \quad (12a-c)
\]

where \( F_e \) is the engine force amplitude, and the displacement amplitudes \( X_e \) and \( X_s \) are taken to be complex quantities since the phases of \( x_e(t) \) and \( x_s(t) \) are different from that of \( F(t) \). Considering only the frequency dependent properties for \( K_e \) and \( K_s \), and further in terms of fundamental harmonics, we obtain that

\[
K_e(j\omega, X, T) \approx K_e(j\omega), \quad K_s(j\omega, X, T) \approx K_s(j\omega), \quad (13a, b)
\]

Substituting equations (12) and (13) into equation (11), we may obtain the frequency domain equations

\[
\begin{bmatrix}
    K_e - m_\omega^2 & -K_e \\
    -K_e & K_s + K_e - m_\omega^2
\end{bmatrix}
\begin{bmatrix}
    X_e \\
    X_s
\end{bmatrix}
= \begin{bmatrix}
    F_e \\
    0
\end{bmatrix}.
\]  

(14)

In vehicle dynamics, the occupant ride quality and vehicle durability are often quantified in terms of the sprung mass and engine mass acceleration amplitudes \( \ddot{X}_e \) and \( \ddot{X}_s \), respectively:

\[
\ddot{X}_e = -\omega^2 X_e, \quad \ddot{X}_s = -\omega^2 X_s, \quad (15a, b)
\]
From equations (14) and (15), we may obtain the following accelerance frequency response functions:

\[
\frac{\dot{X}_e^*}{F_e^*}(\omega) = \frac{m,m_\omega^4 - (K_* + K_*^*)\omega^2}{m,m_\omega^4 - (m, K_* + m, K_*^*)\omega^2 + K_* K_*^*},
\]

\[
\frac{\dot{X}_s^*}{F_s^*}(\omega) = \frac{-K_* \omega^2}{m,m_\omega^4 - (m, K_* + m, K_*^*)\omega^2 + K_* K_*^*}.
\]

As a result, we are assuming a linear response at each excitation frequency. It follows that the frequency variant \(K_*^j\omega\) and \(K_*^j\omega\) may be decomposed as

\[
K_*^j\omega = K_1^j\omega + jK_2^j\omega,
\]

\[
K_*^j\omega = K_1^j\omega + jK_2^j\omega,
\]

where \(K_1\) and \(K_2\) are real parts, and \(K_2\) and \(K_2\) are imaginary parts. Substituting equation (18) into equations (16) and (17), we may obtain the engine and sprung mass accelerances \(a_e^*\) and \(a_s^*\) as follows:

\[
a_e^j\omega = a_e^j\omega e^{j\phi_e(\omega)} = \frac{\dot{X}_e^*}{F_e^*}(\omega) = \frac{R_{Ne} + jI_{Ne}}{R_D + jI_D},
\]

\[
a_s^j\omega = a_s^j\omega e^{j\phi_s(\omega)} = \frac{\dot{X}_s^*}{F_s^*}(\omega) = \frac{R_{Ns} + jI_{Ns}}{R_D + jI_D},
\]

where

\[
R_D = m,m_\omega^4 - (m, K_1 + m, K_3)\omega^2 + K_1 K_3 \quad K_2 K_3 - K_2 \quad K_2 K_3,
\]

\[
I_D = -(m, K_2 + m, K_2 + m, K_2)\omega^2 + K_1 K_2 + K_2 K_2 - K_2 K_2,
\]

\[
R_{Ne} = m_\omega^4 - (K_1 + K_3)\omega^2, \quad I_{Ne} = -(K_2 + K_2)\omega^2,
\]

\[
R_{Ns} = -K_2 K_2 \omega^2, \quad I_{Ns} = -K_2 K_2 \omega^2.
\]

Equations (19) and (20) state that the engine and sprung mass accelerance moduli \(a_e^*\) and \(a_s^*\) can be determined when the variation of each complex spring rate with \(\omega\) is known.

Now, in correlating \(K_*^j\omega\) of the engine mount with its \(K_*^j\omega\), it is presumed that \(K_*^j\omega\) determined from the component test is equivalent to \(K_*^j\omega\) in the vehicle model. Comparing equation (9d) with equation (18a), we obtain that

\[
K_1(\omega) = K(\omega) \cos[\phi(\omega)], \quad K_2(\omega) = K(\omega) \sin[\phi(\omega)].
\]

The suspension system is represented by a Voigt model with \(K_1\) and \(K_2\). Similarly,

\[
K_1(\omega) = k, \quad K_2(\omega) = b_2 \omega.
\]

We may now calculate \(z_e(\omega)\) and \(z_s(\omega)\) for both rubber and hydraulic mounts; \(K_1(\omega)\) and \(K_2(\omega)\) are obtained from equation (22), with \(K_*\) shown in Figures 3(b)–(d). The main premise here is to assume that \(F_*\) varies at each \(\omega\) to produce the same \(X\) as applied in equation (3). Vehicle model parameters corresponding to a typical medium sized passenger car are listed in Table 1.

<table>
<thead>
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<th>Table 1</th>
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<tbody>
<tr>
<td><strong>Vehicle model parameters</strong></td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>(m_1) (kg)</td>
</tr>
<tr>
<td>----------------</td>
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<tr>
<td>122.3</td>
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Figure 6. Accelerance spectra predicted by quasi-linear vehicle model for the rubber mount (—–) and the passive hydraulic mount (——): (a) low frequency engine mass accelerance; (b) low frequency sprung-mass accelerance; (c) high frequency sprung-mass accelerance.

\[ a_e(v) \text{ and } a_s(v) \] are shown at lower frequencies in Figures 6(a) and (b). Obviously, from equations (19) and (20), \[ a = 0 \text{ as } \omega \to 0. \] We note that the hydraulic mount has no effect on the sprung-mass resonance mode occurring around 1 Hz associated with \( k_s \) and \( b_s \). In the engine resonance mode occurring around 9 Hz, we observe excellent resonance control characteristics of the hydraulic mount over the rubber mount. This fact results from the higher \( K_{e2} \) of the hydraulic mount [11]. In particular, one performance limitation of the passive hydraulic mount is identified as follows. Beyond the engine resonance mode, the hydraulic mount yields a higher \( a_s \) than the rubber mount. It results from the upper chamber pressure build-up phenomenon, and may cause deterioration of occupant ride quality. We note that \( K_{e1} \) increases rather abruptly beyond the peak loss angle frequency at 10 Hz. In turn, it will raise the natural frequency of the vehicle model, and consequently yield a higher \( a_e \). An ideally effective anti-vibration mount should possess a dynamic stiffness that either remains constant or increases slowly with \( \omega \) [12].

\[ a_s(v) \] is shown at higher frequencies in Figure 6(c). Up to 150 Hz, the passive mount yields as good, or even better, vibration isolation properties than the rubber mount. However, we observe a problematic aspect of the passive mount beyond 150 Hz: it yields a higher \( a_s \) than the rubber mount. This fact results from a steep increase in \( K_{e2}(v) \). As a consequence, the passive mount may cause deterioration of the isolation properties of the engine mount and in turn could cause high “boom” noise levels. This observation is consistent with limited vehicle tests, in which the performance of the passive mount has been found to be worse than that of the low damping rubber–metal mount at higher frequencies [3]. Note that \( a_s(\omega) \approx 1/m \), as \( \omega \to \infty \). As described in section 3.3, the fluid inertial effect is the “culprit” for the high frequency problems, but it is inevitable as long as the top element excites the fluid vibration.

4. LOW FREQUENCY NON-LINEAR CHARACTERISTICS OF PASSIVE HYDRAULIC MOUNT

It was shown in section 3 that the hydraulic mount exhibits non-linear dynamic properties with deflection amplitude. Next, we account for the internal components of the
hydraulic mount and formulate its lumped parameter non-linear mathematical model on the basis of fluid system equations. The scope of analysis is, however, limited to the low frequency range; i.e., up to 50 Hz.

4.1. NON-LINEAR MATHEMATICAL MODEL OF HYDRAULIC MOUNT

The low frequency lumped parameter model of the hydraulic mount is shown in Figure 7(a). The fluid impedances for the upper and lower chamber inertances are neglected. Let \#1, \#2, \#i and \#d be the control volumes for the upper chamber, lower chamber, inertia track and decoupler, respectively. Applying the first order fluid system dynamics to \#i and \#d, and volumetric continuity to \#1 and \#2, the governing non-linear equations may be expressed in matrix form as

\[
\begin{bmatrix}
I_i & 0 & 0 & 0 \\
0 & I_d & 0 & 0 \\
0 & 0 & C_i(V_1, p_1) & 0 \\
0 & 0 & 0 & C_2(V_2, p_2)
\end{bmatrix}
\begin{bmatrix}
q_i \\
q_d \\
p_i \\
p_d
\end{bmatrix}
+ \begin{bmatrix}
R_i(\Delta p_i, q_i) & 0 & 1 & -1 \\
0 & R_d(\Delta p_d, q_d) & 1 & -1 \\
-1 & -1 & 0 & 0 \\
1 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
q_i \\
q_d \\
p_i \\
p_d
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
-A_p \dot{x}
\end{bmatrix},
\]

(24)

Figure 7. The lumped parameter model of the regular hydraulic mount shown in Figure 1(a). (a) Fluid system variables and control volume \#d. (b) Various stages of decoupler switching dynamics: —–, \#i; ——, \#d.
where \( I_i \) and \( I_d \) are the time-invariant fluid inertances of \# i and \# d, \( q_i(t) \) and \( q_d(t) \) are the volume flow rates through \# i and \# d, \( p_1(t) \) and \( p_2(t) \) are fluid pressures of \# 1 and \# 2, the pressure drop \( \Delta p_i(t) = p_2(t) - p_1(t) \), \( \Delta p_d(t) \) is the pressure drop inside \# d, \( R_i(\Delta p_i, q_i) \) and \( R_d(\Delta p_d, q_d) \) represent the non-linear fluid resistances associated with \# i and \# d, and \( C_i(V_1, p_1) \) and \( C_d(V_2, p_2) \) represent the non-linear compliances of \# 1 and \# 2, respectively. Note that \( C_i(V_1, p_1), C_d(V_2, p_2) \) and \( R_i(\Delta p_i, q_i) \) must be measured in order to characterize non-linear system properties of the hydraulic mount; these are reported in our prior article [9].

The switching mechanism of the decoupler is a highly non-linear function of the decoupler free-volume gap \( V_{gap} \), \( V_d(t) \), \( p_1(t) \) and \( p_2(t) \), where

\[
V_{gap} = A_d A_d, \quad V_d(t) = \int_{t_i}^{t} q_d(t) \, dt, \quad (25a, b)
\]

and where \( t_i \) is the time at the instant that the decoupler is open, as explained below. Various stages of the low frequency decoupler switching dynamics are described with reference to Figure 7(b), which shows the starting transient pressure time histories of the regular mount \((A_d = 3.0 \text{ mm})\) as excited with \( X = 1.0 \text{ mm} \) at 14 Hz.

**Stage 1:** suppose that the decoupler is centered between the upper plate and the lower plate at \( t = 0 \). The fluid displaced by the upward \( x \) may begin to flow entirely through the decoupler, since \( R_d \) is negligible in comparison with \( R_i \). When \( V_d < 0.5V_{gap} \), the decoupler is open. This is called the “decoupled state”, since the inertia track is decoupled from the hydraulic mount dynamics. It follows that \( p_1 \approx p_2 \) and \( q_i \approx 0 \) in this stage.

**Stage 2:** at the moment \( V_d = 0.5V_{gap} \), the decoupler sits under the upper plate. Thus \( q_i \approx 0 \) thereafter, since the decoupler is virtually closed. This is called that “closed state” since the inertia track is coupled to the mount dynamics. From this moment on till the decoupler is open again, the fluid displaced farther by the upward \( x \) may flow entirely through the inertia track in the direction from the lower to the upper chambers; i.e., \( q_i \neq 0 \). It follows that \( p_1 < p_2 \) during this stage and \( p_1 \) is usually lower than \( p_{sw} \) (i.e., experiencing vacuum), depending on the magnitude of \( X \).

**Stage 3:** next, at the moment \( p_1 \geq p_2 \) in conjunction with the downward \( x \), the decoupler is open again, yielding the decoupled state. It follows that \( p_1 \approx p_2 \) and \( q_i \approx 0 \) during this stage.

**Stage 4:** at the moment \( V_d = V_{gap} \), the decoupler sits on the lower plate. It is closed again, causing the flow \( q_i \) in the direction from the upper to the lower chambers while \( q_d = 0 \). This is the coupled state and \( p_1 > p_2 \) during this stage.

**Stage 5:** next, at the moment \( p_2 \geq p_1 \) in conjunction with the upward \( x \), the decoupler is open again, yielding the decoupled state. It follows that \( p_1 \approx p_2 \) and \( q_i \approx 0 \) during this stage.

**Stage 6:** at the moment \( V_d = V_{gap} \), the decoupler sits under the upper plate again. It is closed now, causing the flow \( q_i \) in the direction from the lower to the upper chambers while \( q_d = 0 \). This is the coupled stage and \( p_2 > p_1 \) during this stage.

Thereafter, stages 3–6 repeat themselves as long as \( X \) is large enough to produce the fluid flow through the inertia track, as illustrated in Figure 7(b). Note that stages 1 and 2 exist only during the initial transient state. Provided that \( x \) is small enough to yield \( V_d < V_{gap} \) through the entire period of excitation, the decoupler is never closed at any moment and \( q_i \approx 0 \). Therefore, \( m \) is virtually supported by the low damping top element since the lower rubber bellow is very compliant; this yields the desired vibration isolation properties during the steady state operation. A simulation model is formulated by analyzing equation (24) and incorporating the decoupler switching mechanism, as described overleaf.
4.1.1. Inertia track dynamics

The inertia track consists of two distinct paths \( \# i_1 \) and \( \# i_2 \), since a small clearance exists between the blocking marker and the lower plate [9]. The momentum equations for two parallel paths \( \# i_1 \) and \( \# i_2 \) are

\[
\begin{align*}
    p_2(t) - p_1(t) &= I_1 \dot{q}_1(t) + \Delta p_{i_1}(t) q_1(t) [q_1(t)], \\
    p_2(t) - p_1(t) &= I_2 \dot{q}_2(t) + \Delta p_{i_2}(t) q_2(t) [q_2(t)],
\end{align*}
\]

where the positive values \( \Delta p_{i_1}(t) \) and \( \Delta p_{i_2}(t) \) are the pressure drops associated with the mainly turbulent flow \( q_1(t) \) and \( q_2(t) \). The fluid inertances \( I_{i_1} \) and \( I_{i_2} \) of \( \# i_1 \) and \( \# i_2 \) are defined as

\[
I_1 = k \rho_g l_{i_1}/A_{i_1}, \quad I_2 = k \rho_g l_{i_2}/A_{i_2},
\]

where \( k = 4/3 \) for laminar flow and 1 for turbulent flow, \( l_{i_1} \) and \( l_{i_2} \) are the lengths, \( A_{i_1} \) and \( A_{i_2} \) are the cross-sectional areas of \( \# i_1 \) and \( \# i_2 \) respectively, and \( \rho_g \) is the glycol fluid density.

4.1.2. Decoupler dynamics

Assuming turbulent flow through the decoupler orifice,

\[
\Delta p_{d}(t) = \frac{1}{C_{de} A_{de}} \left( \frac{\rho_g}{2} \right) q_d(t),
\]

where \( C_{de} \) is the discharge coefficient and \( A_{de} \) is the effective decoupler area representing the switching mechanism:

\[
A_{de} = A_d, \quad \text{for the decoupled state}; \quad A_{de} \approx 0, \quad \text{for the coupled state}.
\]

The momentum equation for \( \# d \) is

\[
p_2(t) - p_1(t) = I_d \dot{q}_d(t) + \Delta p_{d}(t) q_d(t) [q_d(t)],
\]

where the fluid inerance \( I_d = \rho_g l_d/A_d \), with \( l_d \) being the length of \( \# d \). The first order fluid dynamics of \( \# d \), as given by equation (31), is nearly undamped during the decoupled state since \( \Delta p_{d} \) generated by equations (28) and (29) is very small. Such undamped (or switching) decoupler dynamic characteristics cause pressure ripples in the decoupled state, as depicted in Figure 7(b). There are obviously three distinct flow paths between \( \# 2 \) and \( \# 1 \): \( q_{i_1} \), \( q_{i_2} \) and \( q_d \). However, in the decoupled state, \( q_{i_1} \approx q_{i_2} \approx 0 \) since \( \Delta p_{d} \) is negligible in comparison with \( \Delta p_{i_1} \) or \( \Delta p_{i_2} \) for a given \( q_d \). On the other hand, in the coupled state, \( q_d \approx 0 \) since \( \Delta p_{d} \) given by equations (28) and (30) is much greater than \( \Delta p_{i_1} \) or \( \Delta p_{i_2} \). For simulation details, see references [13].

4.1.3. Volumetric continuity of fluid chamber

The upper and lower chamber volume increments \( V_1(t) \) and \( V_2(t) \), from the condition that \( p_1 = p_2 = p_{\text{atm}} \), are given as

\[
\begin{align*}
    V_1(t) &= V_1^0 + V(t) - A_e [x_e(t) - x_e(t)], \\
    V_2(t) &= V_2^0 - V(t),
\end{align*}
\]

\[
V(t) = \int_{t_0}^{\infty} [q_{i_1}(t) + q_{i_2}(t) + q_d(t)] \, dt,
\]

where \( V_1^0 \) and \( V_2^0 \) are volume increments under the static equilibrium, and \( V(t) \) is the total fluid volume transferred between \( \# 2 \) and \( \# 1 \).
4.2. EXPERIMENTAL VERIFICATION OF NON-LINEAR MODEL

The mathematical model is verified in the frequency domain with regard to dynamic stiffness spectra of the regular mount \((A_d = 0.7 \text{ mm})\), as shown in Figure 8. In extracting \(F_{F(t)}\), the Fourier filter algorithm is used in simulation, while the Fourier series analysis is applied to the sampled \(F_{F(t)}\) signal in experiment [9]. Note that both methods yield basically identical results. For a small amplitude excitation \((X = 0.1 \text{ mm})\), the predicted and measured \(K(v)\) and \(f_K(v)\) match very well.

The spectra levels are fairly low since the decoupler is totally open; \(X_d = 0.16 \text{ mm}\) from equation (1). When \(X\) is increased to 1.0 mm, the predicted \(K(v)\) and \(f_K(v)\) are lower than the measured spectra, especially beyond the peak loss angle frequency. These discrepancies may be due to several simplifying assumptions made in our low frequency model [9]. In addition, the highly non-linear decoupler dynamics could be more complicated than our lumped parameter formulation. As a whole, the mathematical model predicts the frequency variant and amplitude sensitive dynamic properties of the hydraulic mount reasonably well up to 50 Hz. Nevertheless, there is plenty of room for improvement in non-linear modelling [13].

4.3. ANALYSIS OF VEHICLE MODEL WITH NON-LINEAR MOUNT

Now the mechanical impedance loading effect of the hydraulic mount on vehicle dynamics can be taken into account. Applying Newton’s second law to the vehicle model of Figure 2(b), we obtain the dynamic equations of motion in matrix form,

\[
\begin{bmatrix}
   m_e & 0 \\
   0 & m_s
\end{bmatrix}
\begin{bmatrix}
   \ddot{x}_e \\
   \ddot{x}_s
\end{bmatrix}
+ \begin{bmatrix}
   b_e & -b_e & -b_s & b_s \\
   -b_e & b_s + b_e & -b_e & b_e
\end{bmatrix}
\begin{bmatrix}
   \dot{x}_e \\
   \dot{x}_s
\end{bmatrix}
+ \begin{bmatrix}
   k_e & -k_e & -k_s & k_s \\
   -k_e & k_s + k_e & -k_e & k_e
\end{bmatrix}
\begin{bmatrix}
   x_e \\
   x_s
\end{bmatrix}
= \begin{bmatrix}
   F + u \\
   -u
\end{bmatrix},
\]

(34)

**Table 2**

<table>
<thead>
<tr>
<th>Specifications of passive hydraulic mount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia track geometry: (A_i = A_s = 0.2726 \text{ cm}^2)</td>
</tr>
<tr>
<td>Channel code (A): (l_i = 2.5 \text{ cm}, \ l_s = 18.6 \text{ cm})</td>
</tr>
<tr>
<td>Channel code (N): (l_i = 16.1 \text{ cm}, \ l_s = 5.0 \text{ cm})</td>
</tr>
<tr>
<td>Channel code (Q): (l_i = 19.1 \text{ cm}, \ l_s = 2.0 \text{ cm})</td>
</tr>
<tr>
<td>Fluid chamber parameters under (m_e = 122.3 \text{ kg}): (P_i = 0.992 \text{ cm}^3, \ P_s = 28.251 \text{ cm}^3, \ \bar{p} = 116.4 \text{ kPa})</td>
</tr>
<tr>
<td>Top element and decoupler parameters: (A_i = 50.27 \text{ cm}^2, \ A_s = 25.0 \text{ cm}^2)</td>
</tr>
<tr>
<td>Glycol fluid property: (\rho_f = 1059 \text{ kg/m}^3)</td>
</tr>
</tbody>
</table>
where
\[ u(t) = A_p \{ p_1(t) - \bar{p} \}, \]  

and where \( \bar{p} \) is the chamber fluid pressure under the static equilibrium condition. All of the governing equations are solved simultaneously by using a direct time domain integration software [14]. The hydraulic mount specifications are listed in Table 2. Both steady state and transient dynamic performance characteristics of various passive mounts are evaluated and compared with one another [15]. Unless stated otherwise, \( D_d = 0.7 \) mm for the regular mount and the inertia track code is \( N \); referring to Table 2, various codes denote different \( l_1 \) and \( l_2 \).

4.3.1. Harmonic response

The harmonic engine force of uniform amplitude, \( F(t) = 100 \sin \omega t (N) \), is given as the excitation from 3 to 20 Hz. In Figure 9(a) is shown the sprung mass acceleration amplitude spectrum \( \ddot{X}_s(\omega) \) for the rubber mount and the hydraulic mounts with \( D_d = 0, 0.7 \) and 1.4 mm; the fundamental harmonic \( \ddot{x}_s(t) \) is obtained by using the Fourier filter algorithm. A high peak resonance occurs at 9.2 Hz for the low damping rubber mount. The regular hydraulic mount with \( D_d = 0.7 \) mm clearly shows its superior dynamic performance: resonance control and vibration isolation beyond the engine resonant frequency. The resonance level is significantly reduced by the fluid damping dynamics of the inertia track, with the peak \( \ddot{X}_s \) decreasing from 3.3 m/s² to 0.7 m/s². On the other hand, for \( f \geq 15 \) Hz, the decoupler action yields a low \( \ddot{X}_s \), similar to the response produced by the rubber mount. It turns out that \( X < X_d = 0.16 \) mm, or the decoupler is totally open, for \( f \geq 15 \) Hz. One problematic aspect of the regular mount is observed between 11 Hz and 15 Hz, where the hydraulic mount yields a higher \( \ddot{X}_s \) than the rubber mount. This performance limitation may result from the stiffening phenomenon of the hydraulic mount as explained in section
builds up beyond the resonant frequency of the inertia track dynamics, and some natural frequencies of the vehicle engine mounting system will obviously be raised. As \( A_d \) increases from 0.7 to 1.4 mm, the resonance control characteristic deteriorates and the decoupler begins to open totally at a lower frequency, as expected. As \( A_d \) is increased further, the dynamic response may eventually approach that yielded by the rubber mount. In addition, a larger \( A_d \) produces more discontinuity in the response spectrum. Interestingly, the inertia track mount \((A_d = 0 \text{ mm})\) yields a different dynamic response from the regular mount. Below 15 Hz, this mount produces a lower \( X_s \) than the regular mount and rubber mount. The regular mount is not effective in absorbing engine resonance in the decoupled state for \( x < X_e \). However, beyond 15 Hz, \( X_s(\omega) \) for the inertia track mount gradually increases with \( \omega \), unlike the regular mount where it decreases. This may be the reason why the inertia track mount is not suitable as an engine mount by itself; its vibration isolation property is obviously inadequate. In summary, the inertia track mount has excellent resonance control characteristics, while the low damping rubber mount exhibits superior vibration isolation property. In Figure 9(b) are shown the starting transient responses for the regular mount, presenting the resonance control characteristics at 7 Hz. Note that \( X_e(t) \) and \( X_s(t) \) are considerably non-sinusoidal; the irregular shape of \( X_e(t) \) results from the decoupler switching dynamics.

4.3.2. Impulse response

An ideal impulse is simulated by assigning an initial velocity to \( m_e \); \( \dot{x}_e(0) = 0.1 \text{ m/s} \). Top element parameters are taken by their nominal values at 10 Hz; \( k_r = 2.84 \times 10^5 \text{ N/m} \) and \( b_r = 174 \text{ N} \cdot \text{s/m} \). In Figure 10(a) the rubber mount and the regular mount are compared with regard to the impulse response or shock absorption properties. Although the hydraulic mount produces a higher first peak of \( X_s \), it quickly dampens the subsequent response. Furthermore, as shown in Figure 10(b), the inertia track mount is more effective in absorbing the shock excitation. The regular mount is in the decoupled state after the initial transient state; thus \( q \approx 0 \) and fluid damping does not arise. The oscillation frequency of \( X_e(t) \) during the decoupled state coincides with the engine resonant frequency (9.2 Hz) which is exhibited in the impulse response of the rubber mount, as shown in Figure 10(a). On the other hand, the inertia track mount can function as a shock absorber at any moment, since it is always in the coupled state.

![Figure 10](image-url)
4.4. VIBRATION TEST OF ENGINE MOUNTING SYSTEM

An electrodynamic vibration testing system is employed in order to support the vehicle model simulation qualitatively. As shown in Figure 11, a motion input is applied to the engine mounting system through the shaker table (capacity 4900 N sine peak, \( \pm 5.0 \) cm stroke). Both steady state and transient dynamic performances of the regular hydraulic and low damping rubber mounts are examined and compared with one another. The engine mass, made up of iron weights, is supported by low friction linear bearings, with one at each side such that only vertical motion is feasible. The equivalent \( m_e \), including the masses of two linear bearings and the connecting structure, is 110 kg. Harmonic or shock excitation \( \ddot{x}_e(t) \) is applied through the shaker table and the dynamic response \( \ddot{x}_o(t) \) of \( m_e \) is recorded. Both signals are measured with piezoelectric accelerometers [16]. The LVDT measures the engine displacement relative to the table during shock tests. A strain-gage type pressure transducer is installed at the wall of the mount top element in order to measure \( p_1(t) \). Its sensing diaphragm is flush with the wall. All signals are acquired by and stored in a spectrum analyzer. These are then transferred to an off-line 386SX-PC by using the Standard Data Format Utilities software [17]. Note that the effective dynamic system here is similar to that employed in the vehicle model simulation, but not identical, because of strong dynamic interactions between the mount and shaker table (the dynamic model parameters of which are not known). Therefore, experimental data will be used to verify the phenomenon as opposed to a quantitative validation.

4.4.1. Harmonic test

A sinusoidal voltage command signal is fed to the power amplifier, with \( \ddot{x}_e(t) \) and \( \ddot{x}_o(t) \) being continuously monitored by the spectrum analyzer. As a benchmark for obtaining the frequency response for this non-linear system, the input acceleration amplitude \( \ddot{X}_i \) of the fundamental harmonic \( \ddot{x}_i(t) \) is maintained at a constant value of 1.7 m/s\(^2\) (r.m.s. value = 1.2 m/s\(^2\)). In Figure 12(a) are shown the output acceleration amplitude spectra \( \ddot{X}_o(\omega) \) and phase angle spectra \( \phi_{o,\omega} \) for three hydraulic and rubber mounts from 3 to 20 Hz. For the rubber mount, \( \ddot{X}_r \approx 10 \) m/s\(^2\) at the resonant frequency of around 7 Hz. The
hydraulic mounts exhibit resonance control characteristics similar to those described in Figure 9(a) for the engine force excitation. Their plots of $\phi_m(\omega)$ indicate that the hydraulic mount is highly damped. The variations of $\tilde{X}_o(\omega)$ and $\phi_m(\omega)$ with codes $A$, $N$ and $Q$ confirm that $l$ is a key design parameter to tune the dynamic properties of an engine mounting system. The problematic aspect of the passive hydraulic mount due to the upper chamber pressure build-up is also observed. The hydraulic mounts with codes $N$ and $Q$ exhibit vibration isolation characteristics for $f \geq 16$ Hz, whereas the mount with code $A$ provides vibration isolation for $f \geq 17$ Hz. The decoupler may stay decoupled when the vibration isolation properties are exhibited. In particular, for the hydraulic mount, the measured $\tilde{X}_o$ is relatively higher than the predicted $\tilde{X}_i$ with respect to the rubber mount response in each case. For instance at 20 Hz, $\tilde{X}_o$ of code $N$ is 50% higher than that of the rubber mount in Figure 12(a), whereas $\tilde{X}_i$ of code $N$ is only 10% higher than that of the rubber mount in Figure 9(a). This discrepancy may result from the undesirable side effect of the chamber fluid inertia, which is not included in our theory. Note that the inertia track mount is not employed in this vibration test. However, it is believed that this mount will produce performance characteristics similar to those seen in the vehicle model simulation: superior resonance control but poor vibration isolation properties.

In Figure 12(b) are depicted the steady state responses $\ddot{x}_o(t)$, $\ddot{x}_i(t)$ and $p_1(t)$ at 8 Hz in the resonance control region for the mount with code $N$. In the plot of $p_1(t)$, we observe decoupler dynamics stages 3–6. Very small pressure ripples are exhibited during decoupled states, as in the vehicle model simulation. In particular, even though the voltage command signal is sinusoidal, $\ddot{x}_i(t)$ is highly non-sinusoidal due to dynamic interactions between the

Figure 12. Measured harmonic responses of the engine mounting system. (a) Output acceleration spectra, given $\ddot{X}_i = 1.7 \text{ m/s}^2$: — $\theta$ —, rubber mount; — $\square$ —, hydraulic mount (code $A$); — $\triangle$ —, hydraulic mount (code $N$); — $\triangle$ —, hydraulic mount (code $Q$). (b) Steady state time histories for the hydraulic mount (code $N$, $A_v = 0.7$ mm) at 8 Hz.
heavier $m_e$ and the lighter shaker table, in addition to the decoupler switching dynamics. Comparing Figure 12(b) with Figure 9(b), we observe that $\ddot{x}_s(t)$ is similar to $\ddot{x}_d(t)$, whereas $\ddot{x}_o(t)$ looks like $\ddot{x}_e(t)$.

4.4.2. Shock test

A step voltage command signal is fed to the power amplifier. The hydraulic (code $N$) and rubber mounts are compared in Figure 13(a) with regard to $\ddot{x}_o(t)$. As in Figure 10(a) for the vehicle model simulation, the hydraulic mount produces a higher first peak, but it quickly dampens the subsequent response. The effect of different inertia track codes on the shock response was not significant. In particular, even for the rubber mount, the motion of $m_e$ settles down more quickly than observed in the simulation. This is because the vertical motion of $m_e$ is restricted by the guide rod. The corresponding $-x(t) = x_i(t) - x_o(t)$ are compared in Figure 13(b). The shapes of $x(t)$ are similar to those observed in simulation, and the hydraulic mount quickly dampens the engine mounting system as well.

5. A NEW BROADBAND ADAPTIVE HYDRAULIC MOUNT

Given the results of the preceding sections, a few conclusions can be drawn regarding the performance limitations of various passive mount configurations. Specifically, the inherent nature of the regular hydraulic mount is that the engine resonance around 10 Hz must be initiated before the mount is able to dissipate any engine mounting vibratory energy. In other words, fluid damping related with the inertia track flow cannot be generated during the relative engine motion corresponding to the decoupler gap. Yet, on the other hand, other limitations are associated with the undesirable side effect of fluid inertia. For instance, noise, vibration and harshness (NVH) problems of the regular mount result from fluid resonances at frequencies beyond 100 Hz. Furthermore, as seen in section 4.4.1, the fluid inertial effect influences the vibration isolation properties of the regular mount even around 20 Hz. To overcome such problems, a new adaptive mount is proposed.

5.1. PROPOSED ADAPTIVE MOUNT SYSTEM

A low damping rubber mount is most suitable for isolating the engine disturbance force from the vehicle frame at frequencies beyond the engine mounting resonance. On the other hand, the inertia track mount is most appropriate in controlling the engine resonance amplitude and absorbing the shock excitation. Accordingly, the basic premise behind the proposed broadband adaptive system is to let the hydraulic mount function as a rubber mount for the purpose of vibration and acoustic isolation, and as an inertia track mount for resonance control and shock absorption. This adaptive system is comprised of two
modules: a mechanical actuation system and an electronic controller [18]. As shown in Figure 14, the adaptive system can operate in either a “hard” or “soft” state. The rubber sheet, installed just under the top element, and the lower rubber bellow are connected to engine intake-manifold vacuum through two two-position, three-way on–off solenoid valves which are controlled by an ECM. Since intake-manifold vacuum reaches down to 27–44 kPa (absolute) or 17–22 inch Hg vacuum during the normal vehicle operation, it may serve as the vacuum source for our adaptive system. A block diagram of the microcontroller system for controlling the electro-pneumatic solenoid valves is shown in Figure 15. Various on-board sensors may provide the electronic control signals for the ECM. The control signals inform the ECM that the hard state is required, during engine load changes such as vehicle start-up, transmission shifting, and abrupt acceleration or deceleration, and during changes in maneuvering conditions such as braking, cornering and travelling on bumpy roads. The ECM turns off valve 2 and turns on valve 1. As a result, the adaptive mount basically becomes an inertia track mount to produce a high level of damping. Note that, by processing various sensor signals, the ECM can energize the solenoid valves before

Figure 14. The operation and arrangement of the proposed adaptive hydraulic mount: (a) hard state; (b) soft state.
the engine bounce actually takes place. This kind of predicted control is feasible since a
time lag exists between the event of a disturbance source and the engine bounce, due to
vehicle dynamics. For instance, regarding the detection of engine load changes, we may
utilize the throttle position signal, the selector level position signal of the electronically
controlled automatic transmission, and its shift control valve signals [1], etc. On the other
hand, when the ECM learns from the control signals that the soft state is required, i.e.,
during engine idling or cruising, it turns off valve 1 and turns on valve 2. As a result, the
engine motion cannot excite fluid vibration, and the adaptive mount essentially becomes
a rubber mount to produce a low level of dynamic stiffness and damping. A bench set-up
has been prepared in our laboratory to demonstrate the operational feasibility of the
adaptive system successfully. A power supply and vacuum pump simulated the automotive
battery and intake-manifold vacuum. The microprocessor unit and valve driver employed
for the ECM are the Motorola 6809 [19, 20] and Texas Instruments L293 [21], respectively.

5.2. PERFORMANCE CHARACTERISTICS OF ADAPTIVE SYSTEM

The dynamic performance of an adaptive system is evaluated analytically. The dynamic
equations of motion for the vehicle model installed with the adaptive mount are given by
equation (34), with \( u = 0 \) in the soft state. In order to determine the interrelationship
between \( u(t) \) and \( x(t) \), we examine equation (24) from the standpoint of linear system
theory with \( q_e = 0 \). Manipulating in the Laplace domain, we obtain that

\[
U(s) = -\left( \frac{C_1 I_1 A_1 s^2 + C_2 R A_1 s + A_2}{C_1 C_2 I_1 s^2 + C_1 C_2 R s + (C_1 + C_2)} \right) X(s),
\]  

(36)

where the Laplace operator \( s = j \omega \). Consequently, in the hard state, the adaptive mount
functions as a phase-reversed mechanical band-pass filter of the \( x(t) \) signal or a frequency
shaped dynamic compensator. The on–off control mode for \( u(t) \) is shown in the block
diagram of Figure 16.

In Figure 17 adaptive and passive mounts are compared with regard to the harmonic
response \( \bar{X}(\omega) \) and \( \bar{z}(\omega) \) over the entire frequency range. At lower frequencies in Figure
17(a), the adaptive mount functions in the hard state below 15 Hz, but reverts to the soft
state beyond 15 Hz. We observe superior resonance control characteristics of the adaptive
mount over the passive mount. Even its vibration isolation properties beyond 15 Hz are as desirable as those of the passive mount. Consequently, the adaptive mount provides a high damping to control low frequency vibration problems, such as engine shake and bounce, and yet it exhibits a sufficiently low dynamic stiffness at medium frequencies in order to control engine idling vibration. At higher frequencies in Figure 17(b), the adaptive mount, functioning in the soft state, yields superior vibration isolation characteristics over the passive mount. An ideal impulse response should look like Figure 10(b), since the adaptive mount functions in the hard state. Therefore, in addition to yielding excellent harmonic responses, the adaptive mount should improve both ride quality and vehicle durability for the shock excitation during an engine load change. Note that the actual transient response will depend on the switchover response speed of the adaptive mount. This hardware-related aspect is not pursued further in this study. Accordingly, the specific algorithms for switchover conditions and in situ performance must be determined and evaluated in a future study.

6. SMALL SCALE ACTIVE CONTROL MOUNT

In the interest of comparing the dynamic performance of the adaptive hydraulic mount with that of an active engine mount, a full state feedback control law is formulated on the basis of linear optimal control theory [22]. Suppose that a small scale electromechanical or electrohydraulic actuator with a wide bandwidth response provides the active control force $u(t)$ in Figure 2(b); thus the actuator dynamics are neglected. In state space notation, define the state vector $x(t)$ and output vector $y(t)$ of the vehicle model as follows where superscript $T$ denotes matrix transposition,

$$
\dot{x}(t) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} x_e - x_s & \dot{x}_e & x_e & \dot{x}_e \end{bmatrix}^T,
$$

$$
y(t) = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & \ddot{x}_s \end{bmatrix}^T,
$$

Figure 16. The block diagram for the fluid system of the adaptive mount.

Figure 17. Harmonic responses for three mounting modes in the vehicle model: ---, passive mount; ---, adaptive mount, ——, active mount. (a) $\ddot{x}_s(\omega)$ at lower frequencies; (b) $x_e(\omega)$ at higher frequencies.
The plant state equation and output equation are

\[ \dot{x}(t) = Ax(t) + Bu(t) + GF(t), \quad y(t) = Cx(t) + Du(t), \]  

where

\[ A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_r/m_r & -b_r/m_r & 0 & b_r/m_r \\ 0 & 0 & 0 & 1 \\ k_i/m_i & b_i/m_i & -k_i/m_i & -(b_i + b_r)/m_i \end{bmatrix}, \]

\[ B = [0 \quad 1/m_r \quad 0 \quad -1/m_i]^{T}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ k_i/m_i & b_i/m_i & -k_i/m_i & -(b_i + b_r)/m_i \end{bmatrix}, \quad D = [0 \quad 0 \quad 0 \quad -1/m_i]^{T}. \]

In designing the full state feedback law, a quadratic performance index \( J \) is taken to account for various aspects of vehicle performance such as ride quality, mount deflection, and energy expenditure:

\[ J = \frac{1}{2} \int_0^\infty \left[ W_1(y_1/y_m)^2 + W_2(y_2/y_m)^2 + W_3(y_3/y_m)^2 + W_4(y_4/y_m)^2 + W_5(y_5/y_m)^2 \right. \]
\[ + W_6(u/u_m)^2 \big] \, dt \]
\[ = \frac{1}{2} \int_0^\infty [y^{T}Qy + uRu] \, dt, \]

\[ Q = \begin{bmatrix} W_1/y_m^2 & 0 & 0 & 0 & 0 \\ 0 & W_2/y_m^2 & 0 & 0 & 0 \\ 0 & 0 & W_3/y_m^2 & 0 & 0 \\ 0 & 0 & 0 & W_4/y_m^2 & 0 \\ 0 & 0 & 0 & 0 & W_5/y_m^2 \end{bmatrix}, \quad R = W_6/u_m^2, \]  

where \( W_i (i = 1, \ldots, 6) \) are the relative weighting factors, and the subscript \( m \) denotes the maximum value of the root mean square of penalized variables. Observe that, in equation (41b), \( Q \) weighs \( y \) instead of \( x \). The linear control system design software computes a corresponding cross-weighting matrix [23]. This system is completely controllable and observable [24]. Next, suppose that we leave out the engine disturbance term \( GF(t) \) in equation (39) so as to solve a deterministic linear quadratic regulator problem, or apply the stochastic linear regulator problem. Then, we obtain the following optimal feedback law which minimizes \( J \), where \( G \) is the steady state feedback gain matrix:

\[ u(t) = -Gx(t). \]  

In order for the small scale active mount to yield the impulse response \( (x_2(0) = 0\cdot1 \text{ m/s}) \) comparable to that produced with the adaptive mount under a reasonable \( u(t) \), the following parameters are selected for \( J \): \( W_1 = W_2 = W_3 = W_4 = W_5 = W_6 = 1\cdot0, \)
The static stiffness of the suspension system is still $k_s$, whereas the quasi-static stiffness of the engine mount decreases to $k_r (k_s + g_1)/(k_s - g_3) = 6.2 \times 10^4$ N/m for $k_r = 6 \times 10^5$ N/m. Note that, for this active system, natural frequencies and damping ratios are as follows: $f_{1a} = 1.1$ Hz and $\zeta_1 = 0.25$ related to the sprung-mass bounce mode, and $f_{2a} = 6.4$ Hz and $\zeta_2 = 0.53$ related to the engine bounce mode.

In Figure 17 is shown the harmonic response $X_s(\omega)$ and $a_s(\omega)$ for the active mount in comparison with those of the passive and adaptive hydraulic mounts, over the whole frequency range. At lower frequencies in Figure 17(a), with $G$ given by equation (43), we observe that the dynamic performance of the active mount is as desirable as that of the adaptive mount, in terms of resonance control and vibration isolation capabilities. The amplitude of $u(t)$ is 100 N at 5 Hz. We observe that the active engine mount has no influence on the sprung-mass bounce mode, as explained in Figure 6(a). The level of $X_s(\omega)$ may decrease further if a suitable frequency-shaped $J$ is formulated with regard to $\ddot{x}(t)$; however, this can only be achieved at the expense of sacrificing the high frequency performance [23]. The method of frequency-shaped cost functionals is especially useful for the vibration control of narrow-band dynamic systems or for rejecting the narrow-band disturbance [7]. Recall the frequency-shaped dynamic properties of the adaptive mount, as given in equation (36). At higher frequencies in Figure 17(b), the vibration isolation properties of the active mount are not comparable to those of the adaptive mount. Since the bandwidth of actuator dynamics is always limited in practice, we reason that the high frequency performance of the active mount will be more problematic. Fully adaptive dynamic stiffness control active mounts with broadband disturbance rejection capabilities to be used for aircraft engines [26] are obviously beyond the scope of this study. As observed from Figure 18, the active and adaptive mounts produce similar impulse responses in terms of $\dot{x}_s(t)$ and $\dot{x}_e(t)$, where $u(0) = -330$ N for the active control. When the additional design features of the adaptive system, such as hardware simplicity, robustness, reliability and very low power requirements, are considered, we conclude that our adaptive mount is indeed superior to passive engine mounts and comparable to the small scale active mount.

7. CONCLUSIONS

This analytical and experimental study of passive and adaptive hydraulic mounts has made two major contributions to the state of the art. First, quasi-linear and non-linear
analyses of the passive mount in a simple vehicle model have been introduced for the first time, while making clear several issues regarding the dynamic behavior of the mount. With regard to the regular passive mount, in addition to understanding its resonance control, vibration isolation and shock absorption capabilities, specific performance limitations related to the upper chamber pressure build-up and the undesirable side effect of the chamber fluid inertia have been identified. Further research is, however, needed to develop high frequency models, as well as for an improved understanding of non-linear characteristics. Second, the conceptual development and hardware implementation have been presented for a new broadband adaptive hydraulic mount system which employs a vacuum pressure already existing in the engine intake manifold, and two solenoid valves. Even though technical prospects and practical aspects appear promising, the actual performance still must be examined through vehicle tests. Related research objectives which need to be addressed in a future study are as follows: (i) to determine the threshold engine speed at which the switching between the hard and soft states takes place, and their dependence on vehicle conditions, i.e., specific algorithms for switchover conditions; (ii) where and how to install the adaptive mount under the engine transmission block; and (iii) whether the engine intake-manifold vacuum system needs to be boosted, since a loss of intake-manifold vacuum occurs during vehicle acceleration or climbing.

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REFERENCES


APPENDIX: LIST OF SYMBOLS

- $A_d$, $A_e$ decoupler area and equivalent piston area of top element (m$^2$)
- $b_d$, $b_s$ damping coefficient of top element and suspension system (N · s/m)
- $d_o$ orifice diameter of single-orifice mount (m)
- $F$, $F_T$ excitation engine force and transmitted force (N)
- $F_o$ engine force amplitude (N)
- $F_{T1}$ fundamental harmonic of transmitted force (N)
- $f$ frequency (Hz)
- $G$ feedback gain matrix
- $J$ quadratic performance index
- $j = \sqrt{-1}$
- $K$, $K^*$ dynamic stiffness modulus and cross-point dynamic stiffness (N/m)
- $K^*, K^*$ complex spring rate of engine and sprung masses (N/m)
- $k_e$, $k_s$ elastic stiffness of top element and suspension system (N/m)
- $m_e$, $m_s$ a portion of engine mass and sprung mass (kg)
- $p_u$, $p_l$ pressure in upper and lower chambers (Pa)
- $q_d$, $q_i$ volume flow rate through decoupler and inertia track (m$^3$/s)
- $t$ time (s)
- $u$ hydraulic reaction force or active control force (N)
- $X$ amplitude of excitation displacement (m)
- $X_d$ relative engine displacement corresponding to $A_d$ (m)
- $\dot{X}_o$ output acceleration amplitude of engine mass (m/s$^2$)
- $\ddot{X}_e$ sprung-mass acceleration amplitude at fundamental harmonic (m/s$^2$)
- $x$, $\dot{x}$ relative engine displacement or excitation stroke (m)
- $\ddot{x}$ relative engine velocity or excitation velocity (m/s)
- $x_e$, $x_s$ engine and sprung-mass displacement (m)
- $\ddot{x}_e$, $\ddot{x}_s$ engine and sprung-mass acceleration (m/s$^2$)
- $\dddot{x}$ input acceleration of shaker table (m/s$^3$)
- $\dddot{x}_o$ output acceleration of engine mass (m/s$^3$)
- $a_e$, $a_s$ engine and sprung-mass accelerance modulus (kg$^{-1}$)
\( A_d \)  decoupler gap (m)
\( \phi_k \)  loss angle or dynamic stiffness phase (degrees)
\( \phi_{x_0} \)  phase lead of \( x_{i0} \) with reference to \( \dot{x}_{i0} \) (degrees)
\( \omega \)  circular frequency (rad/s)
\# d, \# i \)  control volume for decoupler and inertia track
\# 1, \# 2 \)  control volume for upper and lower chambers

**Superscripts**

- variable at static equilibrium
* complex variable related to fundamental harmonic
\( ^\top \)  matrix transposition