1 INTRODUCTION

Recently, Tiwari et al. described a series of laboratory experiments that must be undertaken to characterize various nonlinearities of hydraulic engine mounts. Essentially they refined the experimental methods that were first proposed by Kim and Singh and successfully extended the nonlinear formulation with empirically obtained functions or curve-fits to the prediction of responses to ideal transient excitations. Yet another recent article by Geisberger et al. has suggested that a detailed experiment must be constructed before nonlinear model parameters are adequately estimated. Such experimental approaches are necessary for research studies but they pose significant difficulties for mount manufacturers and users (vehicle designers) as they may have tens or even hundreds of engine mount designs at their disposal but do not have the luxury of time, or even the facility, to fully characterize the parameters using the suggested research procedures. What is ideally desirable would be an approach that employs limited (and off the shelf) information such as measured data in terms of dynamic stiffness spectra \( K(f, X) \) over the frequency \( f \), (Hz) range of interest at certain displacement excitation amplitudes \( X \), (mm). This article proposes to fill this void by developing a new estimation method that would quickly develop linear or quasi-linear models with reasonable accuracy over both lower (typically up to 50 Hz) and higher frequency (say from 50 to 300 Hz) regimes, as well as in time domain.

2 PROBLEM FORMULATION

2.1 Lumped fluid model

Hydraulic engine mounts are often designed to provide improved stiffness and damping characteristics which vary with frequency and excitation amplitude. Figure 1(a) illustrates a typical schematic of the mount; refer to Singh, et al. for a detailed description of the internal parts, their functions and basic parameters. Such mounts are usually modeled by lumping the fluid system into several control volumes as shown in Figure 1(b). System parameters include the fluid compliances \( C_1 \) and \( C_2 \) of the top (#1) and bottom (#2) chambers, stiffness \( k_1 \) and \( k_2 \) of the elastomeric rubber element (#r), fluid resistance \( R \) and inertance \( I \) of the inertia track (#i), inertance \( I_d \) and resistance \( R_d \) of the decoupler (#d). Further, the dynamic displacement excitation \( x(t) \) is applied under a mean load \( F_m \), and the force \( F_d(t) \) transmitted to the rigid base is often viewed as a measure of mount performance. What will be using both fluid system parameters (such as compliances, \( C_1 \) and \( C_2 \), in pressure and volume units) and mechanical system parameters (such as stiffness \( k_1 \) in force and displacement units). Refer to Singh et al. for details. Continuity equations for the lower and upper chambers of Figure 1(b) yield the following equations where \( q_1(t) \) and \( q_2(t) \) are the flow rates through the inertia track and decoupler respectively, \( A_r \) is the effective piston area, and \( p_1(t) \) and \( p_2(t) \) are dynamic pressures in the top and bottom chambers respectively.

\[
A_r \dot{x}(t) - q_1(t) - q_2(t) = C_1 \dot{p}_1(t), \quad q_1(t) + q_2(t) = C_2 \dot{p}_2(t)
\]

(1a,b)

Momentum equations for the decoupler and inertia track are derived as:
The thin rubber membrane that forms the lower fluid chamber (#2) has a very high compliance \( C_2 \). Thus the absolute lower chamber pressure \( p_t^2 (t) \) and the static equilibrium pressure \( \bar{p} \) can be approximated by the atmosphere pressure \( p_{atm} \). Therefore, the dynamic pressure \( p_2^2 (t) = p_t^2 (t) - \bar{p} (t) \approx 0 \) can be ignored for the sake of simplicity. Measured results\(^1,2,4\) confirm that \( p_2 (t) \) is usually negligible compared with \( p_1 \), i.e. \( p_1 (t) = p_t^2 (t) \).

### 2.2 Mount nonlinearities

The following nonlinearities are usually found in most applications: (a) chamber compliances \( C_1 (p_1) \) and \( C_2 (p_2) \), (b) flow resistances \( R_i (q) \) and \( R_d (q_d) \) through the inertia track and decoupler respectively, (c) decoupler switching action and associated flow \( q_d (t) \), and (d) vacuum phenomenon in the upper chamber\(^2,4\). Kim and Singh\(^2\) and then Tiwari et al.\(^1,4\) have described the mathematical or empirical relationships based on deliberate bench experiments. Further, Geisberger et al.\(^3\) used a hydraulic cylinder with a two-chamber vessel to test various components including upper or lower fluid chambers, inertia track and decoupler. The take-apart sub-structure is clamped between vessel sections and a servo-controlled actuator is used to drive the system. The pressure differential signals are then acquired to estimate nonlinear parameters, among which the decoupler sub-structure was identified as a key to high frequency resonance. They also suggested a continuous function that describes the switching effect and flow leakage through the decoupler.

### 2.3 Objectives

In our proposed approach, we first assess the following constraints from the perspective of system user or manufacturer: (a) the mount is viewed as a black-box component with very limited information provided by the mount vendors to protect their proprietary designs; (b) only steady state \( \tilde{K}(f, X) \) data are available (see section 3 for details and limitations); (c) experimental facilities to conduct bench tests as suggested by researchers\(^1-4\) are not available; and (d) time is of essence since the product design cycles are now very stringent. Accordingly we develop new or refined estimation procedures that are illustrated in Figure 2, and focus on the quasi-linear model in this article. Note that our method would be able to estimate parameters over two frequency regimes: (i) the lower frequency regime (typically up to 50 Hz) that is usually controlled by the inertia track resonance, and (ii) the decoupler resonance that dominates over the higher frequency regime (say from 50 to 300 Hz).
Specific objectives include the following. First, develop linear and quasi-linear $\tilde{K}(f, X)$ formulations at lower frequencies to quantify the inertia-augmented and amplitude-sensitive damping $R(X)$ and asymmetric (nonlinear) characteristics of $C(f, X)$. Second, propose higher frequency models that describe the decoupler resonance and estimate damping $R(X)$ (usually difficult to measure with conventional experiments). Third, discuss the effects of excitation amplitude ($X$) on mount parameters and illustrate the utility of quasi-linear models. Fourth, predict the transient response to a realistic displacement excitation profile and compare predictions with $F'_i(t)$ and $p_i(t)$ measurements.

3 STEADY STATE DYNAMIC STIFFNESS MEASUREMENTS

All mount vendors and users employ the dynamic stiffness testing procedure, corresponding to the ISO standard 10846. Commercial machines are readily available though they may not be able to accommodate non-sinusoidal tests. The mount (along with the fixture) is usually placed in an elastomer test machine and a sinusoidal displacement excitation $x(t) = X \cdot \sin(2\pi ft)$ with peak to peak (p-p) amplitude $X$ at frequency $f$ is applied, under a compressive preload $F_m$ to produce the mean displacement $x_m$. The complex-valued, cross point dynamic stiffness $\tilde{K}(f, X) = |K_e(f, X)|e^{i\phi} = Ke^{i\phi}$ is measured where $K_e$ is the amplitude of force transmitted at $f$, $K(f, X)$ is the stiffness modulus and $\phi(f, X)$ is the loss angle. In this procedure, a Fourier filter is used to assess the response only at $f$ though other frequencies (such as super-harmonics) may be present. Thus, it is difficult to directly quantify the nature and extent of nonlinearities based on the above procedure.

Figure 3 shows one set of $\tilde{K}(f, X)$ data for mount D under $F_m = -1200$ N (compressive). Tests were conducted up to 50 Hz with an increment of 2.5 Hz, say corresponding to $X = 0.1, 0.2, 0.3, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 2.75$ and 3 mm (p-p). Measured results are plotted in a 3-D form with horizontal axes representing $f$ and $X$ respectively. We use surface-interpolation to find a continuous 2-D function $\tilde{K}(f, X)$ with desirable $\Delta f$ and $\Delta X$ resolutions to estimate data at intermediate points. Several interpolation methods such as bi-linear, cubic spline and bi-cubic interpolations were attempted using Matlab’s interpolation routines. Figure 4 shows the surface-fit result using the bi-linear interpolation with $\Delta f = 0.5$ Hz and $\Delta X = 0.02$ mm.

4 MATHEMATICAL MODEL OF A PASSIVE HYDRAULIC MOUNT AT LOWER FREQUENCY

4.1 Cross vs. driving point stiffness

The cross point dynamic stiffness of section 3 is given by the force $F_i(t)$ transmitted to the base that is shown in Figure 5(a), where $F_m$ is the static force, $F'_i(t)$ is the absolute force and $p_i(t) = p_i(t) - \bar{p}$.

\[
F'_i(t) = F_m + F_r(t), \quad F_m = k_r x_m + A_r (\bar{p} - p_{\text{true}}), \quad (3a,b)
\]

\[
F_r(t) = k_r x(t) + b_r \dot{x}(t) + A_r p_i(t)
\]  
(3c)

Thus far we have examined the displacement excitation $x(t)$ which yields cross point dynamic stiffness $\tilde{K}$. Suppose we were to apply a dynamic force $F(t)$ at the driving point and evaluate response $x(t)$. This would give us the driving point dynamic stiffness. The effective (but fictitious) dynamic force $F(t)$ at the driving point can be viewed by rewriting (3a-c) as:

\[
F(t) = F'_i(t) = F_m + F(t), \quad F(t) = m_r \ddot{x}(t) + k_r x(t) + b_r \dot{x}(t) + A_r p_i(t)
\]  
(3d,e)
By comparing (3c) with (3e):
\[ F(t) = F_p(t) + m_r \ddot{x}(t) \]  
(3f)

Observe that \( F(t) \) includes the additional inertia term corresponding to the rubber element mass \( m_r \). However, \( m_r \ddot{x}(t) \) is negligible at lower frequencies due to a small value of \( m_r \). This implies that \( F(t) \approx F_p(t) \) and \( F/X(f) \approx F_p/X(f) \).

Experimental work of Lee et al.9 confirms that the driving and cross point dynamic stiffnesses at lower frequencies are virtually the same for most mounts.

4.2 Analogous mechanical model of fixed decoupler mount at lower f

The fixed decoupler type mount can be analyzed as a sub-set of the complete system by inserting \( q_d = 0 \) or \( R_d \rightarrow \infty \).

Define the effective parameters of the analogous mechanical system of Figure 6 as: effective velocity of inertia track fluid \( \dot{\dot{x}}_e(t) \), effective mass of inertia track fluid column \( m_e \), effective viscous damping of inertia track fluid \( b_e \), equivalent stiffness of upper chamber compliance \( k_1 = A_1^2/R_e \), equivalent stiffness of lower chamber compliance \( k_2 = A_2^2/C_2 \). From (1) and (2), we get:
\[ \dot{p}_e(t)A_e = k_2 \dot{x}_e(t), \quad p_e(t)A_e = k_2 x_e(t) \]  
(4a, b)

Note that (4b) and (4d) are necessary conditions of (4a) and (4c) respectively so that a numerical error in terms of the mean (dc) components is introduced accordingly. However, this error is trivial for sinusoidal responses. It is shown that the effective mass and viscous damping of the fluid inside the inertia track increase proportionally to the square of the area ratio as listed in Table 1, where \( A_1 \) and \( A_2 \) are the cross sectional areas of the decoupler and inertia track, respectively, the reason is because the velocity of the fluid inside the orifice as well as the force that accelerates the fluid column is amplified in proportion to the specific cross-sectional area. Therefore, the effective fluid mass of the inertia \( m_e \) is of the same order of magnitude as the engine mass \( m_e \) (corresponding to the quarter car model5), and the resulting hydraulic mount is highly damped when compared with a conventional rubber mount. This mechanism has been referred to as the “velocity amplifying dynamic damper” effect by Sugino et al.10 or “inertia-augmented damping” phenomenon by Singh et al.5.

Assume \( m_r \) over the lower frequency regime and transform equations (5, 6) into the Laplace (s) domain with zero initial conditions. The following driving point dynamic stiffness \( K_{nm} \) is obtained (in 3rd/2nd order form) corresponding to the mechanical model of Figure 6.

![Diagram of a fixed decoupler mount model](image-url)
TABLE 1—Parameters of an analogous mechanical system corresponding to fixed or free decoupler type hydraulic mount.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression based on physical values</th>
<th>Effective value (in mechanical system units)</th>
<th>Amplification ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia track fluid mass</td>
<td>( m_\infty = A_1^2 I_1 )</td>
<td>( m_\infty = A_1^2 I_1 )</td>
<td>( A_1^2 / A_2^2 )</td>
</tr>
<tr>
<td>Inertia track damping</td>
<td>( b_\infty = A_1^2 R_1 )</td>
<td>( b_\infty = A_1^2 R_1 )</td>
<td>( A_1^2 / A_2^2 )</td>
</tr>
<tr>
<td>Decoupler fluid mass</td>
<td>( m_m = A_1^2 J_1 )</td>
<td>( m_m = A_1^2 J_1 )</td>
<td>( A_1^2 / A_2^2 )</td>
</tr>
<tr>
<td>Decoupler damping</td>
<td>( b_m = A_1^2 R_m )</td>
<td>( b_m = A_1^2 R_m )</td>
<td>( A_1^2 / A_2^2 )</td>
</tr>
<tr>
<td>Upper chamber stiffness</td>
<td>---</td>
<td>( k_2 = A_1 / C_1 )</td>
<td>---</td>
</tr>
<tr>
<td>Lower chamber stiffness</td>
<td>---</td>
<td>( k_2 = A_1 / C_1 )</td>
<td>---</td>
</tr>
</tbody>
</table>

\[
K_{m32}(s) = k_2 + k_1 + b_s s - \frac{k_1^2}{m_\infty s^2 + b_\infty s + k_1 + k_2} \quad (7a)
\]

The corresponding static stiffness is \( k_2 + k_1/k_1 + k_2 \). Further, assume \( C_2 > 100 C_1 \) so that \( k_2 + k_1 \approx k_1 \). Simplify \( K_{m32} \) to the following expression:

\[
K_{m32}(s) = k_2 + k_1 + b_s s - \frac{k_1^2}{m_\infty s^2 + b_\infty s + k_1} \quad (7b)
\]

Assume \( b_s = 0 \) and rewrite (7b) in a fraction form. Then the numerator polynomial can be converted into an equivalent form \( s^2 + (b_{\infty} / m_{\infty}) s + k_1 k_2 / (k_1 + k_2) \) which is the characteristic equation of a mechanical oscillator with mass \( m_{\infty} \), damping coefficient \( b_{\infty} \) and spring rate \( k_1 k_2 / (k_1 + k_2) \) as shown in Figure 7(a). Likewise, the denominator polynomial \( s^2 + 2 \zeta_2 \omega_2 s + \omega_2^2 \) can be converted into \( s^2 + (b_{\infty} / m_{\infty}) s + k_1 / (m_{\infty}) \), which implies that \( \omega_2 \) is the natural frequency (in rad/s) of a fluid Helmholtz resonator with compliance \( C_1 \) and inertance \( I_1 \) as shown in Figure 7(b).

4.3 Scope and limitations of the mechanical model

Recall the fluid model of Figure 1, and rewrite the transmitted force \( F_r(t) \) expression as follows using (3e), (2a) and (3f) for the fixed decoupler \( R_q = 0 \)

\[
F_r(t) = k_r x(t) + b_r \dot{x}(t) + A_1 I_1 \dot{q}_1(t) + A_1 R_1 q_1(t) \approx F(t) \quad (8a)
\]

Thus \( F_r(t) \) has three major components: (a) \( k_r x(t) + b_r \dot{x}(t) \) corresponding to the forces transmitted via the rubber element, as represented by a visco-elastic (Voight type) model; (b) inertia force \( A_1 I_1 \dot{q}_1(t) \) of the fluid column that can be equated to \( m_m \ddot{x}_m(t) \) for the model of Figure 6, and (c) viscous damping force \( A_1 R_1 q_1(t) \) generated in the inertia track that could be represented by \( b_m \ddot{x}_m \). Thus, use the mechanical model defined earlier to express as:

\[
F_r(t) = k_r x(t) + b_r \dot{x}(t) + m_m \ddot{x}_m(t) + b_m \ddot{x}_m \quad (8b)
\]

However, from the schematic of the mechanical model of Figure 6, the effective transmitted force \( F_{m3}(t) \) is found as follows:

\[
F_{m3}(t) = k_r x(t) + b_r \dot{x}(t) + k_2 x_m + b_r \ddot{x}_m \approx k_r x(t) + b_r \dot{x}(t) + b_r \ddot{x}_m \quad (9a)
\]

\[
F_{m3}(t) = k_r x(t) + b_r \dot{x}(t) + A_1 R_1 q_1(t) \quad (9b)
\]

Comparison of equations (8) and (9) clearly shows that \( F_{m3}(t) \) under-estimates \( F_r(t) \) by neglecting the inertia force of the inertia track fluid column which is transmitted through the frame. In other words, the dynamic forces (inertia and damping forces) caused by the pressure difference \( (p_1(t) - p_2(t)) \) are directly transmitted to the vehicle frame in the fluid model.
but only the damping force is transmitted to the fixed base in the mechanical model as shown in Figure 7(a). Nonetheless, the analogous method can still be used by approximating the driving point dynamic stiffness of the mechanical model as the cross point dynamic stiffness of the fluid model. This would allow us to estimate parameters using the $K_{m3}$ transfer function. Spectral contents can be obtained by setting $s = j2\pi f$ where $j = \sqrt{-1}$ and $f$ is in Hz.

5 ESTIMATION OFAMPLITUDE DEPENDENT PARAMETERS AT LOWER FREQUENCIES

5.1 Fixed decoupler mount

Using the curve-fit $\tilde{K}(f, X)$ over the lower frequency range (typically 0-50 Hz as shown in Figure 4), a continuous transfer function (in the s domain) is estimated in a $3^{rd}/2^{nd}$ order form.

$$K_{m3}(s) = \frac{n_1s^3 + n_2s^2 + n_3s + n_0}{d_2s^3 + d_1s^2 + d_0s + d_0}$$

Comparison of equations (7b) and (10) shows that $d_2$, $d_1$ and $d_0$ are proportional to $m_{ie}$, $b_{ie}$ and $k_{ie}$ respectively. Define a scaling factor $\delta$ and assume the static stiffness $\gamma = k_{ie} + k_{am} / |k_{ie} + k_{am}| \approx k_{ie} + k_{am} \approx k_{ie}$. Now, we estimate the system parameters as:

$$k_{ie} = \frac{n_0}{d_0}, \quad b_{ie} = \frac{n_2}{d_0}, \quad \delta_\epsilon = \frac{d}{d_0}$$

$$k_{ie} = \delta_\epsilon \delta, \quad k_{am} = \frac{d_{2}}{d_{0}} k_{ie}, \quad m_{am} = \frac{d_{1}}{d_{0}} k_{ie}, \quad m_{ie} = \frac{d_{1}}{d_{0}}$$

Here $\delta$ is a device-specific adjustment ratio (around 1) which could be used to tune $\delta$ for best possible curve-fit results. For instance, the value of $\delta$ for mount D is found to be around 1.16. Effective parameters are estimated from measured data for the fixed decoupler mount under varying $X$. Amplitude dependent results are listed in Table 2 and compared with nominal values (in mechanical system units). Here, $k_{am}$ is the measured value for the rubber element that is obtained by draining the fluid out of the hydraulic mount. Since both $k_{am}$ and $b_{ie}$ vary slightly with $f$, the measured rubber mount data are averaged over the lower frequency range. The nominal chamber stiffnesses satisfy the assumption $k_{ie} = 4.6$ N/mm $<$ $k_{ie} = 438$ N/mm. Figure 8 shows that predicted $\tilde{K}(f, X)$ spectra using mechanical model with estimated parameters correlate well with measurements. Observe the following interesting results from Table 2.

First, both $k_{am}$ and $k_{ie}$ decrease with an increase in $X$ from 0.3 mm to 3 mm, implying a softening effect in the rubber element under increased amplitude. For a specific $X$, the value of $k_{ie}$ is approximately 14% higher that that of $k_{am}$. This can be explained in terms of the additional stiffness that is introduced by the fluid under an increased static pressure. Consequently, the value of $k_{ie}$ should be regarded as the upper boundary of $k_{am}$.

Second, the estimated $b_{ie}$ value decreases slightly with an increase in $X$ even though the measured rubber damping values are almost independent of $X$. The nominal value can thus be regarded as an averaged value within a ±30% error bound.

Third, estimated $m_{am}$ values are consistent with each other. As predicted by the mechanical model, $m_{am}$ is around 46 kg, which is about 1/3 or 1/4 of the engine mass (for a quarter car model) depending on the mounting system. This value is, however, 50% larger than the calculated nominal value. Recall that the inertia track is modeled in our formulation as a straight pipe without any end corrections; thus the effective value elongates the geometric length by up to 33%\textsuperscript{12}. Therefore, it is implied that turbulence takes place within the inertia track\textsuperscript{12}.

Fourth, similar to the $m_{am}$ value, the estimated fluid damping $b_{am}$ is nearly independent of $X$ though it overestimates the measured result by roughly 23%. Since $m_{am}$, $b_{am}$ and $k_{am}$ are proportional to the coefficients of the characteristic equation, this error is partially introduced by an over-estimation of $m_{am}$. This result also quantifies the inertia augmented damping $b_{ie}$ which is 6 to 10 times more significant than the pure rubber damping $b_{ie}$.

Finally, many hydraulic mount formulations\textsuperscript{3,10,14,15} assume that the amplitude-dependence of dynamic stiffness is due to the decoupler action. This assumption implies that the dynamic properties of a fixed decoupler mount will be insensitive to $X$. However, such is not found in the measurement of Figure 3. Amplitude dependency is illustrated in Table 2 by the estimated $k_{ie}$ and its value at $X = 0.3$ mm is nearly three times the value obtained at $X = 3$ mm. The difference could be explained by the vacuum phenomenon\textsuperscript{12}, which is associated with the release of pre-dissolved gas in the fluid during the expansion process. Vacuum introduces an additional compliance to the upper chamber and it is more dominant at higher $X$, resulting

### Table 2—Estimated parameters of a fixed decoupler mount (D). Baseline values are: $k_{ie} = 432$ N/mm, $b_{ie} = 270$ Ns/m, $C_{ie} = 2.5 \times 10^{11}$ m$^3$/N, $C_{am} = 2.4 \times 10^{-11}$ Nm$^2$/N, $R_{ie} = 1.4 \times 10^{10}$ Ns/m$^2$ and $A_{ie} = 8.4 \times 10^{-3}$ m$^2$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal Value</th>
<th>$X$ (mm) p-p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Symbols (units)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_{ie}$ (N/mm)</td>
<td>432</td>
<td>462</td>
</tr>
<tr>
<td>$k_{am}$ (N/mm)</td>
<td>432</td>
<td>528</td>
</tr>
<tr>
<td>$b_{ie}$ (Ns/m)</td>
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<td>295</td>
</tr>
<tr>
<td>$b_{am}$ (Ns/m)</td>
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<td>726</td>
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<tr>
<td>$m_{am}$ (kg)</td>
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<td>2056</td>
</tr>
<tr>
<td>$m_{ie}$ (kg)</td>
<td>30.8</td>
<td>46.3</td>
</tr>
</tbody>
</table>
in a more significant decrease in the estimated $k_1$ value. A bi-linear $C(p_1)$ model was suggested by Kim and Singh\(^2\) as well as Tiwari \textit{et al}.\(^{1,4}\) and the $\Delta p_1/\Delta V_1$ relationship was measured from a bench experiment. As a simplification in our work, a quasi-linear model is proposed where $k_1$ is modeled as an empirical function of $X$ based on estimated effective parameters. Then, the following model is utilized to describe the frequency-sensitive and amplitude-dependent dynamic stiffness of a fixed decoupler mount in the lower frequency regime. For the sake of simplicity, all parameters are assumed to be constants other than the $k_1(X)$ function, which could be obtained by interpolating $X$ in a continuous manner.

$$K_{s_3} (s, X) = b_s s + k_s + k_1(X) - \frac{k_1^2(X)}{m_s s^2 + b_s s + k_1(X)}$$  \hspace{1cm} (12)

### 5.2 Free decoupler mount

Next consider the free decoupler mount ($R_d(t) \neq 0 \text{ or } q(t) \neq 0$) of Figure 1. When the mount is subject to an excitation with high $X$, the decoupler remains closed most of the time and the resonance induced by inertia track typically dominates.
over the lower frequency regime. Consequently the governing system should dynamically behave similar to a fixed decoupler mount at lower $f$. Thus the fixed decoupler mount algorithm could be used to reasonably curve-fit $\tilde{K}(f, X)$ spectra at lower $f$, provided the estimated parameters must be viewed as a consequence of the linearization of all non-linear phenomena including the decoupler switching mechanism, vacuum effect and turbulence. Figure 9 shows sample results for a free decoupler mount (D) where the predicted stiffness spectra correlate well with measurements for $X \geq 1.0$ mm. Results are summarized in Table 3, where the estimated parameters for $X \geq 1.5$ mm are found to be consistent with each other. Abrupt changes are, however, observed when $X$ increases from 1.0 to 1.5 mm, which exhibits a significant increase in $b_u$ from 1317 to 2600 N/m. This implies a shift in the operational state: The inertia track is partially coupled ($q_d(t) \neq 0$) under $X = 1.0$ mm, and it is totally coupled ($q_d(t) = 0$) under $X \geq 1.5$ mm. Assuming the inertia track flow $q_d(t)$ is uncoupled up to a specific excitation amplitude $X_p$, the decoupler gap length $g_d$ is geometrically related to this $X_p$ by $g_d = \eta X_p A_d/A_u$, where

\begin{align*}
\phi &= \text{The inertia track} \quad (\text{a}) X = 1.0 \text{ mm p-p} \\
\phi &= \text{The inertia track} \quad (\text{b}) X = 1.5 \text{ mm p-p} \\
\phi &= \text{The inertia track} \quad (\text{c}) X = 2.0 \text{ mm p-p} \\
\phi &= \text{The inertia track} \quad (\text{d}) X = 3.0 \text{ mm p-p}
\end{align*}

Fig. 9 — $\tilde{K}(f, X)$ results of a free decoupler mount (D) with different $X$ values. Key: — measured $K(s)$, —— $K_{\tilde{K}}(s)$ curve-fit result, ---- predicted using mechanical model $K$. 
TABLE 3—Estimated parameters of a free decoupler mount (D). Baseline values are: \( k = 432 \text{ N/m} \), \( b = 270 \text{ Ns/m} \), \( C_1 = 2.5 \times 10^{-11} \text{ m}^3/\text{N} \), \( C_2 = 2.4 \times 10^{-11} \text{ m}^3/\text{N} \), \( R = 1.4 \times 10^5 \text{ Ns/m} \), and \( A = 8.4 \times 10^4 \text{ m}^2/\text{N} \).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal value</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
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<tbody>
<tr>
<td>( k_m ) (N/mm)</td>
<td>432</td>
<td>439</td>
<td>428</td>
<td>420</td>
<td>414</td>
<td>408</td>
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<tr>
<td>( k ) (N/mm)</td>
<td>432</td>
<td>458</td>
<td>406.5</td>
<td>400</td>
<td>391</td>
<td>397</td>
</tr>
<tr>
<td>( b ) (Ns/m)</td>
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<td>715</td>
<td>480</td>
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<td>259</td>
</tr>
<tr>
<td>( k_i ) (N/mm)</td>
<td>438</td>
<td>233</td>
<td>320</td>
<td>296</td>
<td>376</td>
<td>250</td>
</tr>
<tr>
<td>( b_i ) (Ns/m)</td>
<td>1534</td>
<td>1317</td>
<td>2600</td>
<td>2617</td>
<td>2625</td>
<td>2582</td>
</tr>
<tr>
<td>( m_i ) (kg)</td>
<td>30.8</td>
<td>37.8</td>
<td>50.8</td>
<td>47.9</td>
<td>48.0</td>
<td>50.3</td>
</tr>
</tbody>
</table>

\( \eta \leq 1 \) is a factor that accounts for several effects including the fluid accommodated by the upper chamber and leakage flow through the inertia track. From results of Table 3, it is inferred that \( 1.0 \leq X \leq 1.5 \text{ mm} \). Given the nominal values, \( A_1 = 3.31 \times 10^7 \text{ m}^2/\text{N} \), \( A_2 = 1.96 \times 10^6 \text{ m}^2/\text{N} \), and \( \eta = 0.6 \), the decoupler gap length is estimated as \( 1.01 \leq g \leq 1.51 \text{ mm} \), which can be further narrowed down by acquiring additional \( \tilde{K}(f, X) \) measurements. A careful comparison between simulated and measured \( \tilde{K}(f, X) \) spectra yields the value of \( g = 1.1 \text{ mm} \).

When \( X \) is 1.5 mm or higher, the estimated \( k, b, \) and \( m \) vary slightly with \( X \) and their values are comparable to those of the fixed decoupler mount. Nevertheless, the estimated \( b \) is higher than the value found for a fixed decoupler by roughly 30%. This strongly suggests that the decoupler switching mechanism introduces additional damping to the inertia augmented fluid damping. Further, the effective \( k \) varies with \( X \), which could also be explained by the vacuum effects. This variation, however, is not as significant as the one observed for the fixed decoupler mount. This implies that the decoupler switching mechanism alleviates the softening effect introduced by the vacuum formation (in the expansion process).

5.3 Visco-elastic (Voight type) model parameters of decoupled mount

For a free decoupler mount, when \( X \) is small enough so that the decoupler stays open, the fluid flows essentially through the decoupler gap and the inertia track is completely “decoupled”. In this case, the rubber stiffness and damping tend to dictate dynamic properties. The \( \tilde{K}(f, X) \) spectrum is relatively flat since the inertia track resonance over the lower frequency regime is absent. The Voight type visco-elastic model\( ^{\text{11}} \) is used to curve-fit the \( \tilde{K}(f, X) \) data typically up to 50 Hz in terms of effective spring \( (k_e) \) and damper \( (b_e) \) that are in parallel as shown in Figure 10. The resulting dynamic stiffness is given by \( \tilde{K}(f, X) = k + j2\pi f \cdot b \), where both \( k \) and \( b \) weakly depend on \( f \) and \( X \). Alternately, we can curve fit any measured \( \tilde{K}(f, X) \) spectra simply by assuming the Voight model but this may not provide the proper physical interpretation. Parameters of mount \( D \) are estimated in Table 4 corresponding to \( X < 1.0 \text{ mm} \). Figure 11 compares the measurement with the Voight model with parameters \( k_e \) and \( b_e \). Several “irregular” peaks are observed in measured \( \tilde{K} \) at \( X = 0.5 \text{ mm} \), which implies that the mount undergoes a transition between the “decoupled” and “coupled” conditions.

5.4 Comparison with other estimation schemes

A brief discussion of some prior estimation methods, based on measured, \( \tilde{K}(f, X) \), is as follows. Jeong and Singh\( ^{\text{13}} \) utilized the measured \( \tilde{K}(f, X) \) data to develop a mount model by employing a nonlinear synthesis method. This resulted in a nonlinear time domain model that was based on a linear model with frequency-dependent parameters. Several empirical functions were defined to characterize the amplitude-dependent properties. However, the coefficients of the dynamic transfer function were numerical values without clearly defining their physical significance. Also, such values are obtained by observing the effects of parametric variations on the dynamic stiffness. In our work, we overcome these limitations by using effective mechanical parameters (such as \( k_e \)) that could be directly related to the system working principles (such as the

![Fig. 10 — Voight model at lower f when the inertia track is completely decoupled.](image-url)

TABLE 4—Estimated parameters of a free decoupler mount (D) using the visco-elastic (Voight type) model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( X \text{ (mm) p-p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols (units)</td>
<td>0.3</td>
</tr>
<tr>
<td>( k ) (N/mm)</td>
<td>468</td>
</tr>
<tr>
<td>( b ) (Ns/m)</td>
<td>338</td>
</tr>
</tbody>
</table>
vacuum phenomenon) and mount design concepts. Tsujiuchi et al. estimated the compliances of $C_1$ and $C_2$ by using the measured dynamic characteristics. However, they assumed that the inertia track dimensions are known at the design stage, and the decoupler-induced resonance was not considered. Colgate et al. developed two separate linear models corresponding to large and small $X$ values. They also employed a piecewise linear simulation and then used the equivalent linearization technique to explain the amplitude-dependence. However, only the “decoupling” nonlinearity is considered to categorize the two linear models, and the system parameters are assumed to be essentially invariant with $f$ and $X$. Their piecewise linear model is based on the following assumptions: (a) the decoupler gap length $g_d$ is known, (b) large and small amplitude models are available and (c) the time history of $x_d$ (decoupler displacement) and $V_b$ (bulge displacement of the rubber element) can be predicted. A switching mechanism is defined, in time domain, by comparing $|\dot{x}_d|$ vs. $g_d$ and sign($V_b$) vs. sign($x_d$). However, given limited measurements of $\tilde{K}(f,X)$, prediction of $x_d$ is usually difficult although $V_b(t)$ can be estimated from $x(t)$. Also, significant amount of time domain simulation data must be generated to yield a modest amount of frequency domain spectral contents. In our quasi-linear model, most parameters are estimated as a function of $X$ although some (such as $k_1$ and $b_1$) are approximated as constants due to their weak amplitude-dependence for the sake of simplicity. Further, we employ $X$ as an indicator of the operational conditions so that different system parameters are assigned depending on the transitions between the “coupled” and “decoupled” stages. Since both operational modes are conveniently characterized by the linearized system parameters that are estimated from $\tilde{K}(f,X)$ tests, a complicated switching model based on $x_d(t)$, $V_b(t)$, and $g_d$ is deemed unnecessary for most applications.

6 MATHEMATICAL MODEL OF MOUNT AT HIGHER FREQUENCIES

6.1 Formulation

When a free decoupler mount is excited at higher $f$ (say up to 300 Hz), the flow through the inertia track or decoupler could be highly turbulent. In our formulation, the inertia track is modeled as a long tube with two flanged open ends, and thus the effective length ($l$) can be calculated by considering the end corrections $\Delta l^2$: $l = l + 2\Delta l = l + 2 \times 0.85r$, where $l$ and $r$ are the geometric length and hydraulic radius of the inertia track. Further, consider mount G with $r = 5.17$ mm and $l = 236$ mm and calculate the resonance associated with the half-wavelength mode as $f_{1/2} = c_s/(2l)$, where $c_s$ is the sound speed. This frequency is found to be around 3,025 Hz, which is well beyond the upper limit of the frequency range of interest. Consequently, the distributed fluid effects can be neglected and the lumped fluid/mechanical model assumption is valid up to at least 300 Hz.

Since the amplitude of displacement excitation is small ($X < 0.1$ mm) over higher frequency regime (due to the test machine limitations and because of lower displacements found in vehicles at high $f$), the governing system should be almost linear. For excitations with $X < X'_s$, the decoupler gap is assumed to open all the time. Thus, the inertia track is
“decoupled” (or removed as \( q_i(t) \to 0 \)) from the system. This provides a basis for the high frequency model.

Recall the earlier assumption of section 4 that approximated measured cross point spectra by the driving point dynamic stiffness of the fluid model, one should note that dominant inertial and standing wave effects may occur at higher \( f_1 \). In such cases, a precise prediction of the response at higher \( f \) may demand the incorporation of both driving point and transfer stiffnesses. However, the commercial test equipments measure only the cross point and most machines have an upper frequency limit (typically 200 to 500 Hz). Consequently, we still apply the same assumption to estimate parameters for the decoupler resonance. Nevertheless, one should be aware of the errors between driving point and transfer stiffnesses as the frequency increases.

### 6.2 Derivation of the high frequency model

The governing equations of a free decoupler mount can be obtained from equations (1-3) by setting \( q_i(t) = 0 \) (or \( R_i(t) \to \infty \)) due to the “decoupling” effect. Additionally, describe the fluid inertia (\( I_d \)) and resistance (\( R_d \)) of the decoupler and then define them in mechanical system units as

\[
\begin{align*}
\frac{b_{d0}}{b_{d0}} &= R_d A_d^2, \\
\frac{m_{d0}}{m_{d0}} &= I_d A_d^2.
\end{align*}
\]

A simplified high frequency (typically 50-300 Hz) model is shown in Figure 12 in the mechanical system form. A 4th/2nd order linear dynamic stiffness transfer function is obtained as

\[
K_{2nd}(s) = m_d s^2 + b_d s + k_d + k_i \frac{k_i^2}{m_d s^2 + b_d s + k_d}
\]  \hspace{1cm} (14)

This simplified model correlates well (despite some errors in phase) with measured data for mount Q as shown in Figure 13.

### 6.3 Broadband model

It was earlier shown that the effective mass of fluid either inside the inertia track or around the decoupler increases in proportional to the square of the area ratio as listed in Table 1. This is because the force that accelerates the fluid follows by recognizing \( k_1 \gg k_2 \). Refer to Table 1 for definitions of \( m_{d0} \) and \( b_{d0} \).

![Fig. 12—Simplified mechanical (2DOF) model for higher frequencies.](image)

![Fig. 13—\( \tilde{K}(f,X) \) of mount Q at higher frequencies. (a) Measured results with \( X = 0.1 \) mm. (b) Predicted results using the linear model of Fig. 12. Key for (a): excitation: “-” with “--” without bell plate structure.](image)
7 ESTIMATION OF PARAMETERS AT HIGHER FREQUENCIES

The inertial effects of the primary rubber element become increasingly dominant at higher \( f \). Thus another degree of freedom associated with \( m_{p} \) should be included in the mount model. Rewrite equations (10) and (11) with curve-fitted parameters in terms of \( K_{\omega}(s) \) which is a \( 4^{th}/2^{nd} \) order transfer function where the coefficients \( d_{i} \) and \( d_{0} \) are proportional to \( m_{w}, b_{w} \) and \( k_{i} \).

\[
K_{\omega}(s) = \frac{n_{0}s^{4} + n_{1}s^{3} + n_{2}s^{2} + n_{3}s + n_{0}}{d_{i}s^{2} + d_{0}s} 
\]

(18)

Assuming an empirical scaling factor \( \theta \), estimate the analogous physical parameters as follows where \( \gamma \) is defined as an adjustment ratio that varies with \( X \). The \( \gamma \) ratio is used to fine tune factor \( \theta \) to achieve best possible curve-fit results.

\[
k_{i} = \frac{n_{0}}{d_{0}}, \quad m_{p} = \frac{n_{1}}{d_{2}}, \quad b_{i} = \frac{d_{1}m_{w} - n_{1}d_{2}}{d_{2}^{2}}, \quad b_{0} = d_{1}\left(\frac{n_{1}d_{2} - b_{i}}{d_{0}}\right) 
\]

(19a-h)

For the sake of illustration, parameters are estimated using (19) and listed in Table 6 for mount G. Moreover, \( \tilde{K}(f, X) \) measured at lower frequencies with \( X = 1 \) and 2 mm are also curve-fitted and parameters are estimated using (10) and (11). Figures 14 and 15 compare measured, curve-fitted and predicted \( \tilde{K}(f, X) \) spectra at lower and higher \( f \) respectively. Predictions match measurements well in both frequency regimes.

Several interesting conclusions can be drawn by examining the parameters of Table 6. First, the estimated value of \( m_{p} \) is comparable to \( m_{w} \), which confirms the necessity of including \( m_{p} \) in the \( K_{\omega}(s) \) expression of (19). This conclusion is confirmed by Figure 15 that shows that \( K_{\omega}(s) \) model cannot adequately predict the higher frequency resonance (around 150 Hz). Second, estimated value of \( k_{i} \) is around 730 N/mm. Thus the rubber is stiffer at higher \( f \) since its stiffness was estimated around 420 N/mm at lower \( f \). For the sake of verification, the fluid is drained out of the hydraulic mount and the cup-shaped primary rubber element is tested up to 300 Hz (in Figure 16) with \( X \) ranging from 0.5 to 1 mm. Measurements confirm

<table>
<thead>
<tr>
<th>TABLE 5—Mechanical system parameters of mount G curve-fitted from inertia track resonance at lower ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Symbols (units)</td>
</tr>
<tr>
<td>( k_{i} ) [N/mm]</td>
</tr>
<tr>
<td>( b_{i} ) [Nm/s]</td>
</tr>
<tr>
<td>( m_{w} ) [kg]</td>
</tr>
</tbody>
</table>

Consequently, the broadband performance of a hydraulic mount is determined by the characteristic equation, whose coefficients assume binary values depending on the \( f \) range as stated above. Two key natural frequencies (in Hz) thus emerge:

\[
f_{i} = \frac{A_{r}}{2\pi A_{i}} \sqrt{k_{i}} \quad \text{Inertia track resonance frequency (at lower \( f \))} \quad (16)
\]

\[
f_{d} = \frac{A_{w}}{2\pi A_{i}} \sqrt{k_{i}} \quad \text{Decoupler flow to piston area ratio (at higher \( f \))} \quad (17)
\]

This broadband model reveals the following “decoupling” working principles of the hydraulic mount. For an excitation at lower \( f \) with a large \( X \), the inertia track acts essentially as a dominant damper (tuned around \( f_{i} \); as \( q_{i} = 0 \)); it is intentionally designed as such. For an excitation at higher \( f \) with a small \( X \), the decoupler effectively short-circuits the inertia track (\( q_{i} \rightarrow 0 \)) and produces a resonance at \( f_{d} \). For an excitation with a moderate \( X \) close to the decoupling amplitude \( X_{d} \), the mount works in a combination of two states since \( q_{i} \neq 0 \) and \( q_{d} \neq 0 \). In this case, one must employ a true nonlinear model and solve the governing equation in time domain. Thus some caution must be exercised to identify its true nature before any quasi-linear estimation algorithm could be applied. Nevertheless, the dynamic behavior could still be roughly approximated with a quasi-linear model.

<table>
<thead>
<tr>
<th>TABLE 6—Mechanical system parameters at higher ( f ), estimated for mount G using the model of Fig. 10 and measured data of Fig. 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Symbols (units)</td>
</tr>
<tr>
<td>( k_{i} ) [N/mm]</td>
</tr>
<tr>
<td>( b_{i} ) [Nm/s]</td>
</tr>
<tr>
<td>( m_{w} ) [g]</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
</tbody>
</table>
which brings an additional stiffness into the system. An membrane does not completely decouple the inertia track, properties are indeed desirable from the vibration isolation perspective. Fourth, when  

This is because more fluid would flow through the inertia track under a higher \( X \) since less fluid remains around the decoupler in the top chamber. This also explains the “flip” characteristics of the higher frequency resonance \( (f_d) \) near 160 Hz as shown in Figure 17. Finally, the adjustment ratio \( \gamma \) is somewhat proportional to \( X \). Therefore, it could be used to quantify the amplitude dependence of the \( f_d \) value.

8 PREDICTION OF TIME DOMAIN RESPONSE GIVEN A REALISTIC EXCITATION AND EXPERIMENTAL VALIDATION

A realistic displacement profile \( x(t) \) was measured in a front wheel drive vehicle during the gear shift event. The transient record contains approximately three seconds of
data, and oscillatory displacements (from 7.5 to 13.5 Hz) are superimposed with a time-varying mean displacement $x_m(t)$ that increases from -4 to -10.5 mm. The $x'(t) = x_m(t) + \sum X \sin(2\pi f t + \phi_i)$ profile as shown in Figures 18 and 19 is applied to the elastomer test system to excite both fixed and free decoupler mounts (D); details of this experiment are beyond the scope of this article. In this particular case, the hydraulic mounts seem to essentially control the motion by providing additional inertia-augmented damping. Therefore, the quasi-linear model $K_{12}(s, X)$ of section 5 that was estimated from measured $\tilde{K}(f, X)$ is used to predict the transmitted force $F(t)$ to a realistic $x'(t)$. All amplitude dependent parameters are chosen according to the maximum oscillatory amplitude of $X = 1$ mm (p-p), which occurs immediately after the saw tooth-like waveform (around $t = 2.8$ sec in Figures 18 and 19). Therefore, the quasi-linear modes corresponding to $X = 1$ mm are employed to predict the transient responses of Figures 18 and 19. Better predictions could still be obtained by using a piecewise model whose parameters vary according to changes in $X$, as witnessed in $x'(t)$. Further, it is desirable to predict the dynamic pressure $p(t)$ in the upper chamber. Given the assumption of $k_i \gg k_m$, a simplified $P/X$ transfer function could be derived:

$$
\frac{P}{X}(s) = \frac{k_i}{A_n} \cdot \frac{m_n s^2 + b_n s}{m_n s^2 + b_n s + 1}
$$

(20)

It is seen that the mean (dc) term is not present in the numerator polynomial and thus $p(t) = 0$ at $f = 0$ since $C_{\infty} \rightarrow \infty$. All parameters of (20) other than $A_n$ could be easily estimated from the quasi-linear model, and $A_n$ could be either provided by the mount manufacturer or roughly measured given the cross-section of the mount. The resulting formulation is solved in time domain since the excitation consists of transients and steady state events. Simulations using equations (7) and (20) are compared with measurements in Figures 18 and 19 for fixed and free decouplers respectively. Predicted $F(t)$ and $p(t)$ seem to capture the overall behavior including the oscillations that are observed in measured responses. For the free decoupler mount, our predictions essentially constitute the linearized approximations of all nonlinear phenomena including the additional damping effects introduced by the decoupler switching action. Nonetheless, some discrepancies are observed in the mean responses ($F_{\text{mean}}(t)$) when the preload is increased to around 3000 N or beyond ($x_m \geq 9$ mm). This is because equations (7) and (20) are derived using the Laplace transform method, resulting in a possible dc bias. Further
a time-varying mean load; this work will be presented in a future article along with analytical predictions of some transient responses. Based on the theory presented in this article, an interactive simulation software has been developed that is already being used to examine or characterize some practical mounts. Finally, we intend to develop computational procedures that could incorporate quasi-linear or nonlinear mount formulations into large scale finite element or multi-body dynamic models of vehicles.

10 ACKNOWLEDGEMENT

We acknowledge the experimental efforts of M. Tiwari and J. Sorenson from 1999 to 2002 (on mount D). Those studies were financially supported by the Ford Motor Company. Gratitude is also expressed to J. Lopez Tellez of General Motors for mount G measurements.

11 REFERENCES


9 CONCLUSION

Chief contribution of this study has been the development of new estimation procedures that employ measured sinusoidal dynamic stiffness data of fixed or free decoupler mount over low and high frequency regimes to characterize amplitude and frequency dependent parameters. Compared with the previously reported laboratory experiments, our estimation method requires minimal experimental effort. It can be efficiently implemented by mount manufacturers or vehicle designers to quickly develop quasi-linear models and then predict responses to mount excitations with reasonable accuracy. The main limitation of the proposed method is that the predicted response is based on the linearized approximations around the operating points and all nonlinearities are characterized by operation-dependent parameters. Consequently, the estimated quasi-linear model should not be viewed as the true non-linear formulation and as such it might not capture all of the significant events in time domain. An example was shown where the pressure build-up effect, under a realistic excitation, was not predicted as shown in Figures 18 and 19. Better prediction would obviously require an improved nonlinear \( \dot{C}(p) \) model that should incorporate the multi-staged stiffness characteristics including the dynamic stiffening effect corresponding to

Fig. 19—\( F_x(t) \) and \( p(t) \) responses to a realistic displacement profile \( x(t) \) when applied to a free decoupler mount (D). Key: — Measurement, --- Prediction using quasi-linear model \( (X = 1 \text{ mm}) \)

discussion about the effect of such bias error and measurements is beyond the scope of this article.