Examination of multi-dimensional vibration isolation measures and their correlation to sound radiation over a broad frequency range

Rajendra Singh*, Seungbo Kim

Acoustics and Dynamics Laboratory, Department of Mechanical Engineering and The Center for Automotive Research, The Ohio State University, Columbus, OH 43210-1107, USA

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Abstract

This article examines alternate vibration isolation measures for a multi-dimensional system. The isolator and receiver are modelled by the continuous system theory. The source is assumed to be rigid and both force and moment excitations are considered. Our analysis is limited to a linear time-invariant system, and the mobility synthesis method is adopted to describe the overall system behavior. Inverted ‘L’ beam and plate receivers are employed here to incorporate the contribution of their in-plane motions to vibration powers and radiated sound. Multi-dimensional transmissibilities and effectivenesses are comparatively evaluated along with power-based measures for the inverted ‘L’ beam receiver and selected source configurations. Further, sound pressures radiated from the inverted ‘L’ beam receiver are calculated and correlated with power transmitted to the receiver. Interactions within the ‘L’ beam receiver are also analyzed and measures that could identify dominant transfer paths within a system are examined. Sound measurements and predictions for the inverted ‘L’ plate receiver demonstrate that a rank order based on free field sound pressures, at one or more locations, may be regarded as a measure of isolation performance. Measured insertion losses for sound pressure match well with those based on computed results although further study is needed in relation to some discrepancies shown in the results. Finally, several emerging research topics are identified.

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1. Introduction

Force or velocity transmissibility and related concepts are widely used, mostly for unidirectional motions [1–5] but have been extended to multi-dimensional systems [6]. Also, the force or velocity effectiveness term, the ratio of transmitted force or velocity with an isolator to the one without the isolator, has been employed for both uni-directional [1,2,4] and multi-dimensional problems [7,8]. However, for a multi-dimensional problem, the units of translational and rotational quantities are not compatible [9–11]. For this reason, the concept of vibration power has been used to assess transmission to receivers [11–16]. Application of such measures includes structural discontinuities [17,18], vibration isolators [11–16], and bearings [19]. The transmission efficiency, the ratio of input vibration power to transmitted power, has been studied for propagating waves at the interface of infinite structures [17,18]. Input and transmitted powers have also been compared for some finite structures [15,20,21].

For a multi-dimensional system, strong interactions occur between the coupled degrees of freedom. For example, Cremer and Heckle [17] claim that the moment mobility must be dealt with via a matrix in the presence of flexural motions. They have also explained that the coupling mobility in flexural motions may suppress the bending waves under some conditions. This concept has been implemented to attenuate vibration input to a single structure [9]. Further, some separation of transfer paths has been experimentally investigated [22]. Nonetheless, a proper quantification or interpretation of dynamic interactions that occur among coupled structural paths has not been addressed. Furthermore, the quantification of structure-borne energy and source strengths still remain as key obstacles in many cases [23–27]. Satisfactory resolution of such research issues would require appropriate vibration transmission measures. However, such measures for a multi-dimensional system are not well understood and often left to the discretion of user [1,7,28]. In this article, we address this particular issue with emphasis on a multi-dimensional isolator (of distributed parameters) in the presence of a compliant receiver.

2. Problem formulation

The problem is defined via Figs. 1 and 2, in the context of source, path (isolator) and receiver. The scope of this study is limited to the analysis of a linear time-invariant (LTI) system with a single isolator (defined in terms of distributed parameter). In this multi-dimensional system, the source is described by a rigid body and two alternate compliant receivers, namely the inverted ‘L’ beam and plate structures, are utilized for analytical and experimental studies. Harmonic force and moment excitations are applied to the source, up to 2.5 kHz. Mobilities of each component are analytically or computationally obtained, and then the mobility synthesis method is employed to predict the harmonic response of the overall system, based on the formulation reported in our earlier article [29]. Source characteristics that have been previously investigated are also utilized here [12]. Chief objectives of this study include: (1) Examine alternate measures of vibration isolation performance for a multi-dimensional system and quantify the vibration transmission for several system configurations. (2) Calculate and measure the sound generated from the ‘L’ structure receiver and correlate results with vibration isolation measures. Key concepts will be illustrated via experimental and analytical studies on selected isolators.
3. Vibration isolation measures

3.1. Multi-dimensional formulation

Interfacial forces \( \mathbf{F} \) in Fig. 1 are as follows when an external force vector is applied at location 1. Note that \( \alpha_{ij} \), \( \beta_{ij} \) and \( \gamma_{ij} \) represent the mobility matrices of source, path and receiver, respectively; a detailed analysis has been reported in an earlier paper by the same authors [29]. Here, all formulations are in the frequency domain though the ubiquitous \( (\omega) \) term has been dropped for the sake of brevity:

\[
\mathbf{F}_{P2} = \left[ [\alpha_{22} + \beta_{22}] - \beta_{23}[\beta_{33} + \gamma_{33}]^{-1}\beta_{32} \right]^{-1}\alpha_{21}\mathbf{F}_{S1},
\]
Also, velocities (V) at intermediate locations when an external force is applied at 1 are as follows where M is the system mobility matrix:

\[V_1 = M_{11} F_{S1} = [\alpha_{11} + \alpha_{12} [\beta_{23} [\beta_{33} + \gamma_{33}]^{-1} \beta_{23} - [\beta_{22} + \beta_{22}]]^{-1} \alpha_{21} F_{S1}, \]

\[V_2 = M_{21} F_{S1} = [\alpha_{21} - \alpha_{22} [\alpha_{22} + \beta_{22}] - \beta_{23} [\beta_{33} + \gamma_{33}]^{-1} \beta_{23}]^{-1} \alpha_{21} F_{S1}, \]

\[V_3 = M_{31} F_{S1} = \gamma_{33} [\beta_{33} + \gamma_{33}] - \beta_{32} [\alpha_{22} + \beta_{22}]^{-1} \beta_{23}^{-1} \beta_{32} [\alpha_{22} + \beta_{22}]^{-1} \alpha_{21} F_{S1}. \]

### 3.2. Force and velocity transmissibility matrices

The multi-dimensional force transmissibility matrix (TR\(F\)) can be defined from Eq. (1b) and likewise the velocity transmissibility matrix (TR\(V\)) can be developed using Eqs. (2a) and (2c) as follows where/represents a quotient operation for matrices:

\[TR_F = F_{R3} / F_{S3} = [[\beta_{33} + \gamma_{33}] - \beta_{32} [\alpha_{22} + \beta_{22}]^{-1} \beta_{23}]^{-1} \beta_{32} [\alpha_{22} + \beta_{22}]^{-1} \alpha_{21}, \]

\[TR_V = V_{R3} / V_{S3} = M_{31} M_{11}^{-1}. \]
For uni-directional motions, the transmissibility ($TR$) is a non-dimensional scalar quantity, and therefore the diagonal terms of $TR$ are non-dimensional. But, the units of off-diagonal terms vary since each off-diagonal component represents a specific coupling between related forces and motions. Further, the modified force transmissibility matrix ($TR_f^b$) can be defined as follows by using Eq. (1b) for $F_{R3}$ and substituting the null matrix for $\beta$ and $\gamma$ into Eq. (1a) for $F_{p2}^b$; here the superscript $b$ represents the blocked force [2]:

$$TR_f^b = F_{R3}/F_{p2}^b = [(\beta_{33} + \gamma_{33}) - \beta_{32}[\alpha_{22} + \beta_{22}]^{-1}\beta_{23}]^{-1}\beta_{32}[\alpha_{22} + \beta_{22}]^{-1}\alpha_{22}. \quad (4)$$

Similarly, the modified velocity transmissibility matrix ($TR_f^v$) can also be introduced as follows by using Eq. (2c) and noticing $V_{s2} = \alpha_{21}F_{S1}$; here the superscript $f$ denotes free velocity at the source output [2]:

$$TR_f^v = V_{R3}/V_{s2}^f = \gamma_{33}[\beta_{33} + \gamma_{33}] - \beta_{32}[\alpha_{22} + \beta_{22}]^{-1}\beta_{23}]^{-1}\beta_{32}[\alpha_{22} + \beta_{22}]^{-1}. \quad (5)$$

Again, the units of off-diagonal terms in modified transmissibility matrices vary even though the diagonal terms are non-dimensional.

### 3.3. Force and velocity effectiveness matrices

Multi-dimensional representation of the effectiveness concept can be extended to isolators that are described by uncoupled spring elements [7]. To illustrate this measure, consider the interfacial force and response at location 3 for a system without any isolator. These can be obtained by synthesizing only the source and receiver ($\alpha$ and $\gamma$); details may be found in literature [29]:

$$F_{R3,\text{without}} = [\alpha_{22} + \gamma_{33}]^{-1}\alpha_{21}F_{S1}, \quad V_{R3,\text{without}} = \gamma_{33}[\alpha_{22} + \gamma_{33}]^{-1}\alpha_{21}F_{S1}. \quad (6a, b)$$

The force effectiveness matrix $\Xi_F$ is obtained by using Eqs. (1b) and (6a) for the with and without isolator cases, respectively:

$$\Xi_F = F_{3,\text{with}}/F_{3,\text{without}}$$

$$= [(\beta_{33} + \gamma_{33}) - \beta_{32}[\alpha_{22} + \beta_{22}]^{-1}\beta_{23}]^{-1}\beta_{32}[\alpha_{22} + \beta_{22}]^{-1}[\alpha_{22} + \gamma_{33}]. \quad (7)$$

Further, relating Eq. (1c) to Eq. (6b) leads to the velocity effectiveness matrix $\Xi_V$:

$$\Xi_V = V_{3,\text{with}}/V_{3,\text{without}}$$

$$= \gamma_{33}[\beta_{33} + \gamma_{33}] - \beta_{32}[\alpha_{22} + \beta_{22}]^{-1}\beta_{23}]^{-1}\beta_{32}[\alpha_{22} + \beta_{22}]^{-1}[\alpha_{22} + \gamma_{33}]^{-1}. \quad (8)$$

### 3.4. Vibration power

The time-averaged vibrational power input $\Pi_{IN}$ to a structure for a uni-directional motion is defined as follows, given a point harmonic excitation $F e^{i\omega t}$ and the resultant velocity $V e^{i\omega t}$ at the driving point where $F$ and $V$ are complex-valued amplitudes:

$$\Pi_{IN}(\omega) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} F(t)V(t) \, dt = \frac{1}{2}|F||V| \cos \phi$$

$$= \frac{1}{2} \Re[\tilde{F}(\omega)\tilde{V}(\omega)] = \frac{1}{2} \Re[\tilde{F}(\omega)\tilde{F}^*(\omega)]. \quad (9)$$
Here, Re and superscript * denote the real value and the conjugate of a complex quantity, respectively. Similarly, the harmonic power $\Pi$ at any location is as follows, given harmonic excitation $\mathbf{F}e^{i\omega t}$ and response velocity $\mathbf{V}e^{i\omega t}$ vectors where the superscript $T$ denotes the transpose of a vector:

$$\Pi(\omega) = \frac{1}{2} \text{Re}[\mathbf{F}^T(\omega)\mathbf{V}^*(\omega)] = \frac{1}{2} \text{Re}[\mathbf{V}^T(\omega)\mathbf{F}^*(\omega)].$$

Power transmitted by each path $(i)$ at a connection of interest is related to the mobility matrix $(M)$ of that component by

$$\Pi_i = \frac{1}{2} \text{Re}[\mathbf{F}_i \mathbf{V}_i^*] = \frac{1}{2} \sum_j \text{Re}[\mathbf{F}_i \mathbf{F}_j^* M^*_j],$$

where $\mathbf{F}_i$ and $\mathbf{F}_j$ are the $i$th and $j$th components of excitation at the junction and $M_j$ represents the $ij$ mobility component which relates the $i$th velocity and the $j$th force where $M$ is the driving point mobility matrix of subsequent structure. For the sake of illustration, consider a planer motion that is represented by $\mathbf{F} = [f_x f_y q_z]^T e^{i\omega t}$ and $\mathbf{V} = [v_x v_y w_z]^T e^{i\omega t}$. Total power $(\Pi_{\text{Total}})$ transmitted to the structure by force $(f_x, f_y)$ and moment $(q_z)$ components is

$$\Pi_{\text{Total}} = \frac{1}{2}f_x^2 \text{Re}[M_{xx}] + \frac{1}{2} f_x f_y \text{Re}[M_{xy}^*] + \frac{1}{2} f_x q_z \text{Re}[M_{xz}^*]$$

$$+ \frac{1}{2} f_y^2 \text{Re}[M_{yy}] + \frac{1}{2} f_y f_z \text{Re}[M_{yz}^*] + \frac{1}{2} f_y q_z \text{Re}[M_{zy}^*]$$

$$+ \frac{1}{2} q_z^2 \text{Re}[M_{zz}] + \frac{1}{2} q_z f_x \text{Re}[M_{zx}^*] + \frac{1}{2} q_z f_y \text{Re}[M_{zy}^*].$$

Therefore, the input $(\Pi_{\text{IN}})$ power at driving point $(k)$ and the transmitted $(\Pi_{\text{TR}})$ power at connection point $(l)$ are, respectively,

$$\Pi_{\text{IN}} = \frac{1}{2} \text{Re}[\mathbf{F}_{\text{IN}}^T \mathbf{V}_l^*] = \frac{1}{2} \text{Re}[\mathbf{F}_{\text{IN}}^T (\mathbf{M}_{kl} \mathbf{F}_{\text{IN}})^*],$$

$$\Pi_{\text{TR}} = \frac{1}{2} \text{Re}[\mathbf{F}_{\text{TR}}^T \mathbf{V}_l^*] = \frac{1}{2} \text{Re}[\mathbf{F}_{\text{TR}}^T (\mathbf{M}_{kl} \mathbf{F}_{\text{TR}})^*],$$

where $\mathbf{M}_{kl}$ and $\mathbf{M}_{kl}$ are the mobility matrices of the combined structure, and $\mathbf{M}_{kl}$ is the driving point mobility matrix of the subsequent structure only. Since several coupling mobility terms between forces and moments exist, each component of the transmitted power may be positive or negative. However, the power components that are related to the diagonal terms in the driving point mobility of a receiver are always positive since the real part of the diagonal terms in the driving point mobility and the mean-square transmitted force are always positive. Note that the power component for each degree of freedom is interrelated to other power components since the coupled terms in Eq. (12) are associated with two motion or excitation variables in different degrees of freedom. In our study, the sum of power components that are related to coupling mobility of a receiver, as given by Eq. (12), is defined as the coupling power and such coupling powers may be positive or negative.

### 3.5. Mean-square force and velocity $\Psi_W^2$

The mean-square force $\Psi_F^2$ and velocity $\Psi_V^2$ at the interface (location 3 in Fig. 1) between isolator and receiver for uni-directional motions are, respectively:

$$\Psi_F^2 = \langle F_3^2(t) \rangle_t = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} F_3^2(t) \, dt = \frac{1}{2} \text{Re}[\hat{F}_3 \hat{F}_3^*] = \frac{|F_3|^2}{2},$$

$$\Psi_V^2 = \langle V_3^2(t) \rangle_t = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_3^2(t) \, dt = \frac{1}{2} \text{Re}[\hat{V}_3 \hat{V}_3^*] = \frac{|V_3|^2}{2},$$
\[ \Psi^2_V = \left< V_3^2(t) \right>_t = \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} V_3^2(t) \, dt = \frac{1}{2} \text{Re} \left[ \tilde{V}_3 \tilde{V}_3^* \right] = \frac{|V_3|^2}{2}. \]  

Here, \( \left< \cdot \right>_t \) and \( |\cdot| \) represent time-averaged and absolute values respectively. The \( \Psi^2_F \) and \( \Psi^2_V \) terms are meaningful for uni-directional motions but their relative comparisons are inappropriate for multi-dimensional motions since the units of \( \Psi^2 \) terms are not compatible between translational and rotational directions. For instance, it is not suitable to compare the mean-square force (or velocity) variable, as a weighting factor. By adopting the quantity \( \Psi^2_W \) that can hold equivalent units for dissimilar variables is proposed via Eq. (15). By employing \( \Psi^2_W \), a rank ordering of transfer paths is feasible for multi-dimensional motions. In this study, we choose the driving point mobility \( M \) (or its reciprocal that is impedance \( Z \)), corresponding to the force (or velocity) variable, as a weighting factor. By adopting the \( M \) or \( Z \) of receiver at location 3, define the \( \Psi^2_{WF} \) and \( \Psi^2_{WV} \) for uni-directional motions as

\[ \Psi^2_{WF} = \frac{1}{2} \text{Re} \left[ \tilde{F}_3 \tilde{F}_3^* M_{33} \right] = \frac{|F_3|^2}{2} \text{Re} [M_{33}], \quad \Psi^2_{WV} = \frac{1}{2} \text{Re} \left[ \tilde{F}_3 \tilde{V}_3 Z_{33} \right] = \frac{|V_3|^2}{2} \text{Re} [Z_{33}]. \]  

The \( \Psi^2_{WF} \) and \( \Psi^2_{WV} \) have the units of power and are always positive since the real parts of driving point mobility and impedance are positive. Further, the \( \Psi^2_{WF} \) (or \( \Psi^2_{WV} \)) represents the power term consisting of force (or moment) and corresponding velocity. The total weighted mean-square force \( \Psi^2_{WF,Total} \) and velocity \( \Psi^2_{WV,Total} \) can be defined at multi-dimensional interfacial location (3) in Fig. 1 as

\[ \Psi^2_{WF,Total} = \frac{1}{2} \text{Re} \left[ \tilde{F}_3^T \text{diag}[M_{33}] \tilde{F}_3^* \right], \quad \Psi^2_{WV,Total} = \frac{1}{2} \text{Re} \left[ \tilde{V}_3^T \text{diag}[Z_{33}] \tilde{V}_3^* \right]. \]  

For a planer motion example that was presented in the previous section, the \( \Psi^2_{WF,Total} \) and \( \Psi^2_{WV,Total} \) are expanded as

\[ \Psi^2_{WF,Total} = \frac{1}{2} |f_x|^2 \text{Re} [M_{xx}] + \frac{1}{2} |f_y|^2 \text{Re} [M_{yy}] + \frac{1}{2} |q_z|^2 \text{Re} [M_{zz}], \]  

\[ \Psi^2_{WV,Total} = \frac{1}{2} |v_x|^2 \text{Re} [Z_{xx}] + \frac{1}{2} |v_y|^2 \text{Re} [Z_{yy}] + \frac{1}{2} |w_z|^2 \text{Re} [Z_{zz}]. \]  

Therefore, \( \Psi^2_{WF,Total} \) is the sum of power components that are related to the driving point mobilities of the receiver, and \( \Psi^2_{WV,Total} \) is the sum of power components that are related to the driving point impedances of the receiver. Also, the individual mean-square force and velocity terms are scalar quantities.

### 3.6. Structure-borne or airborne noise measures

For two adjacent infinite structures, the transmission efficiency \( \Gamma \) (a non-dimensional scalar quantity) has been defined as the ratio of transmitted power \( \Pi_{TR} \) to incident power \( \Pi_{IN} \) [17,18]. For a finite structure, \( \Gamma \) can be defined in the same fashion as

\[ \Gamma = \frac{\Pi_{TR}}{\Pi_{IN}}. \]  

The effectiveness \( \Xi_{II} \) of vibration power (a non-dimensional scalar quantity) can be defined as follows where “with” implies the net power transmitted to receiver with an isolator and “without”
Table 1
Summary of frequency-domain vibration isolation measures for multi-dimensional motions

<table>
<thead>
<tr>
<th>Measure</th>
<th>Definition as a function of $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force and motion transmissibilities [6,8]</td>
<td>$\text{TR}<em>F = \frac{F</em>{33}}{F_{31}}$ and $\text{TR}<em>V = \frac{V</em>{33}}{V_{31}}$</td>
</tr>
<tr>
<td>Modified force and motion transmissibilities</td>
<td>$\text{TR}<em>F^b = \frac{F</em>{33}^b}{F_{31}^b}$ and $\text{TR}<em>V^b = \frac{V</em>{33}^b}{V_{31}^b}$</td>
</tr>
<tr>
<td>Force and motion effectivenesses [7]</td>
<td>$\Xi_F = \frac{V_{3,\text{with}}}{V_{3,\text{without}}}$ and $\Xi_F = \frac{F_{3,\text{with}}}{F_{3,\text{without}}}$</td>
</tr>
<tr>
<td>Mean-square force and velocities at receiver input</td>
<td>$\Psi_F^2 = \langle F_i^2(t) \rangle_i = \frac{1}{2} \text{Re}(F_i F_i^\dagger) = \frac{</td>
</tr>
<tr>
<td>Weighted mean-square force and velocities at receiver input</td>
<td>$\Psi_{WF,\text{Total}}^2 = \frac{1}{2} \text{Re}(\hat{F}<em>{3}^\dagger \text{diag}[M</em>{33}] \hat{F}<em>{3})$ and $\Psi</em>{WV,\text{Total}}^2 = \frac{1}{2} \text{Re}(\hat{V}<em>{3}^\dagger \text{diag}[Z</em>{33}] \hat{V}_{3})$</td>
</tr>
<tr>
<td>Power transmitted to receiver input [9–16,19–21]</td>
<td>$H_{TR}(\omega) = \frac{1}{2} \text{Re}(\hat{F}^T \hat{V}^<em>) = \frac{1}{2} \text{Re}(\hat{V}^T \hat{F}^</em>)$</td>
</tr>
<tr>
<td>Efficiency of vibration power transmitted to receiver input [15,20,21]</td>
<td>$\Pi(\omega) = \frac{H_{TR}}{H_{IN}}$</td>
</tr>
<tr>
<td>Effectiveness of vibration power transmitted to receiver input [20,21]</td>
<td>$\Pi_{II}(\omega) = \frac{H_{TR,\text{with}}}{H_{TR,\text{without}}}$</td>
</tr>
<tr>
<td>Vibration or sound amplitudes at the receiver</td>
<td>$f(\omega), V(\omega)$ or $p(\omega)$</td>
</tr>
<tr>
<td>Insertion loss [1,2]</td>
<td>$IL_{Q_i} = 10 \log_{10} \left( \frac{\Psi_{Q_i,A}^2}{\Psi_{Q_i,B}^2} \right)$ dB</td>
</tr>
</tbody>
</table>

Note:
1. Subscripts 1, 2 and 3 are input, interfacial and output locations of components as shown in Fig. 1.
2. Operator ‘$/'$ is a quotient operation for vectors.
3. Superscripts $b$ and $f$ represent blocked and free boundary conditions at the source output, respectively.
4. Operators $\langle \rangle_i$ and diag represent time averaged quantity and diagonal matrix of the original matrix, respectively.
5. Subscripts ‘with’ and ‘without’ represent a system with an isolator and without any isolator, respectively. Refer to Figs. 1(b) and (c).
6. Subscripts $IN$ and $TR$ represent input at source and transmitted power at receiver input, respectively.
7. $Q_i = p_i(\omega), F_i(\omega)$ or $V_i(\omega)$ at $i$. Also, $A$ and $B$ represent two different physical systems.

refers to the case when the source is rigidly connected to the receiver:

$$\Xi_{II} = \frac{H_{TR,\text{with}}}{H_{TR,\text{without}}}.$$ (19)

Insertion losses ($IL$) for force and velocity at the receiver structure or sound pressure ($p$) in the acoustic medium are defined as follows where $\Psi^2$ is mean-square value of relevant variable $Q_i$:

$$IL_{Q_i} = 10 \log_{10} \left( \frac{\Psi_{Q_i,A}^2}{\Psi_{Q_i,B}^2} \right) \text{ dB}.$$ (20)

One could choose $Q_i$ to be $F_i(\omega)$, $V_i(\omega)$ or $p_i(\omega)$, etc. at location $i$. Further, $A$ and $B$ represent two different physical systems and therefore the $IL$ values are always relative. The aforementioned vibration isolation measures are summarized in Table 1 for multi-dimensional motions; some explanations are also provided along with citation to literature.
4. System with an inverted ‘L’ beam receiver

4.1. System configuration

Various vibration isolation measures are examined for an analytical isolation system with the inverted ‘L’ beam receiver of Fig. 2(a). Note that longitudinal motion of one beam is coupled with flexural motion of the other and thus both contribute to sound radiation. The four isolator attachment cases of Fig. 2(d) are analyzed up to 3 kHz. These cases provide several transmission configurations even though cases 3 and 4 are statically unstable and case 1 is physically meaningful only for three-dimensional motions in terms of its implementation. Nonetheless, vibration transmitted to the receiver is strongly affected by the mount location, and thus by the free velocity of source [12]. Harmonic moment excitations are applied at the mass center of the source for vibratory power-based and sound field measures. Force excitation cases are also examined for transmissibility and effectiveness of force or velocity.

The mobility matrix $M$ of a rigid body, between any two locations $i$ and $j$, can be determined from the inertia properties at mass center $G$ and geometric information. Details may be found in Ref. [29]. Using the small angle ($\theta$) approximation ($\sin \theta \approx \tan \theta \approx \theta$ and $\cos \theta \approx 1$), the resulting expressions are given by

$$
\begin{bmatrix}
V_i \\
V_j
\end{bmatrix} =
\begin{bmatrix}
M_{ii} & M_{ij} \\
M_{ji} & M_{jj}
\end{bmatrix}
\begin{bmatrix}
F_i \\
F_j
\end{bmatrix} =
\begin{bmatrix}
T_i^T M_{GG} T_j \\
T_j^T M_{GG} T_i
\end{bmatrix}
\begin{bmatrix}
F_i \\
F_j
\end{bmatrix},
$$

(21a)

$$
M_{GG} =
\begin{bmatrix}
M_{v,GG} & 0 \\
0 & M_{w,GG}
\end{bmatrix},
$$

(21b)

$$
M_{v,GG} = \text{diag}\left(\frac{1}{m_j \omega}, \frac{1}{m_j \omega}, \frac{1}{m_j \omega}\right),
$$

$$
M_{w,GG} = \frac{1}{J_0} \begin{bmatrix}
I_{m,xx} & -I_{m,xy} & -I_{m,xz} \\
-I_{m,xy} & I_{m,yy} & -I_{m,yz} \\
-I_{m,xz} & -I_{m,yz} & I_{m,zz}
\end{bmatrix}^{-1},
$$

(21c, d)

$$
T_i = \begin{bmatrix}
I & 0 \\
R_i & I
\end{bmatrix},
$$

$$
R_i = \begin{bmatrix}
0 & -h_{yi} & h_{zi} \\
h_{yi} & 0 & -h_{xi} \\
-h_{zi} & h_{xi} & 0
\end{bmatrix}.
$$

(21e, f)

Here, $m$ and $I$ are mass and inertia of a rigid body and $h$ represents a reference location within the rigid body with respect to $G$. The isolator is connected to the inverted ‘L’ beam at $0.75 \ell_H$, where $\ell_H$ is the length of the horizontal beam. This off-center location highlights the effect of coupling mobility of receiver. Note that such a coupling mobility term does not exist for a centrally driven beam (with symmetric boundaries) and for an infinite beam. The isolator of Fig. 2(c) is modelled using the Timoshenko beam theory to describe flexure along with the wave equation for longitudinal motion. Thus, the effects of shear deformation and rotary inertia are included. Mobilities of the Timoshenko beam have been analyzed for two types of solution in an earlier paper by the same authors [21]; the resulting formulations are used here. Material properties such as shear modulus ($G$), the mass density ($\rho$) and the Poisson ratio ($\nu$) of the isolator are listed in Table 2. The modulus of elasticity $E$ for the rubber material is found from the relation...
\[ E = 3G(1 + QT^2), \]  

where \( Q \) is a numerical constant and \( T \) is the isolator shape factor \([3]\). For a circular rubber cylinder, \( Q = 2 \) and \( T \) is equal to \( 2r/(4\ell) \), where radius \( r \) and length \( \ell \) values are listed in Table 2 \([3]\). Also, a frequency-invariant loss factor \( \eta \) of 0.1 is assumed to incorporate damping within the isolator. It is included in the calculation with the complex-valued modulus of elasticity as \( \tilde{E} = E(1 + j\eta) \). Also, a loss factor of 0.002 is used to represent a lightly damped structure and included in \( \tilde{E} \).

The dynamic behavior of the inverted ‘L’ beam is obtained by synthesizing the mobilities of each beam. Harmonic responses of the finite beam in flexure are as follows, where \( k_B \) is bending wave number and \( A_B, B_B, C_B \) and \( D_B \) are arbitrary constants:

\[ Y(x, t) = Y(x)e^{j\omega t} = \{A_B \sin[k_Bx] + B_B \cos[k_Bx] + C_B \sinh[k_Bx] + D_B \cosh[k_Bx]\}e^{j\omega t}. \tag{22} \]

For a horizontal beam (designated as \( H \)) with one end clamped and the other end free, the above is rewritten as follows, with reference to Fig. 3(a), when a harmonic force or moment excitation is
applied within the beam span:

$$Y_1(x, t) = Y_1(x)e^{iot} = A_1B[\sin(k_Bx) - \sinh(k_Bx) + B_1B[\cos(k_Bx) - \cosh(k_Bx)]e^{iot},$$

$$Y_2(x, t) = Y_2(x)e^{iot} = A_2B[\sin(k_B(\ell_H - x)) + \sinh(k_B(\ell_H - x))] + B_2B[\cos(k_B(\ell_H - x)) + \cosh(k_B(\ell_H - x))]e^{iot}.\]$$

Here, $Y_1$ and $Y_2$ are the steady state responses of the beam over spans $0 \leq x \leq x_F$ and $x_F < x \leq \ell_H$, respectively, where $x_F$ and $\ell_H$ are the excitation location and length of the horizontal beam, respectively. Unknowns $A_1, B_1, A_2B$ and $B_2B$ are obtained by applying boundary and excitation conditions. Using the wave equation, the longitudinal velocity of a beam with one end clamped and the other end free is as follows, where $k_B$ and $S$ are longitudinal wave number and section area, respectively [30]:

$$v_{x1}(x, t) = v_{x1}(x)e^{iot} = j\omega \frac{\sin(k_Lx) \cosh(k_L(\ell_H - x))}{SEk_L \cos(k_L\ell_H)} e^{iot}, \quad 0 \leq x \leq x_f,$$

$$v_{x2}(x, t) = v_{x2}(x)e^{iot} = j\omega \frac{\sin(k_Lx) \cosh(k_L(\ell_H - x))}{SEk_L \cos(k_L\ell_H)} e^{iot}, \quad x_f < x \leq \ell_H.$$

For a vertical beam of length $\ell_V$ (designated as $V$) that is excited at the end of beam, mobilities are also used from Ref. [30] and are not repeated here.

Next, driving point mobilities of inverted ‘L’ beam at interface with an isolator (location 3) are obtained using the following, where $\gamma_H$ and $\gamma_V$ are the mobilities of horizontal and vertical beams, respectively, as shown in Fig. 2(a), and the subscripts after comma represent related locations:

$$\gamma_{33} = \gamma_{H,33} - \gamma_{H,34}[\gamma_{H,44} + \gamma_{V,44}]\gamma_{H,43}.\]$$

Then, interfacial forces $F_{R3}$ at location 3 are obtained by using Eqs. (1b) and (25) and by employing the same $\alpha$ and $\beta$ for source and isolator, respectively.

### 4.2. Vibration isolation measures using TR or Ξ

Force and velocity transmissibilities (TR) are computed using the inverted ‘L’ beam receiver, with only one isolation configuration (case 2) of Fig. 2(d). Only non-dimensional diagonal terms in transmissibility matrices are analyzed. The modified transmissibilities ($TR_F$ and $TR_V$) are also compared with their corresponding transmissibilities ($TR_F$ and $TR_V$) since the $TR_F$ and $TR_V$ may provide a better or convenient isolation measure. For example, there are cases such as in internal combustion engines where direct force inputs are difficult to measure, and the $TR_F$ that employs the blocked force may be considered as an alternative. Further, the effects of free velocity on vibration transmission to receiver have been emphasized for a multi-dimensional isolation system [12] and thus the $TR_V$ that employs the free velocity may also be regarded as an important isolation measure for multi-dimensional cases. In such cases, the relationships between the modified and original transmissibilities need to be examined, and the comparison of such transmissibility measures could yield a useful information. First, $TR_F$ and $TR_V$ spectra are shown in Fig. 4. The axial and rotational components of $TR_F$ are almost the same as the corresponding components of $TR_V$ over the entire frequency range. The lateral components of $TR_F$ and $TR_V$
also have almost the same spectral shapes but the lateral $TR_F^b$ is higher than the one of $TR_F$. Therefore, $TR_F^b$ may be regarded as an alternative to $TR_F$ for some cases (axial and rotational components in this case) although one must exercise caution when replacing $TR_F^b$ by $TR_F$.

Next, the velocity $TR_V$ and modified velocity $TR_V^f$ transmissibilities are shown in Fig. 5. The differences between the $TR_V$ and $TR_V^f$ components are more pronounced than the ones between $TR_F$ and $TR_F^b$. This is because the axial components of $TR_V$ and $TR_V^f$ dominate over the entire frequency range and the lateral components of $TR_V$ and $TR_V^f$ are higher than the rotational component of those. Also, the difference between lateral and rotational components is reduced in the $TR_V^f$ calculations.

Finally, the $\Xi_F$ and $\Xi_V$ effectiveness spectra are shown in Fig. 6. Unlike the $TR_V$ and $TR_V^f$, the lateral and rotational components of $\Xi_V$ are almost the same and the lateral component of $\Xi_F$ is slightly larger than the rotational component of $\Xi_F$ up to around 400 Hz, as shown in Fig. 6. Therefore, the difference between $TR_V$ and $\Xi_V$ confirms that the $TR_V$ does not properly represent the isolation effectiveness for the different motional directions, as known previously. However, similar to $TR_V$ and $TR_V^f$, it is observed that the axial component of $\Xi_F$ or $\Xi_V$ dominates beyond 100 Hz.

4.3. Power-based vibration isolation measures

The four different cases of the isolator connection, as shown in Fig. 2(d), are investigated using the same component parameters. Vibration power transmitted to the receiver is computed and
analyzed up to 3 kHz. As mentioned earlier, each power component cannot be decoupled from others because of coupling and therefore may be positive or negative. Vibration power components \( P \) that are normalized with respect to the total power \( P_{\text{Total}} \) transmitted to the inverted ‘L’ beam are shown in Fig. 7. Therefore, the normalized power components shown in this manner add up to unity. However, it is seen from Fig. 7 that the normalized \( P \)s exceed far from unity because of coupling and exhibit somewhat complicated characteristics of the power transmission process. Note that for the inverted ‘L’ beam receiver, the longitudinal motion of beam \( H \) is coupled with the rotational motion of beam \( V \), ultimately with bending motion of the horizontal beam. Therefore, all coupling terms in the driving point mobility of the ‘L’ beam receiver are non-zero and coupling powers exist between lateral (with respect to isolator) and other directions. However, the coupling does not occur between longitudinal and other (bending) directions of a single straight beam receiver. Fig. 7 shows that the rotational \( P_R \) is dominant and large negative power flows in the axial \( y \) (with respect to isolator) direction are observed for cases 1 and 2. This implies that there exists \( P \) flow from a receiver to an isolator in the axial direction. Further, the lateral \( P_L \) of case 2 grows at higher frequencies. For cases 3 and 4, the axial \( P_A \) is dominant except certain frequencies (around 1 kHz), as shown in Figs. 7(c) and (d). At these frequencies in cases 3 and 4, the \( P_R \) dominates and the \( P_A \) flows in reverse manner (from a receiver to an isolator). Overall, Fig. 7 shows that the \( P_A \) and \( P_R \) spectra are almost symmetric with respect to \( P/P_{\text{Total}} = 0.5 \). This indicates that significant coupling occurs between the \( P_A \) (shear direction of horizontal beam receiver) and \( P_R \) components. Although parts of these
components ultimately cancel each other, all of these orthogonal components may be separately considered in the analysis. For example, consider a system consisting of multiple uni-directional paths. Like the case considered here, reverse (negative) \( P \) flows are expected to occur within certain paths and frequencies since the transfer mobility can be positive or negative. Despite the cancellation of coupling powers within some paths, negative or positive power flow through each uni-directional path must be separately understood to properly design a vibration isolation system for minimum structure-borne noise. Likewise, each path of a multi-dimensional isolator may be analyzed independently. However, it may be useful to group and compare powers for the sake of interpretation. For example, shear and rotational motions are associated with each other by the relationship \( w = \frac{dv}{dx} \) and \( f = -\frac{dq}{dx} \) for a beam structure where \( v, w, f \) and \( q \) represent translational and rotational velocities, force and moment, respectively. Therefore, in our study, the bending \( \Pi_B \) is defined as the sum of the \( \Pi_A \) and \( \Pi_R \) components and it is compared with the \( \Pi_f \) in Fig. 8. The \( \Pi_B \) represents the powers in flexural motion of beam \( H \). However, it should be noted that, in general, this description of flexural motion is not appropriate since the receiver dynamics is governed not only by a directly connected structure but also by subsequent structures. Fig. 8(a) shows that the \( \Pi_B \) is dominant except around 300 Hz and the \( \Pi_f \) flows upward to an isolator at higher \( \omega \) for case 1. Further, the \( \Pi_B \) and \( \Pi_f \) dominate at lower and higher frequencies respectively for case 2, and the \( \Pi_B \) dominates at all frequencies for cases 3 and 4, as shown in Fig. 8(b). The negative \( \Pi_f \) values of Fig. 8 indicate the existence of the coupling between
longitudinal and bending motions of the ‘L’ beam receiver. As mentioned previously, the $\Pi_c$ cannot be negative for a system with a single straight beam receiver. Overall, it can be seen from Fig. 8 that strong coupling still exists between bending and lateral directions due to interactions between receiver structures. In this study, the vibration transmission paths could be identified in a more clear manner by grouping the power components in flexural motions. But, caution must be exercised since the longitudinal motion of a receiver may exhibit strong coupling with its bending motions unlike the ‘L’ beam case examined here. For example, the bending waves through an isolator are typically converted to the longitudinal wave of a receiver, and grouping of flexural motions of an isolator rather than the ones of a receiver may be more suitable in some application. Future research should attempt to properly interpret the vibration transmission process and related couplings, and an efficient analysis scheme must be sought via a better understanding of the coupling process.

Fig. 7. Normalized vibration power component with respect to the total power transmitted to an inverted ‘L’ beam receiver given moment excitation: (a) case 1; (b) case 2; (c) case 3; (d) case 4 as shown in Fig. 2(d); (e) four mounting location cases as shown in Fig. 2(d). ——, axial; ———, rotational; ———, lateral.
Next, the weighted mean square force \( \Psi_{WF}^2 \) and coupling power that are normalized with respect to \( \Pi_{Total} \) are calculated and shown in Fig. 9. Recall from Section 3.4, that the coupling power is designated as the sum of the power terms consisting of a force (or moment) and velocity that is generated by the forces (or moments) in different directions. Further, the sum of the \( \Psi_{WF,Total}^2 \) (or \( \Psi_{WV,Total}^2 \)) and the coupling power is equal to the total power \( \Pi_{Total} \) transmitted to the receiver. Fig. 9 shows that the rotational and axial \( \Psi_{WF}^2 \) are dominant for cases 1 and 2 and the axial \( \Psi_{WF}^2 \) dominates for cases 3 and 4. Further, the lateral \( \Psi_{WF}^2 \) becomes important as frequency increases for case 2, like the \( \Pi \) spectra in Fig. 8(b). The \( \Psi_{WF}^2 \) of Fig. 9 also exhibits strong power coupling phenomena. However, \( \Psi_{WF}^2 \) does not provide any information on power flowing in the reverse manner as seen in the \( \Pi \) spectra since \( \Psi_{WF}^2 \) is always positive. This can be seen from the coupling power. The weighted mean-square velocity \( \Psi_{WV}^2 \) is also shown in Fig. 10. The \( \Psi_{WV}^2 \) spectra are quite different from the \( \Psi_{WF}^2 \) for all cases. Note that \( \Psi_{WF}^2 \) and \( \Psi_{WV}^2 \) are the same when a receiver is defined only in terms of a diagonal driving point mobility matrix such as

Fig. 8. Normalized vibration power component with respect to the total power transmitted to an inverted ‘L’ beam receiver given moment excitation: (a) case 1; (b) case 2; (c) case 3; (d) case 4 as shown in Fig. 2(d); (e) four mounting location cases as shown in Fig. 2(d). ——, bending; ————, lateral.
an infinite beam or a finite beam (with symmetric boundaries) with the isolator connected at the center. Like the $\Psi^2_{WF}$, Fig. 10 shows that the rotational and axial $\Psi^2_{WF}$’s are significant for all cases. However, the lateral $\Psi^2_{WF}$ becomes important over a relatively limited frequency range. Further, stronger coupling terms are observed in Fig. 10 than in Fig. 9.

Total vibration powers $\Pi_{Total}$ transmitted to the ‘L’ beam receiver are compared in Fig. 11(a) for the 4 location cases of Fig. 2(d) given unit (1 N m) moment excitation. It is observed that total vibration powers for cases 3 and 4 are almost the same but these are higher than those for cases 1 and 2. Also, the transmitted power is lowest among the cases considered when the isolator is attached to the mass center of the rigid body (case 1). The power efficiencies $\Gamma$ are shown in Fig. 11(b), and a rank order based on $\Gamma$ is similar to the one given by $\Pi_{Total}$. However, $\Gamma$ in case 1 rises and is higher than the one for case 2 as $\omega$ increases. Also, for all cases, overall characteristics

Fig. 9. Weighted mean square force ($\Psi^2_{WF}$) and coupling power normalized with respect to the total power transmitted to an inverted ‘L’ beam receiver given moment excitation: (a) case 1; (b) case 2; (c) case 3; (d) case 4 as shown in Fig. 2(d); (e) four mounting location cases as shown in Fig. 2(d). ——, rotational; —-–-–-, lateral; ???, coupling power.
of \( \Gamma \) rise as \( \omega \) increases even though the value of \( \Pi_{Total} \) decreases. The power effectivenesses \( \Xi_{II} \) are also shown in Fig. 11(c). The patterns for \( \Xi_{II} \) spectra do not exactly match with \( \Pi_{Total} \) for the cases considered. For example, The \( \Xi_{II} \) for case 3 is higher than the one of case 4 at low frequencies but is lower at high \( \omega \). Overall, one must properly select a power-based measure, given the choice between force (or velocity) transmissibility and effectiveness terms. Further, the differences between \( \Pi_{Total} \) and \( \Gamma \) are expected since the source input powers of each case are different although the excitation moment amplitude remains unchanged. Therefore, the source characteristics also need to be identified to examine various design modifications. For example, \( \Gamma \) is suitable when a source generating mechanism is clearly identified. Without a proper understanding of such a source, \( \Pi_{Total} \) must be evaluated for each design change.
4.4. Sound radiation from receiver as a measure of vibration isolation

Sound pressure \( p \) at selected points in free field \((\dd)\) is calculated in order to examine its relationship with vibration power transmitted to the receiver of Fig. 2(a). An inverted ‘L’ beam is chosen as the chief radiating structure and it will incorporate contributions from longitudinal and flexural structural powers. See Fig. 3 for more details of the system configuration. First, interfacial forces \( F_{R4} \) between horizontal \((H)\) and vertical \((V)\) beams are obtained by using \( F_{R3} \)

Fig. 11. Vibration isolation measures with an inverted ‘L’ beam receiver given moment excitation: (a) total transmitted vibration power \( P_{Total} \); (b) efficiency \( \Gamma \) for total transmitted vibration power; (c) effectiveness \( \Xi_H \) for total transmitted vibration power; (d) four mounting location cases as shown in Fig. 2(d). ——, mount location case 1; - - - - - - , case 2; - - - - - - - - , case 3; - - - - - - - - - - - - , case 4.

4.4. Sound radiation from receiver as a measure of vibration isolation

Sound pressure \( p \) at selected points in free field \((\dd)\) is calculated in order to examine its relationship with vibration power transmitted to the receiver of Fig. 2(a). An inverted ‘L’ beam is chosen as the chief radiating structure and it will incorporate contributions from longitudinal and flexural structural powers. See Fig. 3 for more details of the system configuration. First, interfacial forces \( F_{R4} \) between horizontal \((H)\) and vertical \((V)\) beams are obtained by using \( F_{R3} \)
as follows:

$$F_{R4} = [\gamma_{H,44} + \gamma_{V,44}]^{-1}\gamma_{H,43}F_{R3}.$$  \hspace{1cm} (26)

Further, velocity distributions along $x$ and $y$ of beams $H$ and $V$ can be represented as follows:

$$V_H(x) = \gamma_{H,x3}(x)F_{R3} + \gamma_{H,x4}(x)F_{R4},$$  \hspace{1cm} (27a)

$$V_V(y) = \gamma_{V,y4}(y)F_{R4}.$$  \hspace{1cm} (27b)

Assume distributed line source on the beams, and the sound pressure at $\tilde{d}$ from a baffled harmonic source is [31,32]

$$p = \int_{0}^{L} \left[ \frac{j \rho_a c_a k_a}{2\pi d'(x)} v(x) e^{i(\omega t - k_a d'(x))} \right] dx.$$  \hspace{1cm} (28)

Here, $d'$ is a distance between the vibrating point on the beam and the observation point $\tilde{d}$, $b$ is the beam width and $\rho_a$, $c_a$ and $k_a$ are the density, wave speed and wave number of the air medium, respectively. Assuming a far field, such that $d' \gg \ell$, where $d'$ is the reference location at the intersection of two beams in this case, $d'_H$ and $d'_V$ can be approximated as

$$d'_H(x) \approx d + (\ell_H - x) \sin \theta, \quad d'_V(y) \approx d + (\ell_V - y) \cos \theta.$$  \hspace{1cm} (29a, b)

Further, $d'_H$ and $d'_V$ in the denominator of the integrand can be replaced by its approximate value $d$ but not for the exponent in order to maintain the phasing relations [31]. Here, the sound field in the second quarter of Fig. 3(c) is described by a superposition of two independent hemi-spherical spaces corresponding to the horizontal and vertical baffled beam radiators, as shown in Fig. 3. Note that the Green’s function for a $\frac{3}{4}$ free space is needed to describe the exact sound field and hence there may exist a discrepancy between the realistic sound field and the one described here especially at lower frequencies. Consequently, this study focuses on relative measures of two different systems, such as the insertion loss, and accordingly it is assumed that such discrepancies are negligible especially in the sound field located 45° from the corner of the beams. Yet, a more general case that employs an inverted ‘L’ plate and describes a three-dimensional sound field is investigated via computational and experimental studies in Section 5. Therefore, sound pressure contributions from beams $H$ and $V$ are

$$p = p_H + p_V,$$  \hspace{1cm} (30a)

$$p_H = \frac{j \rho_a c_a k_a \ell_{p,H}}{4\pi d} e^{i(\omega t - k_a d - \ell_H)} \int_{0}^{\ell_H} v_{H,y}(x) e^{i k_a x} \sin \theta \ dx,$$  \hspace{1cm} (30b)

$$p_V = \frac{j \rho_a c_a k_a \ell_{p,V}}{4\pi d} e^{i(\omega t - k_a d - \ell_V)} \int_{0}^{\ell_V} v_{V,x}(y) e^{i k_a y} \cos \theta \ dy.$$  \hspace{1cm} (30c)

Here, $v_{H,y}$ and $v_{V,x}$ are the flexural velocities of beams $H$ and $V$, respectively. Sound pressure from beam $H$ is determined as follows where $G$ is a row vector:

$$p_H = \frac{j \rho_a c_a k_a \ell_{p,H}}{4\pi d} e^{i(\omega t - k_a d - \ell_H)} [G_{H1,3}F_3 + G_{H2,3}F_3 + G_{H1,4}F_4],$$  \hspace{1cm} (31a)
\[ G_{H1,3} = [A_{H1,\ell_1}(x_3) \quad A_{H1,\ell_1}(x_3) \quad A_{H1,\ell_1}(x_3)]g_{H1,A} \]
\[ + [B_{H1,\ell_1}(x_3) \quad B_{H1,\ell_1}(x_3) \quad B_{H1,\ell_1}(x_3)]g_{H1,B}, \tag{31b} \]
\[ G_{H2,3} = [A_{H2,\ell_1}(x_3) \quad A_{H2,\ell_1}(x_3) \quad A_{H2,\ell_1}(x_3)]g_{H2,A} \]
\[ + [B_{H2,\ell_1}(x_3) \quad B_{H2,\ell_1}(x_3) \quad B_{H2,\ell_1}(x_3)]g_{H2,B}, \tag{31c} \]
\[ G_{H1,4} = [A_{H1,\ell_1}(\ell_H) \quad A_{H1,\ell_1}(\ell_H) \quad A_{H1,\ell_1}(\ell_H)]g_{H1,A} \]
\[ + [B_{H1,\ell_1}(\ell_H) \quad B_{H1,\ell_1}(\ell_H) \quad B_{H1,\ell_1}(\ell_H)]g_{H1,B}, \tag{31d} \]
\[ g_{H1,A} = \int_0^{x_3} [\sin(k_H x) - \sinh(k_H x)]e^{jk_0 x \sin \theta} \, dx, \tag{31e} \]
\[ g_{H1,B} = \int_0^{x_3} [\cos(k_H x) - \cosh(k_H x)]e^{jk_0 x \sin \theta} \, dx, \tag{31f} \]
\[ g_{H2,A} = \int_{x_3}^{\ell_H} [\sin[k_H(\ell_H - x)] + \sinh[k_H(\ell_H - x)]]e^{jk_0 x \sin \theta} \, dx, \tag{31g} \]
\[ g_{H2,B} = \int_{x_3}^{\ell_H} [\cos[k_H(\ell_H - x)] + \cosh[k_H(\ell_H - x)]]e^{jk_0 x \sin \theta} \, dx. \tag{31h} \]

Similarly, sound pressure from beam \( V \) is
\[ p_\ell = \frac{j\rho_a c^2 k_0^2 p_\ell}{4\pi d} e^{[\pm i(d - \ell)]} [G_{V,\ell}F_4], \tag{32a} \]
\[ G_{V,\ell} = [A_{V,\ell_1}(\ell_1) \quad A_{V,\ell_1}(\ell_1) \quad A_{V,\ell_1}(\ell_1)]g_{V,A} \]
\[ + [B_{V,\ell_1}(\ell_1) \quad B_{V,\ell_1}(\ell_1) \quad B_{V,\ell_1}(\ell_1)]g_{V,B}, \tag{32b} \]
\[ g_{V,A} = \int_0^{\ell_1} [\sin(k_V x) - \sinh(k_V x)]e^{jk_0 x \cos \theta} \, dy, \tag{32c} \]
\[ g_{V,B} = \int_0^{\ell_1} [\cos(k_V x) - \cosh(k_V x)]e^{jk_0 x \cos \theta} \, dy. \tag{32d} \]

Calculated mean-square sound pressures (\( \Phi_p^2 \)) are shown in Fig. 12(a) for four cases. The horizontal beam is connected to the isolator at \( 3\ell_H/4 \) from its clamped end. The field point is located at \( d = 1 \) m from the intersection of two beams, at \( 45^\circ \) from the outer surfaces of each beam, as shown in Fig. 3(b). The rank order associated with four locations and related sound pressure spectral shapes of Fig. 12(a) match the transmitted vibration power spectra of Fig. 11(a) although discrepancies are observed at some frequencies. Sound pressures at other field observation points, \( d = 3 \) m with \( 45^\circ \) and \( d = 1 \) m with \( 85^\circ \), are also shown in Figs. 12(b) and (c). It is observed from Fig. 12 that the rank orders (corresponding to four isolator locations) at different sound field observation points do not change, except at some frequencies although their spectral shapes differ especially beyond 200 Hz. Fig. 13 compares insertion losses based on mean-squared sound pressures (\( IL_{\psi_p^2,1-j} \)) at a field point \( (d = d_1, \theta = \theta_1) \) with the ones of vibration power.
input ($IL_{II,i-j}$) to the ‘L’ beam for the four isolator cases of Fig. 2(d) given the moment excitation. Here, subscripts $i$ and $j$ denote mount cases as shown in Fig. 2(d). Spectral averages of the insertion losses ($IL_{\Psi_p^2,SAvg}$ and $IL_{II,SAvg}$) are also compared in Fig. 13. Fig. 13 shows that the $IL_{\Psi_p^2}$ spectra closely represent the $IL_{P}$ curves although some discrepancies between $IL_{\Psi_p^2,i-4}$ and $IL_{II,4-1}$ (cases 4 and 1) are observed. This suggests that $IL_{\Psi_p^2,i-j}$ may not be equal to $IL_{II,i-j}$ when systems $i$ and $j$ transmit different vibration components. Note that the rotational and axial components dominate the vibration transmission for cases 1 and 4, respectively. Therefore, in such cases, correlations between $IL_{\Psi_p^2}$ and $IL_{II}$ could be enhanced by averaging sound pressures over the entire field.
5. Experimental system with an inverted ‘L’ plate receiver

5.1. System configuration

Similar to the inverted ‘L’ beam receiver, an inverted ‘L’ plate receiver, as shown in Fig. 14, is employed to describe both in-plane and out-of-plane motion transmissions to the receiver. Overall, 4 isolators of Figs. 14(c) and (d) are experimentally studied. The material properties and dimensions of the source, isolator and receiver are summarized in Table 3. Each isolator is located at either the center or edge of mass to realize different vibration isolation configurations. However, the mount location on the receiver side is unchanged. The field point \((\xi_1 = \bar{d})\) for sound pressure measurements \(p(\omega)\) is located at a distance of 0.2 m from the mating edge of two plates, at 45° from the outer surfaces. Sound pressures at other field locations \((\xi)\) and structural velocities at selected points \((\zeta)\) on the receiver plates are also examined, as summarized in Table 4. Fig. 15 shows the modified experimental schematic used for moment driven responses where a phase shifter is used. Table 5 lists the instruments used. Measurement are conducted in an anechoic

![Graphical representation](image-url)
room under the sine sweep excitation (up to 3 kHz). In-phase and 180° out-of-phase forces (in y direction) are separately applied to the edges of the rigid source to simulate the force \( f_y \) and moment \( q_z \) excitations at \( G \), respectively. Forces from two shakers and accelerations at the driving point locations are measured using two impedance heads. Plastic and steel stingers are

---

**Table 3**

<table>
<thead>
<tr>
<th>Property or dimension</th>
<th>( m ) (kg)</th>
<th>( E ) (MPa)</th>
<th>( G ) (MPa)</th>
<th>( \eta )</th>
<th>( \rho ) (kg/m³)</th>
<th>Dimensions (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source (rectangular rigid body)</td>
<td>1.2</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>( \ell = [x, y, z] = [140, 64, 47] )</td>
</tr>
<tr>
<td>Rubber isolator I</td>
<td>—</td>
<td>2.8</td>
<td>0.8</td>
<td>0.2</td>
<td>1000</td>
<td>( \ell_{\text{effective}} = 35 ) ( r_{\text{effective}} = 12 )</td>
</tr>
<tr>
<td>Aluminum isolator I</td>
<td>—</td>
<td>6.688 ( \times 10^4 )</td>
<td>2.4 ( \times 10^4 )</td>
<td>0.001</td>
<td>2723</td>
<td>( \ell_{\text{effective}} = 35 ) ( r_{\text{effective}} = 12 )</td>
</tr>
<tr>
<td>Polypropylene isolator II</td>
<td>—</td>
<td>5</td>
<td>1.6</td>
<td>0.2</td>
<td>1000</td>
<td>( \ell = 27 ) ( r = 7 )</td>
</tr>
<tr>
<td>Aluminum isolator II</td>
<td>—</td>
<td>6.688 ( \times 10^4 )</td>
<td>2.4 ( \times 10^4 )</td>
<td>0.001</td>
<td>2723</td>
<td>( \ell = 27 ) ( r = 7 )</td>
</tr>
<tr>
<td>Inverted ‘L’ plate receiver</td>
<td>—</td>
<td>19.5 ( \times 10^4 )</td>
<td>8.3 ( \times 10^4 )</td>
<td>0.001</td>
<td>7700</td>
<td>( \ell = 400 ) ( t = 1 ) (square horizontal and vertical plates)</td>
</tr>
</tbody>
</table>

Note that rubber and polypropylene properties are approximate.
Table 4
Locations of vibration transmission measures

<table>
<thead>
<tr>
<th>Location no.</th>
<th>Measure</th>
<th>Co-ordinates (m)</th>
<th>‘L’ structure location</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_1 )</td>
<td>Sound pressure</td>
<td>( x = 0.14, y = 0.14, z = 0 )</td>
<td>Acoustic free field</td>
</tr>
<tr>
<td>( \xi_2 )</td>
<td>Sound pressure</td>
<td>( x = -0.18, y = 0.46, z = 0 )</td>
<td>Acoustic free field</td>
</tr>
<tr>
<td>( \xi_3 )</td>
<td>Velocity</td>
<td>( x = -0.15, y = 0, z = 0 )</td>
<td>Horizontal plate</td>
</tr>
<tr>
<td>( \xi_4 )</td>
<td>Velocity</td>
<td>( x = 0, y = -0.15, z = 0 )</td>
<td>Vertical plate</td>
</tr>
</tbody>
</table>

Table 5
List of instruments used for experimental studies

<table>
<thead>
<tr>
<th>Item</th>
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</tr>
<tr>
<td>Microphones</td>
<td>PCB</td>
<td>L130C10</td>
</tr>
<tr>
<td>Impedance heads</td>
<td>PCB</td>
<td>288D01</td>
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<td>Shakers</td>
<td>Labworks</td>
<td>ET-132-2</td>
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<td>Phase shifter</td>
<td>AVC Instrumentation</td>
<td>780M01</td>
</tr>
<tr>
<td>Power amplifiers</td>
<td>Electro-Voice</td>
<td>7300A</td>
</tr>
<tr>
<td>Dynamic signal analyzer (8 channel)</td>
<td>HP</td>
<td>HP3566A</td>
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</table>

Fig. 15. Experimental schematic for the source characteristics study of a system with an inverted ‘L’ plate receiver and sound radiated from receiver plate.
used over low (up to 1 kHz) and high (1–3 kHz) frequency regimes, respectively, since the dynamic forces could not excite the system above 1 kHz with plastic stingers. Typical force signals are shown in Fig. 16. The input forces with almost the same magnitudes and 180° (or 0°) phase difference are maintained throughout the experiments for moment (or force) excitation cases as shown in Fig. 17. However, the phase between two forces deviates slightly from 180° (or 0°) at a system resonance (approximately 600 Hz). This resonance appears to be a result of the experimental system dynamics with two shakers. Forces measured at the driving point locations are used for computational predictions.

The mobilities of the inverted ‘L’ plate structure are obtained by using a commercial finite element (FEA) IDEAS [33] code. Further, interfacial forces and moments between the isolator and receiver are calculated by synthesizing the mobilities of the inverted ‘L’ plate, source and isolator. Then, the plate velocity distribution from the FEA calculation is provided to a commercial boundary element method (BEM) SYSNOISE [34] code to predict the sound radiation. Individual sound fields generated by each plate for interfacial forces and moments are superimposed to determine the resultant sound pressure. Note that direct radiation from either source or isolator is not included in such calculations. Overall, sound pressure and velocity amplitudes at locations of Table 4 are obtained using the FEA and BEM methods. Vibration power ($P_{TR}$) transmitted to the ‘L’ plate and the power ($P_{RAD}$) radiated to the acoustic medium from the receiver, as defined below, are also predicted:

$$P_{RAD} = \frac{1}{2} \int_{A} p v_a^* \, dS.$$  \hspace{1cm} (33)

Here, $p$ and $v_a$ are sound pressure and the particle velocity amplitudes, respectively, at a control surface $S$ [17,32].

5.2. Effect of isolator material

The effect of isolator material properties is quantified in terms of insertion losses ($IL$). Sound pressure ($p$), vibration velocity ($v$) and acoustic power ($P_{RAD}$) are calculated, where,
in dB,
\[ IL_{P_{RAD}} = 10 \log_{10} \left( \frac{\psi_{p,A}^2}{\psi_{p,B}^2} \right), \quad IL_{v_f} = 10 \log_{10} \left( \frac{\psi_{v_f,A}^2}{\psi_{v_f,B}^2} \right), \quad (34a, b) \]
\[ IL_{P_{RAD}} = 10 \log_{10} \left( \frac{\Pi_{RAD,A}}{\Pi_{RAD,B}} \right). \quad (34c) \]
Here, $p_i$ and $v_j$ are sound pressure at acoustic field point $i$ and velocity at receiver structure location $j$, respectively. Further, ‘$A$’ and ‘$B$’ represent the system with an aluminum and rubber (or polypropylene) isolator, respectively. The $IL_p$ and $IL_v$ spectra are obtained from both experimental and computational studies but only the computed results are used for $IL_{\Pi,rad}$. The results are given at the center frequencies of the 1/3 octave band. Each isolator of Figs. 14(c) and (d) is separately examined and the mount location is unchanged for all cases. Experimental and computational results at the response locations ($\xi$) of Table 4 are shown in Figs. 17 and 18 for isolators I and II, respectively, when force ($y$) is applied at the mass center of a rigid body source. Further, the measured force inputs are used for computational studies. Resulting vibration and acoustic measures cannot be normalized with respect to their excitation forces since two different input forces are used. However, it is observed that measured input forces from the shaker stingers to a mass source do not vary much given different system configurations. Figs. 17 and 18 show that vibration and noise transmissions are much reduced over a wide range of frequencies when a rubber isolator with a lower $G$ is used in place of an aluminum isolator. Further, $IL_{\Pi,rad}$ spectra for sound power radiated from the ‘L’ plate receiver match well with the $IL_p$ spectra for sound pressure, especially at location $\xi_1$. However, the experiment results of $IL_p$ do not exhibit as much reduction as the ones computed beyond 500 Hz. One of the reasons is that the actual sound radiated from the receiver is lower than shaker noise beyond 500 Hz, especially when a rubber isolator is located at the center of source. See Figs. 17(b) and 18(b) where the background noise from shakers is also shown with mean-square sound pressure ($\Psi^2_p$). Note that the measured $\Psi^2_p$ of the system shows almost the same level as the $\Psi^2_p$ of shakers as shown in Figs. 17(b) and 18(b). Further, note that $IL_v$ spectra, that are not contaminated by shaker noise, are much higher than $IL_p$ beyond 500 Hz as shown in Figs. 17(c), (d) and 18(c), (d). Observe that the $IL_v$ from experiments reasonably match with predicted $IL_v$ as shown in Figs. 17(e), (f) and 18(e), (f). However, relatively large discrepancies between measured and predicted insertion losses are observed for isolator II of Fig. 18 at some frequencies, say 250–800 Hz. One reason for this may be the interfacial conditions of the isolators. For example, the polypropylene isolator (II) is connected through a plastic thread with the receiver plate and source mass, unlike the other three isolators. Also, refer to Fig. 14 for isolator shapes and material, and note that the rubber isolator I has aluminum interfacial plates at both ends of the isolator, unlike isolator II. Therefore, errors in the measurements of insertion loss, which compares the polypropylene and aluminum isolators (II), may be pronounced for isolator II. Yet, the elastomeric non-linearities could introduce other errors. For example, the polypropylene isolator (II) could have preload and amplitude dependencies and could exhibit spectrally varying material properties, unlike metallic mounts. Note that we have assumed spectrally invariant properties for the four isolator examples. Spectral averages of measured and computed results are shown in Table 6 for insertion losses. Overall, reasonable agreements between computed and experimental results are observed even though some measurements are contaminated by the shaker noise. Further, the root cause of the relatively larger discrepancies for some isolators (for example, the polypropylene isolator case), which may be due to interfacial conditions and/or elastomeric non-linearities, needs to be further studied.

### 5.3. Effect of isolator location

Next, the effects of mount location are examined using the experimental system of Fig. 14. Only the rotational free velocity of the source should exist for the moment excitation case when a
mount is located at the center of the mass source. And, both translational and rotational free velocities occur when the isolator is placed at the edge of the rigid body source [12]. Similar to the previous cases, insertion losses ($IL$) are calculated by using Eqs. (34a)–(34c). In this case, subscript ‘$A$’ and ‘$B$’ refer to the cases when the isolator is placed at the edge and at the center of the mass respectively. First, consider the rubber and polypropylene isolator (I) cases of Figs. 19(a), (c) and 20(a), (c), where both computational and experimental results show that all vibration and sound measures are significantly reduced when an isolator is moved from the edge to the mass center. Like the previous case, measured $IL_p$ spectra do not exhibit as much reduction as the ones computed beyond 500 Hz because the actual sound radiated from the receiver is lower than the background noise from shakers. Results are given in terms of 1/3 octave band center frequencies from 31.5 to 2500 Hz. Only the mean values within each bandwidth are plotted here.
shaker noise beyond 500 Hz, especially when a rubber isolator is located at the center of the source. Further, similar to the previous case, $IL_v$ spectra beyond 500 Hz are much higher than $IL_p$ since measured $IL_v$ are not contaminated by the shaker noise. Next, aluminum isolators (I and II) are examined. Figs. 19(b), (d) and 20(b), (d) show that vibration and acoustic measures are reduced by connecting an aluminum mount at a source location with zero translational free velocity. However, the aluminum isolator cases show less reduction when compared to the rubber or polypropylene isolator case. Further, it is observed in Figs. 19(b) and (d) that measured $IL_p$ values exhibit a reasonable match with computed $IL_p$ since the sound pressures with an aluminum mount are higher than the shaker noise level. However, the aluminum isolator cases show less reduction when compared to the rubber or polypropylene isolator case. Further, it is observed in Figs. 19(b) and (d) that measured $IL_p$ values exhibit a reasonable match with computed $IL_p$ since the sound pressures with an aluminum mount are higher than the shaker noise level. Similar to the mount material case, spectral averages of the measured and computed insertion losses are again shown in Table 6. It is observed that significant reductions in vibration transmission, based on mount locations, are identified by using sound pressure measures for a system with Rubber I or Polypropylene II isolator. Velocity measures at selected locations also provide large reductions in vibration transmitted to a receiver, as seen from Table 6. Further, some reductions in velocity and sound measures are observed for a system with an aluminum isolator. Like the previous case, reasonable agreements between computed and experimental results are observed even though some measurements are contaminated by the shaker noise as discussed before.

Table 6
Spectral averages of insertion losses for the experimental system of Fig. 13

<table>
<thead>
<tr>
<th>Effect</th>
<th>Measure (mean-square value)</th>
<th>Isolator</th>
<th>Computation (dB)</th>
<th>Experiment (dB)</th>
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<tbody>
<tr>
<td>Isolator material given force excitation</td>
<td>Velocity ($\psi^2_v$) at $\xi_3$ or $\xi_4$</td>
<td>Isolator I&lt;sup&gt;a&lt;/sup&gt;</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Isolator II&lt;sup&gt;b&lt;/sup&gt;</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Isolator I&lt;sup&gt;a&lt;/sup&gt;</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Isolator II&lt;sup&gt;b&lt;/sup&gt;</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Isolator location given moment excitation</td>
<td>Sound pressure ($\psi^2_p$) at $\xi_1$ or $\xi_2$</td>
<td>Rubber I&lt;sup&gt;c&lt;/sup&gt;</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Velocity ($\psi^2_v$) at $\xi_3$ or $\xi_4$</td>
<td>Aluminum I&lt;sup&gt;c&lt;/sup&gt;</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Polypropylene II&lt;sup&gt;c&lt;/sup&gt;</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Aluminum II&lt;sup&gt;c&lt;/sup&gt;</td>
<td>9</td>
<td>5</td>
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<tr>
<td></td>
<td></td>
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<td>Aluminum I&lt;sup&gt;c&lt;/sup&gt;</td>
<td>7</td>
<td>8</td>
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<td></td>
<td></td>
<td>Polypropylene II&lt;sup&gt;c&lt;/sup&gt;</td>
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<td>25</td>
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<tr>
<td></td>
<td></td>
<td>Aluminum II&lt;sup&gt;c&lt;/sup&gt;</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

<sup>a</sup>Baseline for insertion loss calculation is System with Rubber isolator I.
<sup>b</sup>Baseline for insertion loss calculation is System with Polypropylene isolator II.
<sup>c</sup>Baseline for insertion loss calculation is System with a mount located at the center of the mass source.
6. Conclusion

6.1. Summary

Several measures of vibration isolation performance have been critically examined for a multi-dimensional system with inverted ‘L’ structure receivers. Non-dimensional components of multi-dimensional transmissibilities and effectivenesses are comparatively evaluated for an inverted ‘L’ beam receiver and four source configurations. Radiated sound pressures resulting from both in-plane and out-of-plane motions of the ‘L’ beam receiver, have also been calculated and correlated with power-based measures. Further, vibration power components transmitted to the ‘L’ beam receiver and their interactions have been analyzed. Our analysis shows that significant couplings occur between different degrees of freedom. In order to efficiently interpret transfer paths of inherent complexity, several measures have been examined. Sound measurements and predictions for the inverted ‘L’ plate demonstrate that a rank order based on free field sound pressures, at one or more properly selected points, could be regarded as a measure of the vibration power transmitted to the receiver. Measured insertion losses for sound pressure match well with those based on computed results, especially on the basis of spectrally averaged values. Finally, the effects of interfacial conditions and elastomeric non-linearities need to be further studied.
6.2. Emerging research issues

Properties of elastomeric and hydraulic isolators typically show frequency dependency and are sensitive to preload and dynamic excitation levels [2,3]. In order to properly predict the isolation behavior of such non-linear systems, one must question the use of measures that are typically derived based on the linear system theory [35]. Several experimental methods have been developed to characterize the stiffnesses of an isolator. Direct force measures have been pursued by simulating ideal boundary conditions [36,37] but such approaches are limited to lower frequencies since the unwanted dynamics of the measurement machine is involved as the frequency increases [36]. Recently, some approximate methods that are based on motion transmissibilities have also been developed [38–41]. However, alternate laboratory measurement methods need to be correlated using appropriate isolation measures.

Simplified isolator models are often employed to describe the dynamic behavior of vibration isolation systems [3,5,11,15,16,42]. These are often longitudinal spring models [3,5,11] though some models include flexural components with or without the cross-axis coupling terms [15,16,42]. However, it is known that the rotational component becomes important and standing wave effects occur within an isolator as the frequency increases [1,4,42]. Therefore, more advanced isolator models are needed to properly describe couplings among multiple and/or multi-dimensional transmission paths [12,24–27]. Both driving and transfer point stiffnesses must be known to properly interpret isolator measures.

Fig. 20. Insertion loss ($IL_v$) of velocity at plate location $\xi_3$ of Table 4 on mount location given moment excitation. (a) Rubber isolator I; (b) aluminum isolator I; (c) polypropylene isolator II; (d) aluminum isolator II. ——, calculated; •, measured. Results are given in terms of 1/3 octave band center frequencies from 31.5 to 2500 Hz. Only the mean values within each bandwidth are plotted here.
Several efforts have been made to quantify the contribution of source to vibration transmission to a receiver [12,23–27]. One may introduce a source descriptor along with an effective mobility concept to quantify the source contribution [23–27]. Further, the role of free source velocity on vibration transmission has been emphasized [12] and recently a “pseudo-force” method has been developed to simulate the free velocities of a source [43,44]. However, a more appropriate quantification of source strength is still required and dynamic interactions between sources and paths would need further investigation, especially in the presence of compliant vibration sources. Finally, future work is required to properly interpret coupling phenomena and to develop efficient transfer path measures for ‘real life’ systems that incorporate multiple isolators. Efforts are also needed to identify appropriate measures for structure-borne and airborne noise paths over a broad range of frequencies.

Acknowledgements

The General Motors Corporation (Noise and Vibration Center) and the Goodyear Tire and Rubber Company (Transportation Molded Products) are gratefully acknowledged for supporting this research.

Appendix A. Nomenclature

\( A, B, C, D \) arbitrary constants
\( b \) width
\( c \) wave speed
\( d \) distance to sound observation point from reference
\( E \) Young’s modulus
\( f \) force amplitude
\( f \) force amplitude vector
\( F \) excitation
\( F \) excitation vector
\( g \) function for sound pressure
\( G \) shear modulus
\( G \) function vector for sound pressure
\( h \) reference location in rigid body with respect to mass center
\( I_s \) area moment of inertia
\( I_m \) mass moment of inertia
\( I \) identity matrix
\( IL \) insertion loss
\( j \) \( \sqrt{-1} \)
\( k \) wave number
\( l \) length
\( m \) mass
\( m \) inertia matrix
$M$ mobility
$\mathbf{M}$ mobility matrix
$p$ sound pressure
$q$ moment amplitude
$q$ moment amplitude vector
$Q$ numerical constant for rubber material
$R$ rotation matrix for the cross vector product
$r$ radius
$S$ area
$S, P, R$ source, path and receiver
$t$ thickness
$T$ transformation matrix
$T$ shape factor
$TR$ transmissibility
$\mathbf{TR}$ transmissibility matrix
$v$ translational velocity
$v$ translational velocity vector
$V$ velocity
$\mathbf{V}$ velocity vector
$w$ rotational velocity
$w$ rotational velocity vector
$X$ displacement in $x$ direction
$Y$ displacement in $y$ direction
$\mathbf{X}$ generalized co-ordinate vector
$x, y, z$ cartesian co-ordinates
$\alpha, \beta, \gamma$ mobilities of components
$\mathbf{\alpha, \beta, \gamma}$ mobility matrices of components
$\Gamma$ structural power efficiency
$\eta$ loss factor
$\theta$ rotational displacement
$\xi$ response location
$\Xi$ effectiveness
$\Xi$ effectiveness matrix
$\Pi$ vibration power (time-averaged)
$\rho$ mass density
$\Psi$ mean-square
$\omega$ frequency, rad/s

Subscripts
$a$ air
$A$ axial (power)
$B$ flexural motion or bending (power)
$G$ mass center
$H$ horizontal beam or plate
### Symbols and Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>IN</td>
<td>input</td>
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<tr>
<td>i, j</td>
<td>indices or reference points in mass</td>
</tr>
<tr>
<td>l</td>
<td>lateral (power)</td>
</tr>
<tr>
<td>L</td>
<td>axial or longitudinal motion</td>
</tr>
<tr>
<td>p</td>
<td>perimeter</td>
</tr>
<tr>
<td>Q</td>
<td>variable</td>
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<tr>
<td>R</td>
<td>rotational</td>
</tr>
<tr>
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</tr>
<tr>
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<td>spectral average</td>
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<td>TR</td>
<td>transmitted out</td>
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<tr>
<td>V</td>
<td>vertical beam or plate</td>
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<td>w</td>
<td>rotational component of mobility matrix of rigid body</td>
</tr>
<tr>
<td>W</td>
<td>weighted</td>
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<td>WF</td>
<td>weighted force</td>
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<tr>
<td>WV</td>
<td>weighted velocity</td>
</tr>
<tr>
<td>with</td>
<td>with isolator</td>
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<td>1, 2, 3, 4</td>
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### Superscripts

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<td>complex valued</td>
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<td>.</td>
<td>time derivative</td>
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<td>complex conjugate</td>
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### Operators

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<tr>
<td>Re</td>
<td>real part</td>
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<tr>
<td>⟨⟩</td>
<td>time-averaged</td>
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<tr>
<td>/</td>
<td>quotient for matrices</td>
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### References

[34] Sysnoise Users Manual Version 5.4. 1999 NIT, Belgium.