INTRODUCTION

Elastomeric components are widely used in vehicle isolation systems such as engine mounts, suspension bushings, shock and strut mounts, cradle and body frame mounts, and exhaust hangers [1]. These components present many challenges in modeling such as amplitude and frequency dependent properties and the dynamic properties of assemblies may differ from those of components due to preload and boundary condition effects [2, 3, 4].

In large scale vehicle simulations, elastomeric components are commonly represented as lumped stiffness and damping elements [5] as the inclusion of the continuous three-dimensional elements and relevant material data is computationally prohibitive and this information may not be available early in design stages. Nevertheless, the internal mass contribution to the dynamic stiffness at higher frequencies cannot be ignored [3, 6-7]. For example, Hamada [3] demonstrates an exhaust hanger design through finite element simulations that component geometry can be used to tune the internal dynamics of the isolator. The internal dynamics of the isolator is also referred to as internal resonances or wave effects [7].

Elastomeric components are often characterized using a cross-point uniaxial, non-resonant sinusoidal measurement [8]. The connection models in commercial simulation software [9-10] seem to incorporate data from such cross-point non-resonant characterization methods. Further, connector models do not include options for separate driving- and cross-point dynamic stiffness. To model such behavior, an analyst is compelled to include a separate substructure to represent the elastomeric component.

In view of these above mentioned complexities, this article attempts to clarify frequency dependent behavior of elastomeric components under the small strain assumption where internal resonances strongly influence the component's dynamic stiffness. This will be achieved by developing detailed finite element models of components and reducing the observed frequency responses to analytical expressions based on simple lumped models. Further, benchmark experiments are conducted to illustrate the proposed concepts and to validate of the models. Finally, a hybrid FRF based substructure vehicle system model [10] is used to evaluate the effect on the sound pressure level sensitivity.

PROBLEM FORMULATION

The scope of this article is limited to elastomeric mounts or bushings where the material is considered to be linear elastic with frequency dependent structural damping. Only the axial direction of a cylindrical component (as displayed in Fig. 1) is examined. The specific objectives of this article are as follows: 1. Design two illustrative cylindrical mounts that are scaled to have identical axial static stiffness of the same material and compare the dynamic behavior, 2. Develop a lumped parameter model that captures the...
underlying physical phenomenon, 3. Conduct benchmark validation experiments; and 4. Examine the effect of the internal resonance on a vibro-acoustic vehicle system model.

Figure 1. Schematic of the mount / bushing computational model. The inner and outer surfaces are kinematically coupled to reference points, $x_1$ and $x_2$, respectively.

A schematic of the computational mount models is illustrated in Fig. 1 and are comprised of cylindrical geometries. The dimension and material properties of the two mounts (cases A and B) are given in Table 1. The dimensions are scaled such that the axial static stiffness is identical between the mounts; thus, the larger mount (A) has a 50% larger diameter and a volume four times greater than that the smaller mount (B). These mounts are subsequently analyzed via a detailed finite element simulation.

Table 1. Dimensions and material properties of the scaled 100 and 67 mm diameter mounts (A and B).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A</th>
<th>Case B</th>
<th>Ratio (A/B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD, mm</td>
<td>100</td>
<td>67</td>
<td>1.5</td>
</tr>
<tr>
<td>ID, mm</td>
<td>32</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>h, mm</td>
<td>78</td>
<td>50</td>
<td>1.56</td>
</tr>
<tr>
<td>mass, kg</td>
<td>0.550</td>
<td>0.136</td>
<td>4.0</td>
</tr>
<tr>
<td>$E$, MPa</td>
<td>10</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$, loss factor</td>
<td>0.10</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$, kg-m$^{-3}$</td>
<td>1000</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>$k$, axial stiffness, N-mm$^{-1}$</td>
<td>1305</td>
<td>1312</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**DYNAMIC STIFFNESS MATRICES VIA COMPUTATIONAL MODEL**

The elastomeric mount models contain only a representative elastomer and the inner and outer surfaces are constrained through use of kinematic couplings each to a single reference point, $x_1$ and $x_2$. The models are used to extract a two-terminal dynamic stiffness matrix at a given frequency ($\omega$, rad/s) as

$$
\begin{bmatrix}
  f_1(\omega) \\
  f_2(\omega)
\end{bmatrix} =
\begin{bmatrix}
  D_{11}(\omega) & D_{12}(\omega) \\
  D_{21}(\omega) & D_{22}(\omega)
\end{bmatrix}
\begin{bmatrix}
  x_1(\omega) \\
  x_2(\omega)
\end{bmatrix},
$$

where each element of the dynamic stiffness matrix is computed by imposing the following ideally fixed boundary conditions

$$
D_{11}(\omega) = \frac{f_1}{x_1} \bigg|_{x_1=0}, \quad D_{12}(\omega) = \frac{f_1}{x_2} \bigg|_{x_2=0}, \quad D_{21}(\omega) = \frac{f_2}{x_1} \bigg|_{x_1=0}, \quad \text{and} \quad D_{22}(\omega) = \frac{f_2}{x_2} \bigg|_{x_2=0}.
$$

Additionally, all other degrees of freedom, except for the actuation direction, at the reference points are set to zero. The reaction forces at the reference points are then output and divided by the input displacement to compute the dynamic stiffness elements. These boundary conditions are equivalent to a displacement controlled stepped sinusoidal test commonly employed to characterize a physical component while the other end is fixed to the base. The cross-point terms, $D_{12}$ or $D_{21}$, are typically reported from a commercial elastomer test machine [8].

Figures 2 and 3 contain the elements of dynamic stiffness matrix and the results are shown normalized with respect to the static stiffness value. Both mounts exhibit similar trends. First, the driving point dynamic stiffness magnitude terms, $D_{11}$ and $D_{22}$, initially start at the static stiffness value and decrease to a minimum value of above the loss factor, 0.10, before reversing and increasing to a value of 10 times greater than the static value. For both mounts, the driving point stiffness terms are not equal; thus, the mounts behave differently depending upon which surface, the inner or outer, is being actuated by a displacement input or constrained. The cross-point terms, $D_{12}$ and $D_{21}$, (shown in Figs. 2b and 3b) are identical (as expected) and do not exhibit the same reduction in dynamic stiffness as is observed for the driving point terms; however, the amplification is observed in the frequency range shown for the larger mount with a 100 mm diameter.

Figure 2. Dynamic stiffness terms for mount A, 100 mm diameter mount; (a) Driving point terms; (b) Cross-point terms; key: $\blacklozenge D_{11}$, $\circ D_{22}$ and $\Delta D_{12}$. Note that $D_{12} = D_{21}$.
When comparing dynamic properties of mounts A and B, the driving point dynamic stiffness term, $D_{11}$, achieves its minimum value at 320 and 680 Hz, respectively, a factor 2.1. This result is explained by the 4.0 ratio of A/B mount mass as shown in Table 1 and is discussed in detail later in this article.

**EXPERIMENTAL VALIDATION**

Controlled benchmark non-resonant test experiments are performed using solid cylindrical elastomeric components and are arranged in a horizontal configuration for characterization in the shear directions which are shown in Fig. 4. The shear configuration is achieved with a pair of cylinders to ensure appropriate boundary conditions. The diameter and height of each cylinder is 25.4 mm. Metallic caps are bonded to both sides of each cylinder and are threaded for ease in assembly. The non-resonant test employs a uniaxial hydraulically actuated, close loop servo-controlled elastomer test machine [8]. A sinusoidal displacement, $x(t)$, is applied to the top side of the specimen with a peak-to-peak displacement of 0.1 mm, and the transmitted force, $f(t)$, at the bottom of the specimen is measured at the same frequency. The dynamic stiffness is defined as:

$$\tilde{k}_d(\omega) = \frac{f}{x} e^{i\omega t} = \left| \tilde{k}_d \right| e^{i\delta} = k + i\h$$  \hspace{1cm} (3)

where $\delta$ is the loss angle, $k$ is the storage stiffness and $h$ is the dissipative stiffness. The dynamic stiffness is rewritten by defining the structural loss factor, $\gamma = \tan \delta$, as

$$\tilde{k}_d(\omega) = k(1 + i\gamma).$$  \hspace{1cm} (4)

Fig. 5 shows the measured cross-point dynamic stiffness for a frequency range up to 600 Hz. At 600 Hz, the test machine is no longer able to hold the specified tolerance for controlled displacement and thus the test is ended for this experiment. The experimental results are normalized by the dynamic stiffness observed at 100 Hz.

A linear finite element model is constructed of the benchmark experiment. The material nonlinearities that dominate the measured frequency dependent behavior of the experiment below 100 Hz are intentionally ignored in the model. The material is modeled as linear elastic with structural (hysteretic) damping that is proportional to stiffness and set equal to the measured loss factor for stiffness. The dynamic elastic modulus ($\tilde{E}$) is scaled to match the component dynamic stiffness measurement at 100 Hz. The measured and simulated cross-point dynamic stiffness values are compared in Fig. 5, and a good agreement is achieved above 100 Hz. The dynamic stiffness is amplified by a factor of two from 100 to 600 Hz in both the simulation and experiment.

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**Figure 3.** Dynamic stiffness terms for Case B, 67 mm diameter mount; (a) Driving point terms; (b) Cross-point terms; key: $\bigcirc D_{11}$, $\bigcirc D_{22}$ and $\bigtriangleup D_{12}$. Note that $D_{12} = D_{21}$.

**Figure 4.** Schematic of the non-resonant test with cylindrical elastomeric elements configured in shear loading.

**Figure 5.** Comparison of cross-point dynamic stiffness term for benchmark experiment shown in Fig. 4; key: $\bigtriangleup$ Measurement, $-$ Finite element simulation.
The simulation model of the benchmark experiment is used to examine the driving-point dynamic stiffness term, $D_{11}$, and is shown in Fig. 6. Following a similar trend as the mount examples, the driving-point stiffness exhibits the reduction in dynamic stiffness to the loss factor value at 490 Hz before the magnitude reverses and increases. To understand this decrease in dynamic stiffness, the deformed shapes of the steady-state response is shown in Figs. 7b and 7c at 10 and 490 Hz. At a lower frequency, the elastomeric element performs as a shear stiffness element; whereas at 490 Hz, the slope of the elastomeric element approaching the displacement actuation boundary is zero. This indicates that the internal mass significantly contributes to the observed dynamic stiffness of the component.

![Figure 6. Comparison of simulated driving- and cross-point dynamic stiffness magnitude terms for benchmark experiment shown in Fig. 4; key: $D_{12}$, $D_{11}$.

a. Configuration

b. Shear element

c. First internal resonance

Figure 7. Simulated magnified deformed shapes of the elastomeric cylinders using FEA; (a) schematic of the experimental setup where vertical dynamic displacement excitation; (b) low frequency behavior where cylinder acts as a shear stiffness element, 10 Hz; (c) behavior at first internal resonance, 490 Hz.

**SIMPLIFIED LUMPED SYSTEM MODEL**

Based upon the above analyses and observations, a lumped parameter dynamic model is developed to compare trends to the detailed finite element mount models. A single-degree-of-freedom model is considered as shown in Fig. 8. The equation of motion is written as

$$[-\omega^2 m + (\tilde{k}_1 + \tilde{k}_2)] Y(\omega) = \tilde{k}_1 X_1(\omega) + \tilde{k}_2 X_2(\omega),$$

(3)

where $m$ is the effective mass, $Y$ is the displacement of mass element, $X_1$ and $X_2$ are the displacements of the massless spring terminals, and $\tilde{k}_1$ and $\tilde{k}_2$ are complex valued elastic spring constants. The imaginary part of the dynamic stiffness spring represents a structural damping mechanism used to represent frequency dependent properties. Next, expressions between the terminal displacements and forces at a given frequency are written as

$$F_1(\omega) = \tilde{k}_1 (Y(\omega) - X_1(\omega)) \quad \text{and}$$

$$F_2(\omega) = \tilde{k}_2 (Y(\omega) - X_2(\omega)).$$

(6a-b)

Finally, the dynamic stiffness matrix between $X_1$ and $X_2$ are determined by using Eqn. 5 and substituting the results for $Y$ in Eqn. 6. The final expression for the dynamic stiffness matrix at a given frequency is

$$[D(\omega)] = \frac{1}{k_1 + k_2 - \omega^2 m} \begin{bmatrix} \tilde{k}_2 - \omega^2 m & -\tilde{k}_1 \tilde{k}_2 \\ -\tilde{k}_1 \tilde{k}_2 & \tilde{k}_1 - \omega^2 m \end{bmatrix},$$

(7)

where $\omega$ is the circular frequency in rad/sec.

![Figure 8. Single-degree-of-freedom model with forces and motions at two terminals.](image)
PARAMETER ESTIMATION

To compare the lumped parameter model with the detailed finite element model, one must first estimate the model parameters. The lumped model of Fig. 8 has four parameters, namely, \( k_1, k_2, \gamma, \) and \( m \). The loss factor is chosen at the measured value at 100 Hz. The expression for the dynamic stiffness matrix, Eq. (7), provides simple formulas to compute the model parameters. First, the driving point stiffness elements are minimized at separate frequency values, thus \( k_1 = \omega^2 m \) and \( k_2 = \omega^2 m \). Next, another equation can be included when one considers the static stiffness, \( k_s \), value between \( x_1 \) and \( x_2 \) as

\[
k_s = \frac{1}{1/k_1 + 1/k_2}.
\]

When combining the equations, the effective mass, \( m \) can be determined as

\[
m = k_1 \frac{\omega^2 + \omega^2}{\omega^2 \omega^2}.
\]

The lumped model representation is compared to the detailed finite element model for the 67 mm mount (B) in Fig. 9. The general trends are captured for both driving- and cross-point dynamic stiffness terms. These are typical results for both the 67 mm (B) and 100 mm (A) mounts for each dynamic stiffness matrix term. The identified parameters for the lumped model are tabulated for detailed finite element models of mounts A and B in Table 2. In each case, \( k_1 \) and \( k_2 \) are not equal. Also, the effective mass, \( m \), is slightly less than the actual mass of the elastomeric mount. When comparing the volumetric effect of a larger mount design, the elastic spring constants remain within 10 to 20% between the different sizes. This may be explained by a different surface area being constrained for each model; however, the effective mass, \( m \), is the main difference between the two mount geometries. The effective mass scales closely to the actual mass ratio of a factor of four.

DYNAMIC COMPLIANCE MATRICES

In frequency response function based substructuring methods [10], the connection properties must be modeled as separate substructures to capture the internal dynamics as described in this article. Thus, it is convenient to express the bushing properties in terms of their dynamic compliance (\( H \)) matrices. To do so, we seek the relationship between applied forces and displacements of the form

\[
\begin{bmatrix}
X_1(\omega) \\
X_2(\omega)
\end{bmatrix} = \begin{bmatrix}
H_{11}(\omega) & H_{12}(\omega) \\
H_{21}(\omega) & H_{22}(\omega)
\end{bmatrix} \begin{bmatrix}
F_1(\omega) \\
F_2(\omega)
\end{bmatrix},
\]

where each element of the dynamic compliance matrix at any frequency is computed under the following ideally free boundary conditions

\[
H_{11} = \frac{X_1}{F_1} \bigg|_{F_2 = 0}, \quad H_{12} = \frac{X_1}{F_2} \bigg|_{F_1 = 0},
\]

\[
H_{21} = \frac{X_2}{F_1} \bigg|_{F_2 = 0}, \quad \text{and} \quad H_{22} = \frac{X_2}{F_2} \bigg|_{F_1 = 0}.
\]

Additionally, all other degrees of freedom, except for the actuation direction, are unconstrained (free). The response displacements at the reference points are then output and normalized by the input force to compute the dynamic compliance elements. These boundary conditions are equivalent to modal testing under freely suspended condition of a physical component. The dynamic compliance terms can be written directly by inverting the dynamic stiffness matrix (\( D \)), Eqn. (7), as

\[
\begin{bmatrix}
H(\omega)
\end{bmatrix} = \frac{-1}{\omega^2 m} \begin{bmatrix}
1 - \omega^2 m/k_1 & 1 \\
0 & 1 - \omega^2 m/k_2
\end{bmatrix}.
\]

To verify the above relationships, the finite element models of the 67 and 100 mm diameter mounts (A and B) are subjected to the boundary conditions described in Eqn. 11a-d and compared with the analytical model of Eqn. 12 (Fig. 10). Similar agreement is achieved as that of the dynamic stiffness matrix terms.
Table 2. Lumped parameter model parameters for simulated mount designs, (A and B).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A 100 mm</th>
<th>Case B 67 mm</th>
<th>Ratio A/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$, N/mm</td>
<td>1880</td>
<td>2054</td>
<td>0.9</td>
</tr>
<tr>
<td>$k_2$, N/mm</td>
<td>4410</td>
<td>3678</td>
<td>1.2</td>
</tr>
<tr>
<td>$\gamma$, loss factor</td>
<td>0.10</td>
<td>0.10</td>
<td>1.0</td>
</tr>
<tr>
<td>Model (effective), $m$, kg</td>
<td>0.47</td>
<td>0.11</td>
<td>4.3</td>
</tr>
<tr>
<td>Physical mass, kg</td>
<td>0.550</td>
<td>0.136</td>
<td>4.0</td>
</tr>
</tbody>
</table>

APPLICATION TO FRF BASED VEHICLE MODEL

To investigate the effect of the internal resonance within a set of elastomeric components, an experimental vibro-acoustic FRF based trim body model is developed for a production minivan where the rear suspension is entirely disconnected from the vehicle body. A 12 degree of freedom input and 19 degree of freedom output vibro-acoustic frequency response function model is constructed via use of commercially available test and modeling software [10]. The trim body inputs are the four rear subframe mount connection locations and represent the translational degrees of freedom. The rotational inputs and responses are intentionally ignored for convenience of the model construction. The body mounts are modeled using dynamic stiffness where the internal resonance is ignored and compared with models containing such resonances as discussed in this article. The hybrid subsystem model contains the trim body experimental representation and is combined with a finite element model of the rear subframe. The remaining rear suspension components are intentionally ignored for this study. Fig. 11 contains the results for examining the normalized sound pressure sensitivity for a dynamic force applied in the vertical direction to the subframe attachment to the lower control. The predictions tend to deviate above 60 Hz. This is the expected result as the internal mass distribution for the elastomeric mount can be ignored at lower frequencies. The difference in the prediction for mounts A and B increases as the internal mass contribution becomes more significant at higher frequencies. Accordingly, it is difficult to state that a larger or smaller size mount provides better performance as issues are application specific and dependent on the frequency range.

CONCLUSION

This article contributes to the state-of-the-art by highlighting the importance of considering the internal mass distribution of elastomeric components over mid to high frequency range from noise and vibration perspective. Dynamic scaling of elastomeric components becomes a critical concern as automotive industry is seeking ways to remove weight from the vehicle to improve fuel efficiency; however, since the internal mass contribution of elastomeric components is often neglected in large-scale structural simulations, the resulting effect on dynamic performance is seldom predicted and thus not well understood.

An illustrative example is developed where two different sized mounts, constructed of the same material and are shaped to achieve the same static stiffness behavior; however, the two mounts exhibit drastically different dynamic behavior. Physical insight is provided through the development of a single-degree-of-freedom model to explain the differences. An internal mount resonance is present and the mounts performance is dependent upon the boundary conditions on the input-output terminals of the mount model. Further, a method to extract the parameters for the lumped model from a detailed finite element mount model is provided. A new controlled benchmark experiment is used to validate the simulated behavior.

Finally, the effect of the internal resonance is examined within the context of a hybrid vibro-acoustic model to describe the dynamics of trim-body and rear subframe system. It is demonstrated that the internal mount resonance exhibits a significant influence of the sound pressure sensitivity for road noise say above 60 Hz. The proposed analytical work is promising for developing simple, yet reasonably accurate methods to design elastomeric components early in the R&D
stage and should be useful in optimization studies. The work is limited to cylindrical mount geometries as further efforts are needed to examine more complex component shapes and geometries.

REFERENCES


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