Estimation of interfacial forces in a multi-degree of freedom isolation system using a dynamic load sensing mount and quasi-linear models

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A new indirect measurement concept is developed to estimate interfacial dynamic forces by employing the hydraulic mount as a dynamic force sensor. The proposed method utilizes a combination of mathematical models and operating motion and/or pressure measurements. A laboratory experiment consisting of a powertrain, three powertrain mounts (including a dynamic load sensing hydraulic mount), a sub-frame, and four bushings is then constructed to verify the proof-of-concept. Quasi-linear fluid and mechanical system models of the experiment are proposed and evaluated in terms of transfer functions and forced sinusoidal responses. The lower chamber pressure in the hydraulic mount is estimated since it is not available from measurements. This leads to an improved estimation of the effective rubber and hydraulic path parameters with spectrally varying and amplitude-sensitive properties up to 50 Hz. Finally, the reverse path spectral method is employed to predict interfacial forces at both ends of the mount by using measured motions and upper chamber pressure signals. Overall, the proposed quasi-linear fluid system model yields better indirect estimates of forces from the measured responses when compared with direct force measurements, through a simpler mechanical system model provides some insights. This work also advances prior component and transfer path type studies by providing an improved multi-degree of freedom system perspective.

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1. Introduction

Precise knowledge of the dynamic forces at sub-system junctions or interfaces is of vital interest in the dynamic and vibro-acoustic design of mechanical systems, vehicles, buildings, and power plants. In general, it is difficult to install force transducers at sub-system junctions without altering interfacial or boundary conditions unless there is a mobility mismatch. Thus, indirect measurement or force reconstruction methods must be adopted to estimate dynamic forces [1–9]. This has been the subject of several recent articles. For instance, Inoue et al. [1] and Gunduz et al. [2] have suggested new or improved transfer path methods to estimate forces in parallel structural paths of a discrete vibratory system. Yap and Gibbs [3] examined the forces at the machine–receiver interface by using the mobility method. Leclere et al. [4] assessed the internal loads on bearings with an inverse transfer function method. Carne et al. [5] utilized frequency response function data to indirectly estimate the input force. Jacquelin et al. [6] employed a deconvolution technique to
reconstruct the dynamic force. Tao et al. [7] identified the excitation force in the center of an engine using the velocity amplitude and phase at the mounting points. Liu and Shepard [8] compared the truncated singular value decomposition and the Tikhonov filter approaches used to enhance the inverse process. Lin and Chen [9] identified the contact stiffness and damping properties of mechanical interfaces. However, most of the available indirect or inverse force estimation methods are valid only for linear time-invariant systems since they employ transfer functions or similar concepts.

In this article, an inherently nonlinear hydraulic mount will be embedded into a multi-degree of freedom system and then utilized as a load sensing device to estimate interfacial forces (at both ends of the mount). In recent work from the component perspective [10,11], forces that are transmitted by a hydraulic mount to a rigid base have been successfully estimated by using linear, quasi-linear, and nonlinear models. In this paper, a system perspective is adopted and used to construct a laboratory experiment to examine the proof-of-concept. The scope of this article, however, is limited to the harmonic interfacial forces (in frequency or time domain) using measured or calculated motions and/or internal pressure signals. Recent articles by Gunduz et al. [2] and Yoon and Singh [10,11] provide a comprehensive review of the relevant literature on the identification of interfacial forces and hydraulic engine mount models, respectively. Additional reviews will be reported as the material is further developed.

2. Problem formulation

Underlying issues can be conceptually illustrated by the generic source–path–receiver system of Fig. 1. This is a modified version of the systems analyzed by Inoue et al. [1] and Gunduz et al. [2]. Here, multiple linear and nonlinear isolators (paths) are shown. Path I has two separate force paths: one path is given by a linear spring \( k_{ES1} \) and viscous damper \( c_{ES1} \); and the other path incorporates mass \( m_1 \) which is connected to the source and receiver by two linear springs \( k_{ES11} \) and \( k_{ES12} \) and dampers \( c_{ES11} \) and \( c_{ES12} \). Path II includes a nonlinear spring \( k_{ES2} \) and damper \( c_{ES2} \), along with a nonlinear fluid path designated by \( p_u(t) \). A linear spring and damper (designated as \( k_{EG} \) and \( c_{EG} \)) also support the source. The receiver is connected to the ground by four linear springs \( k_{SG1}, k_{SG2}, k_{SG3} \) and \( k_{SG4} \) and viscous dampers.

![Fig. 1. Schematic of nonlinear isolation system in the context of source, path and receiver network. Both paths are assumed to include hydraulic mounts, and thus have parallel force paths. In particular, path II is assumed to possess spectrally varying and amplitude-sensitive properties.](image-url)
For the sake of illustration, assume a linear time-invariant (LTI) system, and define interfacial forces in the time domain for path II as follows:

\[ f_{II}(t) = c_{ES2} \ddot{x}(t) + k_{ES2} \ddot{z}(t) + \text{Ar} p_u(t). \]  

(1)

\[ f_{IB}(t) = -c_{ES2} \ddot{z}(t) - k_{ES2} \ddot{x}(t) - \text{Ar} p_u(t). \]  

(2)

Here, \( f_{II}(t) \) and \( f_{IB}(t) \) are the interfacial forces on the source (subscript IT) and receiver (subscript IB) sides, respectively; \( A_r \) is the effective piston area in the fluid path; \( x_S(t) \) and \( x_D(t) \) are the displacements of the source and receiver, respectively; and \( \ddot{z}(t) = x_D(t) - x_S(t) \) is the relative displacement. By transforming Eqs. (1) and (2) into the Laplace domain (s) and assuming that the initial conditions are equal to zero, the path forces are:

\[ F_{IT}(s) = (c_{ES2} + k_{ES2}) \Xi(s) + A_r P_u(s). \]  

(3)

\[ F_{IB}(s) = -(c_{ES2} + k_{ES2}) \Xi(s) - A_r P_u(s). \]  

(4)

When the above equations are compared, the top and bottom forces (in path II) are identical except for the sign as long as there are no inertial elements in the fluid path. This is obviously not the case in many practical devices [12]. In order to estimate interfacial forces on both source and receiver sides, one must recognize and resolve some difficulties. First, the above equations require a precise knowledge of in-situ parameters such as nonlinear stiffness \( k_{ES2} \), damping \( c_{ES2} \) (in the rubber force path), and effective piston area \( A_r \) (in the hydraulic force path) [13,14]. Second, forces could be estimated by using measured motions and/or pressure signals, but then effective (dynamic) stiffness and damping parameters must be known a priori [15]. The latter poses a special difficulty for both elastomeric and hydraulic isolators [13]. For instance, hydraulic engine mounts exhibit spectrally varying and amplitude-sensitive parameters [16].

The specific objectives of this research are as follows: (1) construct and instrument a laboratory experiment corresponding to Fig. 1; one load sensing hydraulic mount will be embedded. The system is excited in the vertical direction by a steady-state sinusoidal force \( f_E(t) \) of frequency \( \omega_0 \), and motions in other directions are ignored; (2) conduct experiments under sinusoidal excitation and measure dynamic accelerations (at different points in the system), fluid pressure (in the top chamber of the load sensing mount), and forces (at selected interfaces); (3) develop 2 and 3 degree of freedom (dof) linear and quasi-linear models (with spectrally varying and amplitude-sensitive parameters as suggested in prior component studies [10,11,16]); this would include the determination of the effective parameters, including upper and lower chamber compliances and (4) estimate the interfacial forces at both ends of the load sensing hydraulic mount by employing mechanical and fluid models of the load sensing mount and compare them with direct force measurements in the frequency and time domains.

3. Experiment with a dynamic load sensing hydraulic mount

Fig. 2 illustrates the laboratory experimental setup with powertrain (assembly of engine and transmission) and sub-frame; the setup is based on the generic multi-degree of freedom isolation system of Fig. 1. The powertrain is connected to the sub-frame by two hydraulic mounts and is supported by a grounded third rubber mount. The sub-frame is supported by four identical elastomeric bushings. The experiment is excited by an electrodynamic shaker located on the powertrain. A signal generator is used to generate multiple excitations. One piezoelectric force transducer located between the shaker and the powertrain measures the excitation force \( f_E(t) \); it is also used as a reference signal. Here, the excitation force is expressed as \( f_E(t) = \text{Re}[\tilde{F}_E e^{j\omega t}] \), where \( \tilde{F}_E = F_E e^{j\omega t} \) is the complex valued excitation amplitude, \( F_E \) is the amplitude of force,
\( \phi_E \) is the phase of excitation force \( f_E(t) \), \( \omega_o \) is the excitation (fundamental) frequency (rad s\(^{-1}\)), and \( \text{Re}[\cdot] \) is the real value operator; a tilde over a symbol implies that is complex valued. The scope of the experiment is limited to 50 Hz with five different excitation amplitudes (1, 10, 50, 100, and 150 N).

The following sensors are employed as illustrated in Fig. 2: two piezoelectric accelerometers located on top of two hydraulic mounts and two accelerometers placed on the sub-frame near the hydraulic mounts; two piezoelectric force transducers located between the hydraulic mounts and the sub-frame for directly measuring the interfacial forces; a piezoelectric force transducer located on top of the load sensing hydraulic mount to directly measure the interfacial forces on the source side; and one piezoelectric pressure transducer within the upper chamber of the load sensing hydraulic mount.

### 4. Development of a linear time-invariant (LTI) mechanical system model

Analogous 2- and 3dof mechanical system models of the experiment are shown in Fig. 3. Voigt models are used to describe two hydraulic mounts, one rubber mount, and bushings. Here, the following symbols are designated: \( m_E \), mass of the powertrain; \( m_S \), mass of the sub-frame; \( c_{HF} \), viscous damping coefficient of the hydraulic mount at the front side; \( c_{HR} \), viscous damping coefficient of the dynamic load sensing hydraulic mount; \( m_{ie} \), effective mass of the inertia track column which is estimated from the fluid system model of the hydraulic mount; \( x_{ie}(t) \), effective velocity of the inertia track fluid; \( k_u \) and \( k_l \), effective stiffness of the upper and lower chambers, respectively; \( k_{Rr} \) and \( c_{Rr} \), rubber stiffness and viscous damping coefficients of the dynamic load sensing hydraulic mount, respectively; \( c_R \), viscous damping coefficient of the rubber mount; and \( k_B \) and \( c_B \), stiffness and viscous damping coefficients of the bushings, respectively. The dynamic load sensing hydraulic mount is described only by stiffness and viscous damping elements in the 2dof model. On the other hand, the 3dof model includes effective mechanical properties (including fluid inertia) as derived from the fluid parameters \([12,17,18]\).

Assume that both systems of Fig. 3 are linear time-invariant (LTI). First, the 2dof model (with subscript 2) is described below. Here, \( x_2(t) \) and \( f_2(t) \) are the displacement and excitation force vectors, respectively, and \( M_2, K_2, \) and \( C_2 \) are the mass, stiffness, and damping matrices.

\[
M_2 \ddot{x}_2(t) + C_2 \dot{x}_2(t) + K_2 x_2(t) = f_2(t), \tag{5}
\]

\[
M_2 = \begin{bmatrix}
m_E & 0 \\
0 & m_S
\end{bmatrix}, \quad K_2 = \begin{bmatrix}
k_R + k_{ES2} & -k_{ES2} \\
-k_{ES2} & k_{ES2} + k_B
\end{bmatrix}, \quad C_2 = \begin{bmatrix}
c_R + c_{ES2} & -c_{ES2} \\
-c_{ES2} & c_{ES2} + c_B
\end{bmatrix}. \tag{6a-c}
\]

\[
x_2(t) = \begin{bmatrix}
x_E(t) \\
x_S(t)
\end{bmatrix}, \quad f_2(t) = \begin{bmatrix}
\text{Re}[f_E e^{i\omega_o t}] \\
0
\end{bmatrix}. \tag{7a,b}
\]

Here, \( k_{ES2} = k_{HF} + k_{HR} \); \( c_{ES2} = c_{HF} + c_{HR} \). Similarly, the 3dof model (with subscript 3) is derived.

\[
M_3 \ddot{x}_3(t) + C_3 \dot{x}_3(t) + K_3 x_3(t) = f_3(t), \tag{8}
\]
5. Development of a quasi-linear (QL) mechanical system model

An interrelationship between the interfacial forces \( \tilde{F}_{IT}(\omega) \) and \( \tilde{F}_{IB}(\omega) \) and internal pressure \( \tilde{P}_u(\omega) \) can be observed from the spectral contents in Fig. 4 with respect to the fundamental and super-harmonic terms. Thus, a quasi-linear (QL) model with effective parameters must be developed to successfully predict the interfacial forces. Two analogous mechanical system models, as illustrated in Fig. 3, will be employed to develop such models. Fig. 3(a) shows that the 2dof model includes the Voigt formulation in terms of \( k_{HR} \) and \( c_{HR} \) elements to represent the load sensing hydraulic mount. However, the 3dof model includes effective mechanical parameters \( k_i \), \( k_t \), and \( c_{ie} \) that effectively describe the internal fluid system.

First, the 2dof model (with subscript 2) is used to define the interfacial forces on both the top and bottom ends of the load sensing hydraulic mount as follows:

\[
f_{IT}^2(t) = c_{HR} \dot{x}_T(t) + k_{HR} x_T(t),
\]

\[
f_{IB}^2(t) = -c_{HR} \dot{x}_B(t) - k_{HR} x_B(t).
\]

Here, \( \dot{x}_T(t) - x_T(t) \) is the relative displacement between \( x_T \) and \( x_B \). By transforming Eqs. (11) and (12) into the Laplace domain \( (s) \) with the assumption that the initial conditions are equal to zero, the interfacial forces are:

\[
F_{IT2}(s) = (c_{HR}s + k_{HR})\tilde{\xi}(s),
\]

\[
F_{IB2}(s) = -(c_{HR}s + k_{HR})\Xi(s).
\]

Fig. 4. Comparison of spectral contents given \( f_L(t) = \text{Re}[\tilde{F}_L e^{it \omega}] \) at \( \omega/2\pi = 23 \text{ Hz} \) with \( |F_L| = 100 \text{ N} \).
Likewise, the interfacial forces with the 3dof model (with subscript 3) are calculated as follows:

\[
\begin{align*}
    \text{f}_{f_3}(t) &= c_{HR} \ddot{x}_3(t) + k_{HR} \ddot{x}_3(t) + k_{u3}(t), \\
    \text{f}_{i_3}(t) &= -c_{HR} \ddot{x}_3(t) - k_{HR} \ddot{x}_3(t) - k_{i3}(t), \\
    \text{f}_{T_3}(s) &= (c_{HR} + k_{HR}) \Xi(s) + k_u \Xi(s), \\
    \text{f}_{B_3}(s) &= -(c_{HR} + k_{HR}) \Xi(s) - k_i \Xi(s).
\end{align*}
\]

Here, the relative displacements are defined as: \( \ddot{x}_u(t) = x_u(t) - x_o(t) \) and \( \ddot{x}_l(t) = x_o(t) - x_d(t) \). From Eqs. (15)–(18), the 3dof model shows that forces \( f_{f_3}(t) \) and \( f_{i_3}(t) \) differ by \( k_{u3}(t) \) and \( k_{i3}(t) \), which signify different relative displacements and effective stiffness values. Thus, the fluid element displacement \( x_u(t) \) must be predicted to estimate \( f_{f_3}(t) \) and \( f_{i_3}(t) \). The relationship between \( x_u(t) \) and other known variables such as measured signals (with subscript \( M \)), including displacements \( x_{EM}(t) \) and \( x_{SM}(t) \) and force \( f_{EM}(t) \), can be derived from Eqs. (5)–(10). Transforming Eqs. (5)–(10) into the Laplace domain and ignoring initial conditions:

\[
\begin{align*}
    [m_E \ddot{x}_E + (c_R + c_{HF} + c_R) s + (k_R + k_{HF} + k_R + k_o)] \Xi(s) = &-[4(c_R + c_{HF}) + (k_R + k_{HF})] \Xi(s) - k_u \Xi(s) = F_E(s), \\
    [m_S \ddot{x}_S + (c_R + c_{HF} + c_R) s + (k_R + k_{HF} + k_R + k_o)] \Xi(s) = &-[4(c_R + c_{HF}) + (k_R + k_{HF})] \Xi(s) - k_u \Xi(s) = 0.
\end{align*}
\]

From Eqs. (17)–(20), the interfacial forces \( F_{f_3}(s) \) and \( F_{i_3}(s) \) are derived as follows:

\[
\begin{align*}
    F_{T_3}(s) &= F_E(s) + (c_{HF} + k_{HF}) \Xi(s) - [m_E \ddot{x}_E + (c_R + c_{HF}) s + (k_R + k_{HF})] \Xi(s), \\
    F_{B_3}(s) &= (c_{HF} + k_{HF}) \Xi(s) - [m_E \ddot{x}_E + (c_R + c_{HF}) s + (k_R + k_{HF})] \Xi(s).
\end{align*}
\]

Next, the interfacial forces in the frequency domain can be calculated by replacing \( s \) with \( j \omega \) in the above equations. Thus, interfacial forces with the 2dof model in the frequency domain are as follows:

\[
\begin{align*}
    \tilde{F}_{T_2}(\omega) &= (i \omega c_{HR} + k_{HR}) \Xi(\omega), \\
    \tilde{F}_{B_2}(\omega) &= -(i \omega c_{HR} + k_{HR}) \Xi(\omega).
\end{align*}
\]

Likewise, the interfacial forces with the 3dof model are described below:

\[
\begin{align*}
    \tilde{F}_{T_3}(\omega) &= \tilde{F}_E(\omega) + (i \omega c_{HR} + k_{HR}) \tilde{X}_3(\omega) - [\omega^2 m_E + i \omega (c_R + c_{HF}) + (k_R + k_{HF})] \tilde{X}_3(\omega), \\
    \tilde{F}_{B_3}(\omega) &= (i \omega c_{HF} + k_{HF}) \tilde{X}_3(\omega) - [\omega^2 m_E + i \omega (c_R + c_{HF}) + (k_R + k_{HF})] \tilde{X}_3(\omega).
\end{align*}
\]

Here, the displacements \( \tilde{X}_E(\omega) \) and \( \tilde{X}_3(\omega) \) are calculated directly from the measured accelerations \( \Psi_{EM}(\omega) \) and \( \Psi_{SM}(\omega) \) by the relationships \( \tilde{X}_{EM}(\omega) = -\Psi_{EM}(\omega)/\omega^2 \) and \( \tilde{X}_{SM}(\omega) = -\Psi_{SM}(\omega)/\omega^2 \) by avoiding double integration in the time domain. Thus, the measured relative displacement \( \tilde{X}_M(\omega) = \tilde{X}_{EM}(\omega) - \tilde{X}_{SM}(\omega) \) is calculated by \( \tilde{X}_M(\omega) = -\Psi_{EM}(\omega)/\omega^2 + \Psi_{SM}(\omega)/\omega^2 \). Fig. 5 shows the measured \( \tilde{X}_M(\omega) \) in the frequency domain under the sinusoidal excitation force \( f_{EM}(t) = \text{Re}[F_E e^{i \omega t}] \) with \( |F_E| = 100N. \)

Recall that Eqs. (19) and (20) are based upon the linear time-invariant (LTI) system concept, and thus their stiffness \( k_{HF}, k_R, k_{HF}, k_R \) and viscous damping coefficients \( c_{HR}, c_{HF}, c_R \) are assigned constant values. To develop quasi-linear models for a 2- or 3dof mechanical system, spectrally varying parameters must be included. Fig. 6 shows measured stiffness and viscous damping coefficient values of hydraulic and rubber mounts from the non-resonant dynamic stiffness testing procedure under the ISO standard 10846 [19]. Though such empirical data (typically supplied by the mount vendors) are limited in scope, some useful information can still be gained. Therefore, the quasi-linear model is developed by employing the following steps: (1) start with mechanical system models where the interfacial forces are derived from the LTI system formulation; (2) embed spectrally varying stiffness and damping elements in terms of \( k_{uM}(\omega) \) and \( c_{uM}(\omega) \) (where \( w \) is the mount index for \( HF, HR, R, \) and \( B \) ) as shown in Fig. 6 into the frequency domain formulation; (3) employ measured motions \( \tilde{X}_{EM}(\omega) \) and \( \tilde{X}_{SM}(\omega) \) in Eqs. (23) and (24) for the 2dof model and Eqs. (25) and (26) for 3dof model; and (4) calculate the time histories of the interfacial forces by using the complex exponential form as shown below. Note that this includes only the fundamental (excitation) frequency term as follows:

\[
\begin{align*}
    f_{f_2}(t) &= |\tilde{F}_{f_2}(\omega)| \text{Re}[e^{i \omega t + \varphi_{f_2}}], \\
    \varphi_{f_2} &= \angle \tilde{F}_{f_2}(\omega) \quad (v = 2 \text{dof or 3dof}), \\
    f_{i_2}(t) &= |\tilde{F}_{i_2}(\omega)| \text{Re}[e^{i \omega t + \varphi_{i_2}}], \\
    \varphi_{i_2} &= \angle \tilde{F}_{i_2}(\omega) \quad (v = 2 \text{dof or 3dof}).
\end{align*}
\]
6. Development of a quasi-linear (QL) fluid system model

The linear system models for mounts were initially developed by Singh et al. [20]. Kim and Singh [21,22] and Tiwari et al. [17] measured nonlinear inertia track resistances and upper and lower chamber compliances using laboratory
These properties were utilized by Adiguna et al. [18] and He and Singh [12] to predict steady state and transient responses. Other hydraulic mount studies include work by Shangguan and Lu [23], Fan and Lu [24], Truong and Ahn [25], Geisberger et al. [26], Mrad and Levitt [27], and Lee and Moon [28]. In particular, this article extends prior studies by Yoon and Singh [10,11] and focuses on in-situ path forces in the context of Fig. 1. The interfacial forces will be divided into rubber and hydraulic force paths, and the quasi-linear model will be developed by embedding the spectrally varying and amplitude-sensitive parameters and by using measured displacement and upper chamber pressure signals.

Fig. 7 illustrates the fluid system model of the dynamic load sensing hydraulic mount (fixed decoupler type). Here, $f_{Ir}(t)$ and $f_{Ih}(t)$ are the top and bottom sides of the rubber path forces, respectively, and $f_{It}(t)$ and $f_{Ib}(t)$ are the top and bottom sides of the hydraulic path forces, respectively. The rubber forces are modeled using $k_{Rr}$ and $c_{Rr}$ elements, which are assumed to be identical on the top and bottom sides of the hydraulic mount. However, the hydraulic path force is assumed to be asymmetric due to the nonlinearities from the fluid parameters. The designation of the fluid parameters in Fig. 7(b) are as follows: $p_u(t)$ and $p_l(t)$ are the upper and lower chamber pressures, respectively; $C_u$ and $C_l$ are the upper (#u) and lower (#l) chamber compliances, respectively; $I_i$ is the inertance of the inertia track (#i); $R_i$ is the resistance of the inertia track; and $q_i$ is the fluid flow through inertia track. The interfacial forces for both the top and bottom sides of the mount are derived with the following sign conventions: $p_u(t)$ and $p_l(t)$ are positive (in compression) corresponding to the upward (positive) motion of $x_E(t)$, $x_S(t)$, $q_i(t)$, $f_{It}(t)$, and $f_{Ib}(t)$; and $p_u(t)$ and $p_l(t)$ are negative (in expansion) for the downward motion of $x_E(t)$, $x_S(t)$, $q_i(t)$, $f_{It}(t)$, and $f_{Ib}(t)$.

$$f_{It}(t) = f_{Ir}(t) + f_{Ith}(t), \quad (31)$$
$$f_{Ib}(t) = f_{Ir}(t) + f_{Ibh}(t), \quad (32)$$
$$f_{Ir}(t) = c_{Rr} \dot{x}(t) + k_{Rr} x(t), \quad (33)$$
$$f_{Ith}(t) = -2A_p p_u(t) + A_p p_l(t), \quad (34)$$
$$f_{Ibh}(t) = -A_p p_u(t) + 2A_p p_l(t). \quad (35)$$

Here, the relative displacement $\xi(t) = x_E(t) - x_S(t)$. Also, the momentum and continuity equations for the hydraulic path with respect to $\xi(t)$ are as follows [17,18,20]:

$$p_u(t) - p_l(t) = I_q(t) + R_q(q_i(t), \quad (36)$$
$$C_u \dot{p}_u(t) = A_q \dot{\xi}(t) - q_i(t), \quad (37)$$
$$C_l \dot{p}_l(t) = q_i(t). \quad (38)$$

Transform Eqs. (31)-(38) into the Laplace domain ($s$) and ignore the initial conditions:

$$F_{It}(s) = F_{Ir}(s) + F_{Ith}(s), \quad (39)$$
$$F_{Ib}(s) = F_{Ir}(s) + F_{Ibh}(s). \quad (40)$$
Several upper and lower chamber pressure transfer functions \( (G_1 - G_3) \) are determined from Eqs. (44)–(46), as expressed below along with the fluid system parameters

\[
G_1(s) = \frac{P_u}{\Delta p_u} = \frac{A_r}{C_u + C_1} \left( \frac{s^2}{\omega_{N1}^2} + \frac{2 \zeta_1 \omega_{N1}}{\omega_{N1}} + 1 \right),
\]

\[
G_2(s) = \frac{P_f}{\Delta p_u} = \frac{1}{(s^2/(\omega_{N1}^2) + (2 \zeta_1 \omega_{N1}) s + 1)},
\]

\[
G_3(s) = \frac{P_l}{\Delta p_u} = \frac{A_r}{C_u + C_1} \left( \frac{s^2}{\omega_{N2}^2} + \frac{2 \zeta_1 \omega_{N2}}{\omega_{N2}} + 1 \right),
\]

\[
\zeta_1 = \frac{1}{2} \sqrt{\frac{C_r R_z}{I_t}}, \quad \omega_{N1} = \sqrt{\frac{1}{C_{138}}},
\]

\[
\zeta_2 = \frac{1}{2} \sqrt{\frac{C_r R_z}{I_t(C_u + C_1)}}, \quad \omega_{N2} = \sqrt{\frac{C_u + C_1}{C_u C_1 I_t}}
\]

Eqs. (34), (35), (42), and (43) reveal asymmetrical dynamic characteristics since the hydraulic path forces \( f_{ITM}(t) \) and \( f_{IBM}(t) \) have different values. The following nominal (and constant) parameters are incorporated for calculations: \( k_r = 2 \times 10^8 \text{ N m}^{-1} \); \( c_r = 496.1 \text{ N s m}^{-1} \); \( l = 4 \times 10^8 \text{ kg m}^{-2} \); \( C_u = 2.5 \times 10^{-11} \text{ m}^5 \text{ N}^{-1} \); \( C_1 = 2.4 \times 10^{-9} \text{ m}^5 \text{ N}^{-1} \); and \( R_z = 2 \times 10^8 \text{ N s m}^{-2} \). Measured motion \( \dot{z}_{im}(t) \) and upper chamber pressure \( p_{im}(t) \) are employed to predict the interfacial forces \( f_{IT}(t) \) and \( f_{IB}(t) \). However, the lower chamber pressure spectrum \( P_l(o, \Omega) \) is still unknown. Accordingly, it must be estimated by using Eqs. (39)–(43). First, by replacing \( s \) with \( io \) in Eqs. (39)–(43), the equations in the frequency domain are expressed in terms of spectrally varying and amplitude-sensitive properties as follows:

\[
\dot{F}_{IT}(o, \Omega) = (io c_r + k_r) \dot{z}(o) - 2 A_r \dot{P}_u(o, \Omega) + A_r \dot{P}_l(o, \Omega),
\]

\[
\dot{F}_{IB}(o, \Omega) = (io c_r + k_r) \dot{z}(o) - A_r \dot{P}_u(o, \Omega) + 2 A_r \dot{P}_l(o, \Omega).
\]

By subtracting Eq. (55) from (54), the lower chamber pressure \( \dot{P}_l(o, \Omega) \) is derived as follows:

\[
A_r \dot{P}_l(o, \Omega) = -A_r \dot{P}_u(o, \Omega) - \dot{F}_{IT}(o, \Omega) + \dot{F}_{IB}(o, \Omega).
\]

Second, the measured data set \( \dot{F}_{ITM}(o, \Omega) \) and \( \dot{F}_{IBM}(o, \Omega) \) from the laboratory experiment can be used to estimate \( \dot{P}_l(o, \Omega) \), in the complex valued form as follows:

\[
\dot{F}_{IT} = \dot{F}_{ITRE} + i \dot{F}_{ITIM}, \quad \varphi_{IT} = \angle \dot{F}_{ITM}(o, \Omega),
\]

\[
\dot{F}_{ITRE} = \text{Re}[\dot{F}_{IT}] = |\dot{F}_{IT}| \cos(\varphi_{IT}),
\]

\[
\dot{F}_{ITIM} = \text{Im}[\dot{F}_{IT}] = |\dot{F}_{IT}| \sin(\varphi_{IT}),
\]

\[
\dot{F}_{IB} = \dot{F}_{IBRE} + i \dot{F}_{IBIM}, \quad \varphi_{IB} = \angle \dot{F}_{IBM}(o, \Omega),
\]

\[
\dot{F}_{IBRE} = \text{Re}[\dot{F}_{IB}] = |\dot{F}_{IB}| \cos(\varphi_{IB}),
\]

\[
\dot{F}_{IBIM} = \text{Im}[\dot{F}_{IB}] = |\dot{F}_{IB}| \sin(\varphi_{IB}),
\]

\[
\dot{P}_{um} = P_{ure} + i P_{um}, \quad \varphi_{Pum} = \angle \dot{P}_{um}(o, \Omega),
\]

\[
P_{ure} = \text{Re}[\dot{P}_{um}] = |\dot{P}_{um}| \cos(\varphi_{Pum}).
\]
\[ \bar{P}_{LM}(\omega, \Xi) = \frac{\bar{F}_{M}}{\bar{C}} \cos(\phi_{PM}) \]

By substituting Eqs. (57)–(65) into Eq. (56), the real and imaginary parts of \( \bar{P}_{LM}(\omega, \Xi) \) are calculated as

\[ P_{RE} = -P_{IM} + \frac{1}{A_{r}} (-F_{IM} + F_{RE}). \quad \text{and} \]

\[ P_{IM} = -P_{RE} + \frac{1}{A_{r}} (-F_{RE} + F_{IM}). \]

Fig. 8 compares estimated \( \bar{P}_{LM}(\omega, \Xi) \) measured \( \bar{P}_{LM}(\omega, \Xi) \) spectra. Note that the \( \bar{P}_{LM}(\omega, \Xi) \) spectrum seems to be similar in terms of dynamic behavior and its range of magnitudes. This means that the load sensing hydraulic mount is also affected by the lower chamber dynamic pressure \( \bar{P}_{R}(\omega, \Xi) \). Next, the effective stiffness and damping coefficient parameters in terms of \( k_{RE}(\omega, \Xi) \) and \( c_{RE}(\omega, \Xi) \), respectively, are estimated from the calculated \( \bar{P}_{LM}(\omega, \Xi) \) as follows:

\[ \bar{F}_{RE}(\omega, \Xi) = (i\omega c_{RE} + k_{RE})\bar{Z}(\omega) = F_{IM} + iF_{RE} + 2A_{r}(P_{RE} + iP_{IM}) - A_{r}(P_{RE} + iP_{IM}). \]

\[ k_{RE}(\omega, \Xi) = F_{IM} + A_{r}(2P_{RE} - P_{IM}). \]

\[ c_{RE}(\omega, \Xi) = \frac{1}{A_{r}} (F_{IM} + A_{r}(2P_{RE} - P_{IM})). \]

Fig. 9 shows the results from Eqs. (69) and (70) by using the measured \( \bar{Z}(\omega, \Xi) \) and \( \bar{P}_{LM}(\omega, \Xi) \). Results show that \( k_{RE}(\omega, \Xi) \) and \( c_{RE}(\omega, \Xi) \) not only include the dynamics of the component itself but also some coupling effects from the sub-system responses, since three resonances are observed in \( k_{RE}(\omega, \Xi) \). Note that \( c_{RE}(\omega, \Xi) \) has the highest value around 11 Hz. Finally, both effective lower and upper chamber compliances \( \bar{C}_{L}(\omega, \Xi) \) and \( \bar{C}_{U}(\omega, \Xi) \), respectively, are
estimated. The $\tilde{C}_{le}(\omega_o, \Xi_M)$ term is expressed in the complex valued form as follows:

$$\tilde{C}_{le}(\omega_o, \Xi_M) = C_{ln} \tilde{\lambda}(\omega_o, \Xi_M) = C_{ln}(\alpha_{pl} + i\beta_{pl}).$$

(71)

Here, $C_{ln}$ is the nominal value of $C_l$, and $\tilde{\lambda}(\omega_o, \Xi_M)$ is the spectrally varying and amplitude-sensitive parameter for $C_l$ with the coefficients $\alpha_{pl}$ and $\beta_{pl}$. From Eq. (48) by replacing $s$ with $i\omega$ and employing Eqs. (71)–(73), the coefficients $\alpha_{pl}$ and $\beta_{pl}$ are estimated as follows:

$$\frac{P_{RE} + iP_{IM}}{P_{RE} + iP_{IM}} = \frac{1}{1 - \omega^2 I Cl\alpha_{pl} + i\beta_{pl}} + i\omega R Cl\alpha_{pl} + i\beta_{pl},$$

(74)

$$\alpha_{pl} = \frac{\alpha_{pl2}\omega_o^2 + \alpha_{pl1}\omega_o}{d_{ct}(\omega_o^2 + R_o^2 \omega_o^2)}, \quad \beta_{pl} = \frac{\beta_{pl2}\omega_o^2 + \beta_{pl1}\omega_o}{d_{ct}(\omega_o^2 + R_o^2 \omega_o^2)}.$$

(75,76)

$$d_{ct} = C_{ln}(P_{RE}^2 + P_{IM}^2).$$

(77)

$$\alpha_{pl2} = I(P_{RE}^2 + P_{IM}^2 - P_{RE}P_{RE} - P_{IM}P_{IM}),$$

(78)

$$\alpha_{pl1} = R(-P_{RE}P_{IM} + P_{IM}P_{RE}),$$

(79)

$$\beta_{pl2} = I(P_{RE}P_{IM} - P_{IM}P_{RE}).$$

(80)
\[ \beta_{pl1} = R_f \left( P_{RE}^2 + P_{IM}^2 - P_{IM} \beta_{RE} - P_{IM} \beta_{IM} \right). \]  

(81)

The \( \tilde{C}_{uu}(\omega, \Xi_M) \) term is also estimated by substituting \( i\omega \) for \( s \) with Eq. (49) and employing the results \( \tilde{C}_{uu}(\omega, \Xi_M) \) from Eqs. (71)-(81). Next \( \tilde{C}_{uu}(\omega, \Xi_M) \) is defined in terms of an empirical spectrally varying and amplitude-sensitive parameter \( \lambda_a(\omega, \Xi_M) \) along with coefficients \( \gamma_{pu} \) and \( \beta_{pu} \):  

\[ \tilde{C}_{uu}(\omega, \Xi_M) = C_{uu}(\gamma_{pu} + i\beta_{pu}), \]  

(82)

\[ \gamma_{pu} = \gamma_u(\omega, \Xi_M), \]  

(83)

\[ \beta_{pu} = \beta_u(\omega, \Xi_M), \]  

(84)

\[ \rho = |\tilde{Z}_M(\omega)|, \]  

(85)

\[ P_{RE1} = |\tilde{P}_{IM}| \cos(\varphi_{pu1} - \varphi_z), \]  

(86)

\[ P_{IM1} = |\tilde{P}_{IM}| \sin(\varphi_{pu1} - \varphi_z). \]  

(87)

Here, \( \tilde{P}_{RE}(\omega, \Xi_M) \) is calibrated by using \( \tilde{Z}_M(\omega) \) as a reference value for the sake of deriving effective values  

\[ \frac{P_{RE1} + iP_{IM1}}{\rho} = A_r \left[ 1 - i\omega^2 \tilde{C}_{uu} \lambda + i\omega \tilde{C}_{uu} \tilde{R}_f \right]. \]  

(88)

Fig. 10 compares the spectral contents of \( \tilde{C}_{uu}(\omega, \Xi_M) \) and \( \tilde{C}_{uu}(\omega, \Xi_M) \). When the ranges of \( |\tilde{C}_{uu}| \) and \( |\tilde{C}_{uu}| \) are compared, their magnitudes are similar, but the phase of \( \tilde{C}_{uu}(\omega, \Xi_M) \) shows a significant dynamic phenomenon above 20 Hz. Thus, the lower chamber dynamics must be modeled under in-situ conditions.

Overall, the QL model with effective fluid parameters \( k_{rel}(\omega, \Xi) \), \( c_{rel}(\omega, \Xi) \), \( \tilde{C}_{uu}(\omega, \Xi) \) and \( \tilde{C}_{uu}(\omega, \Xi) \) is developed based on the reverse path spectral method [29,30] as illustrated in Figs. 11 and 12. Fig. 11 describes the procedure used to determine effective parameters from relevant transfer functions \( G_1(\omega, \Xi) \) and \( G_2(\omega, \Xi) \) and dynamic stiffness \( \tilde{K}_g(\omega, \Xi) \). Note that \( \tilde{P}_{IM}(\omega, \Xi_M) \) is predicted first using \( \tilde{F}_{IT}(\omega, \Xi_M) \) and \( \tilde{F}_{IB}(\omega, \Xi_M) \). Then \( k_{rel}(\omega, \Xi_M) \) and \( c_{rel}(\omega, \Xi_M) \) are identified as illustrated in Fig. 11(a). Fig. 11(b) shows the procedure to determine \( \tilde{C}_{uu}(\omega, \Xi_M) \) and \( \tilde{C}_{uu}(\omega, \Xi_M) \) by employing \( G_1(\omega, \Xi) \) and \( G_2(\omega, \Xi) \). Therefore, the interfacial forces are estimated as suggested by Eqs. (54) and (55)  

\[ \tilde{F}_{IT}(\omega, \Xi) = \tilde{F}_{IT}(\omega, \Xi) + \tilde{F}_{IT}(\omega, \Xi), \]  

(105)

\[ \tilde{F}_{IB}(\omega, \Xi) = \tilde{F}_{IB}(\omega, \Xi) + \tilde{F}_{IB}(\omega, \Xi), \]  

(106)

\[ \tilde{F}_{IR}(\omega, \Xi) = \tilde{K}_g(\omega, \Xi) \tilde{Z}(\omega), \]  

(107)

\[ \tilde{K}_g(\omega, \Xi) = i\omega c_{rel}(\omega, \Xi) + k_{rel}(\omega, \Xi). \]  

(108)

Here, \( \tilde{F}_{IR}(\omega, \Xi) \) is the rubber path force with \( \tilde{K}_g(\omega, \Xi) \) as its dynamic stiffness, and \( \tilde{F}_{IT}(\omega, \Xi) \) and \( \tilde{F}_{IB}(\omega, \Xi) \) are the hydraulic path forces on the top and bottom sides of the hydraulic mount, respectively. The hydraulic path forces \( \tilde{F}_{IT}(\omega, \Xi) \) and \( \tilde{F}_{IB}(\omega, \Xi) \) are...
and \( \tilde{F}_{\text{th}}(\omega, \Xi) \) may be estimated by using two different transfer functions as derived below. First, consider the force transmissibility concept:

\[
\tilde{F}_{\text{th}}(\omega, \Xi) = A_r \left[ \tilde{G}_2(\omega, \Xi) - 2 \\tilde{P}_{\text{ue}}(\omega, \Xi) \right],
\]

(109)

Second, consider the dynamic stiffness concept:

\[
\tilde{F}_{\text{th}}(\omega, \Xi) = A_r \left[ -2 \tilde{G}_1(\omega, \Xi) + \tilde{G}_3(\omega, \Xi) \right] \Xi(\omega),
\]

(111)
\[
\tilde{F}_{th}(\omega, \Xi) = A_3[-\tilde{G}_1(\omega, \Xi) + 2\tilde{G}_3(\omega, \Xi)]\tilde{\Xi}(\omega).
\]  

(112)

Fig. 12(a) and (b) compare both force estimation methods. The dynamic stiffness concept as illustrated in Fig. 12(b) is simpler than the force transmissibility concept since it needs only measured displacement \(\tilde{\Xi}_m(\omega_o)\). Further, the method for both force transmissibility and dynamic stiffness concepts, as illustrated in Fig. 12(a), needs two sets of measured data, \(\tilde{\Xi}_m(\omega_o)\) and \(\tilde{P}_{\text{ad}M}(\omega_o, \Xi_M)\). However, the number of effective parameters is reduced compared with the case of Fig. 12(b). From the frequency domain results, the time histories of interfacial forces with the fluid system model are estimated as

[Diagram of force estimation in frequency domain using the reverse path spectral method: (a) force transmissibility \((\tilde{G}_2e^{i\omega t})\) and dynamic stiffness \((\tilde{K}_{Re})\) concepts; and (b) dynamic stiffness \((\tilde{G}_1e^{i\omega t}, \tilde{G}_3e^{i\omega t} \text{ and } \tilde{K}_{Re})\) concept.]

[Graph showing comparison between measured and predicted interfacial force spectra \(\tilde{F}_\text{fit}(\omega_o, \Xi_M)\) using mechanical or fluid system model, given \(f(t) = \text{Re}[F_2 e^{i\omega t}]\) with \(|F_2| = 100\text{N}\), and by employing measured \(\tilde{\Xi}_m(\omega_o)\) and \(\tilde{P}_{\text{ad}M}(\omega_o, \Xi_M)\). Key: – – , measured; – , 2dof mechanical model; – – , 3dof mechanical model; – – – , fluid system model.]
follows:

\[ f_{IT}(t) = \|\hat{F}_{IT}(\omega_o, \Xi_M)\| \Re\{e^{i(\omega_o t + \varphi_{IT})}\}, \quad \varphi_{IT} = \angle \hat{F}_{IT}(\omega_o, \Xi_M), \]  

(113,114)

\[ f_{IB}(t) = \|\hat{F}_{IB}(\omega_o, \Xi_M)\| \Re\{e^{i(\omega_o t + \varphi_{IB})}\}, \quad \varphi_{IB} = \angle \hat{F}_{IB}(\omega_o, \Xi_M). \]  

(115,116)

7. Experimental validation of quasi-linear (QL) models in frequency and time domains

Figs. 13 and 14 compare measured and predicted interfacial forces by using both the mechanical and fluid system QL models described in Sections 5 and 6. As observed in Figs. 13 and 14, the fluid model predicts the interfacial forces better than the 2- or 3dof mechanical model. Specifically, the fluid model reflects the asymmetrical characteristics well. Observe that \( \hat{F}_{IT}(\omega_o, \Xi_M) \) of the 2dof mechanical model follows the measurement quite well. However, the 2dof model is unable to predict \( \hat{F}_{IB}(\omega_o, \Xi_M) \) since it assumes symmetric forces as evident from Eqs. (23) and (24). The force estimation with the 3dof mechanical model shows the asymmetrical behavior for both the magnitude and phase spectra as observed in Figs. 13 and 14. However, its force magnitudes do not match with direct force measurements since the effective mechanical parameters are not well assessed.

Figs. 15 and 16 show the contributions of the rubber and hydraulic paths. The magnitudes of both the rubber and hydraulic path forces are similar to, but much higher than, the total force. This suggests that the interfacial forces are affected by the relative phase between paths. For instance, Figs. 15(b) and 16(b) show that the \( \hat{F}_{IT}(\omega_o, \Xi_M) \) and \( \hat{F}_{IB}(\omega_o, \Xi_M) \) spectra are more affected by the rubber path force since the phases of \( \hat{F}_{IT}(\omega_o, \Xi_M) \) and \( \hat{F}_{IB}(\omega_o, \Xi_M) \) are the almost same as \( \hat{F}_{IT}(\omega_o, \Xi_M) \) and \( \hat{F}_{IB}(\omega_o, \Xi_M) \).

Fig. 14. Comparison between measured and predicted interfacial force spectra \( \hat{F}_{IB}(\omega_o, \Xi_M) \) using mechanical or fluid system model, given \( f_0(t) = \Re\{f_0 e^{i\omega_0 t}\} \) with \( |f_0| = 100 \text{N}, \) and by employing measured \( \Xi_M(\omega_o) \) and \( P_{IB}(\omega_o, \Xi_M) \). Key: - - - - measured; - - - - 2DOF mechanical model; - - - - 3dof mechanical model; - - - - fluid system model.
Fig. 17 compares measured $f_{IT}(t)$ and $f_{IB}(t)$ with estimated forces in the time domain by using the 2dof mechanical and fluid system QL models. Note that only the fundamental $o$ term is included here. To assess the validity of each model, define an error $e(t)$ between the measured force $f_{TM}(t)$ and predicted force $f_{IT}(t)$ and $f_{IB}(t)$ at any time $t$ as $e(t) = (f_{TM}(t) - f_{IT}(t))/f_{TM}(t)$ ($w = T$ or $B$). The overall root-mean-square error $E_{\text{rms}}$ is then given by $E = \sqrt{1/N_{\text{max}}} \sum_{\nu=1}^{N_{\text{max}}} [e(t)]^2$, where $N_{\text{max}}$ is the maximum number of points in the time domain. For the top forces $f_{IT}(t)$, errors from the 2dof and fluid system models are found to be 71.0 and 9.0 percent, respectively. Errors for the bottom forces $f_{IB}(t)$ from the 2dof and fluid system models are 111.0 and 3.7 percent, respectively. Thus, the fluid system model is more accurate than the 2dof mechanical model. Nevertheless, the fluid system model still shows discrepancies since it does not include any super- or sub-harmonics.

8. Conclusion

This article has made several contributions to the state of the art, as summarized below. First, a new concept was developed to indirectly measure interfacial forces by employing the hydraulic mount as a dynamic force sensor. The proposed method utilized a combination of models and operating motion and/or pressure measurements. Second, a laboratory experiment consisting of a powertrain, three powertrain mounts (including a dynamic load sensing hydraulic mount), a sub-frame, and four bushings was then constructed to verify the proof-of-concept. Third, the lower chamber pressure $p_{l}(t)$ was estimated, as it was not measured in either this study or any prior articles [10,11,12,17,18]. This led to an improved estimation of effective lower chamber compliance $\tilde{C}_{le}(o, \Xi)$ along with $k_{rel}(o, \Xi)$, $c_{rel}(o, \Xi)$ and $\tilde{C}_{ue}(o, \Xi)$. Overall, the proposed fluid system model yielded a better prediction of the interfacial forces, though the mechanical models provided some useful insights. This work also advanced prior component studies [10,11] by providing an improved multi-degree of freedom isolation system perspective.

This article focused on the frequency domain analyses. Limited results in the time domain were also presented. Interfacial force measurements exhibited multi-harmonic terms (Fig. 4) under sinusoidal excitation. As part of future work,
the super-harmonic responses in the time domain should be incorporated for a better estimation of the interfacial forces. Other excitations such as transients [2,18] should be considered as well.

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References


