Development of Refined Clutch-Damper Subsystem Dynamic Models Suitable for Time Domain Studies

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ABSTRACT
This study examines clutch-damper subsystem dynamics under transient excitation and validates predictions using a new laboratory experiment (which is the subject of a companion paper). The proposed models include multi-staged stiffness and hysteresis elements as well as spline nonlinearities. Several example cases such as two high (or low) hysteresis clutches in series with a pre-damper are considered. First, detailed multi-degree of freedom nonlinear models are constructed, and their time domain predictions are validated by analogous measurements. Second, key damping sources that affect transient events are identified and appropriate models or parameters are selected or justified. Finally, torque impulses are evaluated using metrics, and their effects on driveline dynamics are quantified. Dynamic interactions between clutch-damper and spline backlash nonlinearities are briefly discussed.


INTRODUCTION
Vehicle powertrains are usually subjected to transient loads that result from abrupt changes in throttle position, clutch engagement/disengagement, and braking. Such transient events lead to problematic noise and vibration phenomena and excessive impulsive torsional vibrations that are eventually transmitted to the cabin. Few recent papers have employed a lumped parameter approach to address impulsive responses in vehicle drivelines [1, 2, 3, 4, 5, 6]. For example, Crowther et al. [2] have studied the driveline clunk in a rear wheel drive driveline with a 4 degree of freedom (4DOF) nonlinear torsional model. It has been validated experimentally in the time domain using a non-rotating driveline experiment. Their model, with minimal components, included both backlash and friction nonlinearities within the differential. Li and Singh [2] have simplified the driveline system to study transient response associated with the engine start-up process with focus on vibration amplification. They assumed that flywheel motion is not affected by the downstream driveline components due to the massive flywheel inertia, and the downstream system behaves as an integral inertial component during low frequency dynamics of a typical vehicle driveline. Accordingly, they developed a nonlinear single degree of freedom (SDOF) system with a multi-staged clutch damper including hysteresis and multi-staged stiffness elements. This article aims to extend the prior study [7] by proposing alternate models with focus on the identification of damping elements during transient events. In particular, both computational and experimental methods are adopted to examine the transient dynamics of a clutch-damper subsystem. The clutch-damper experiment (published in a companion paper [8]) simulates an impulsive excitation transmitted through the clutch-damper subsystem to the driveline. In the current paper, the applicability of nonlinear hysteresis models for clutch-damper subsystems under transient conditions is explored. Then, key damping sources that affect transient events are identified and a refined model is proposed. Specific objectives of this paper are: (1) Develop a refined nonlinear model and investigate dynamic characteristics that are not fully addressed in the companion paper [8]; and (2) Compare predictions with measurements and define damping mechanisms during the transient events.

NEW CLUTCH-DAMPER EXPERIMENT
Figure 1 shows the physical schematic of a clutch-damper experiment used to examine only transient oscillations for a nonrotating system using a pneumatic actuator for loading (refer to [8] for more details). The clutch-damper assembled on a shaft is supported by two bearings. The front end of the shaft is connected to a torsion bar. The clutch at the flywheel side is grounded to the fixtures that are bolted to a heavy test bed. On the far end of the cantilevered torsion bar, a manual release mechanism is bolted to the bar to which the pneumatic actuator tip is attached. It is released to provide excitation during the experiment. This setup allows various levels of step-like torque excitations (as shown in Fig. 2) to the clutch-damper subsystem as listed in Table 1. Time domain torque responses and vibrational accelerations at various locations are measured for subsequent damping mechanisms identification and validation purposes.
**MINIMAL ORDER NONLINEAR SYSTEM MODELS**

For the sake of simplicity, the powertrain system could be divided into connected subsystems to provide some physical insights regarding the nature of nonlinear problems, as shown schematically in Figure 3. Several approaches have been utilized to examine the impulsive loading responses in drivelines, such as finite element analysis [6], though this method is time intensive. In addition, it is difficult to include accurate damping models since dissipation mechanisms during transient events (unlike steady state conditions) are still poorly understood. The lumped parameter method, with a minimal order, is a powerful technique to model the dynamic response of torsional systems subjected to impulsive loads [2, 3, 4, 5, 6, 7]. Moreover, the inclusion of clearance and friction nonlinearities in this approach is more tractable. Two dynamic models for the clutch-damper subsystem are proposed in this paper: (1) A nonlinear SDOF system with a rigid spline contact model as in [7]; and (2) A 3DOF nonlinear model including a compliant spline contact model.

**Table 1. Transient excitation torque characteristics with reference to Fig. 2.**

<table>
<thead>
<tr>
<th>Experimental Run</th>
<th>$T_1$ [Nm]</th>
<th>$\Delta \tau$ [ms]</th>
<th>$\Delta T$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>645</td>
<td>1</td>
<td>595</td>
</tr>
<tr>
<td>2</td>
<td>1010</td>
<td>1</td>
<td>960</td>
</tr>
<tr>
<td>3</td>
<td>1350</td>
<td>1</td>
<td>1300</td>
</tr>
<tr>
<td>4</td>
<td>1730</td>
<td>1</td>
<td>1680</td>
</tr>
</tbody>
</table>

**MATHEMATICAL FORMULATION**

Four clutch-dampers with multiple hysteresis and stiffness stages, as illustrated in Table 2, are considered. For each case, a torque-angular deflection curve is assumed to be available similar to the quasi-static curve shown in Figure 4 for a typical clutch-damper assembly with a pre-damper. The hysteresis in the system can be estimated by observing the resulting area between loading and unloading traces.

**Table 2. An overview of clutch-damper cases and properties with reference to Figure 4.**

<table>
<thead>
<tr>
<th>Clutch-Damper Case</th>
<th>Pre-damper stage</th>
<th>Operating Stiffness $K_a$</th>
<th>Hysteresis $H_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Yes</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>A2</td>
<td>No</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>A3</td>
<td>No</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>A4</td>
<td>No</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>
The torsional stiffness for each stage can be increased abruptly by applying a preload on the helical springs for a particular stage. On the other hand, adding a pre-damper component to the clutch leads to considerable softening of the first stage stiffness while enlarging the transition to the next stage. A 3DOF nonlinear model including a compliant spline contact model (as displayed in Figure 5a) is compared with a simplified SDOF nonlinear model with a rigid spline model (as shown in Figure 5b). The equations of motion for the 3DOF model in matrix form is

\[ \mathbf{J}\ddot{\theta}(t) + C\dot{\theta}(t) + \mathbf{K}\theta(t) = \mathbf{T}(t). \]  

(1)

where \( \mathbf{J}, \mathbf{C}, \) and \( \mathbf{K} \) are inertia, damping, and stiffness matrices, respectively. Note that the stiffness matrix includes two nonlinear stiffness elements. First, the multi-staged clutch model is represented by the following:

\[ T_{10}(\theta_i(t)) = T_0(\theta_i(t)) + T_p(\theta_i(t)) + H(\theta_i(t), \dot{\theta}_i(t)). \]  

(2)

Here, \( T_0 \) is the torque transmitted through elastic elements (springs), \( T_p \) is the internal clutch preload, and \( H \) is the clutch-damper hysteresis torque (dry-friction torque losses inside the clutch-damper subsystem); these are estimated from the quasi-static torque curve for each clutch.

\[ T_0(\theta_i(t)) = K_{\text{II}} \theta_i(t) + 0.5\left(K_{\text{I}} - K_{\text{II}}\right) \left[\left(\tanh(\sigma(\theta_i(t) - \theta_i) + 1) - \left(\tanh(\sigma(\theta_i(t) - \theta_i) + 1)\right]\right] + 0.5\left(K_{\text{II}} - K_{\text{I}}\right) \left[\left(\tanh(\sigma(\theta_i(t) - \theta_i) + 1) - \left(\tanh(\sigma(\theta_i(t) - \theta_i) + 1)\right]\right] \right. \]  

(3)

Here, \( \sigma \) is a regularizing factor. Define \( T_p \) and \( H \) as follows:

\[ T_p(\theta_i(t)) = 0.5T_p\left[\tanh(\sigma(\theta_i(t) + \theta_i)) + \tanh(\sigma(\theta_i(t) - \theta_i))\right]. \]  

(4)

\[ H(\theta_i(t), \dot{\theta}_i(t)) = \left[0.5H_{\text{II}} + 0.25H_{\text{I}}\tanh(\sigma(\theta_i(t) - \theta_i)) - 0.25H_{\text{II}}\tanh(\sigma(\theta_i(t) + \theta_i))\right] \tanh(\sigma(\theta_i(t) + \theta_i)). \]  

(5)

Note, hysteresis is only effective in the second stage (operating stage) for all clutches considered in this paper. Further, the compliant contact model for the clutch spline with an angular clearance \( b \) is given by the following:

\[ K_{\text{c}}(\delta(t)) = K_{\text{cc}} \delta(t), \text{ and } C_{\text{c}}(\delta(t)) = C_{\text{cc}} \delta(t). \]  

(6)

where, \( \delta(t) = \begin{cases} 1 & |\theta_i - \theta_j| \geq 0.5b \\ 0 & \text{Otherwise.} \end{cases} \)

The spline mesh offset torque [3] is:

\[ T_{\text{sp}} = 0.5\text{sgn}(\theta_i(t))k_3b. \]  

(7)

Now the torque vector could be constructed from excitation torque, clutch hysteresis torque, and a spline mesh torque offset as follows:

\[ \mathbf{T}(t) = [T(1) \quad T_{\text{sp}} \quad T_{\text{sp}} - H(\theta_i(t), \dot{\theta}_i(t)) - T_p(\theta_i(t))]^T. \]  

(8)

where, \( T(t) \) is the step-like external torque as shown in Figure 2. Applying a load starting from zero, the upstream elements (1, 2) are rotated freely through the backlash region with an angular displacement equivalent to \( \pm b \). Therefore, the initial condition displacements need to be adjusted accordingly.

\[ \begin{bmatrix} \dot{\theta}(0) \\ \dot{\theta}(0) \end{bmatrix} = \begin{bmatrix} K_{\text{I}}^{-1}T(0) + \text{sgn}(T(0))b \end{bmatrix} [1 \quad 0]^T \quad \text{and} \quad \dot{\theta}(0) = 0. \]  

(9)

Additionally, non-dimensional variables are introduced below based on the system characteristics such as the second stage stiffness \( K_{\text{II}} \) of the clutch and the first natural frequency \( \omega_1 \).

\[ \tilde{\tau} = 2\pi \frac{1}{\omega_1}, \quad \tilde{T}(T) = \frac{T(T)}{bK_{\text{II}}}, \quad \tilde{\theta}(T) = \frac{\theta(T)}{b}, \quad \tilde{\dot{\theta}}(T) = \frac{\dot{\theta}(T)}{b}. \]  

(10 a-f)

Here, \( \lambda \) is the time period between collision impulses; it will be used as a metric in a latter section. Finally, Equations (1), (2), (3), (4), (5), (6), (7), (8), (9) are implemented and solved by a commercial code [9] as demonstrated in Figures 6 and 7 for the 3DOF system model of Figure 5 (a).

Figure 5. Minimal order nonlinear torsional system models of the experiment of Figure 1. (a) 3DOF nonlinear system; (b) SDOF nonlinear system. Key: \( J_1 \)- lumped inertia of torsion arm, \( J_2 \)- lumped inertia of clutch shaft with spline, \( J_3 \)- lumped inertia of clutch hub, \( K_{\text{I}} \)- torsion arm stiffness, \( K_{\text{II}} \)- spline contact stiffness, \( K_{\text{c}} \)- non-linear clutch stiffness, \( H \)- non-linear clutch hysteresis, and \( C \)- bearing losses.
RESULTS AND DISCUSSION

Figure 8(a) shows a typical comparison between normalized measured and predicted torque transmitted through the input shaft corresponding to the clutch case A1 (see Table 2). Measured normalized angular acceleration observed at shaft inertia is compared with simulations in Figure 8(b). As expected, deviations are observed between simulation and measurements in Figures 8 (a-b) after the first bounce for the SDOF model. The difference between the measured and predicted time length between collision impulses can be attributed to the total effect of the simplified damping models of the clutch as well as ignoring spline contact compliance and damping. Conversely, the spline contact stiffness and the corresponding damping were implemented into the 3DOF model. Here, the prediction matches the measurements better in terms of peak-to-peak, but it still differs from the measurements in terms of the time duration between collision impulses. The 3DOF model including the spline contact is superior to the SDOF model but is still insufficient. Further examinations of the nature of dynamic interaction between spline and multi-staged stiffness (clutch-damper) are conducted using the 3DOF model; predictions during the step down excitation are shown in Figures 8(c-d). Clearly, double-sided impact events take place at first followed by multiple single-sided impacts before full steady state contact is reached. Knowing alternate response regimes provide clues regarding the dissipation mechanisms during the transient events at each individual stage. Based on this analysis, a refined clutch model will be proposed in the next section.
Figure 7. Model template for A2 to A4 clutch-damper cases using a commercial code [2].

Figure 8. Simulation results (in dimensionless form) using 3DOF and SDOF nonlinear models of Figure 5 for clutch-damper case A1 (Table 2) corresponding to $\Delta T = 1680$ Nm. (a) Simulated and measured intermediate torques at input shaft. (b) Simulated and measured angular accelerations at torsion arm. (c) Simulated relative mesh angular displacements for the spline. (d) Simulated relative angular displacements for the clutch-damper. Key for Figs. 8 (a-b): Simulated using SDOF system (blue dotted line), simulated using 3DOF system (black line) and measured (red x markers).
DEVELOPMENT OF A Refined NONLINEAR MODEL

In the previous sections, the transient damping characteristics of the clutch-damper device are approximated using hysteretic damping from measured quasi-static curves. Careful examination of the measurements reveals that the clutch-damper subsystem response amplitudes may not decay in a linear manner. From the measurements shown in Figure 8, it is found that the response decay has two components: linear and exponential, as well as a multistage nature. These observations lead to the identification of the dissipation mechanisms during the transient events as a combination of hysteresis and viscous damping for each clutch stage. Material damping produced during spring coils compression or elongation could contribute to some of this observed viscous damping dissipation [10]. However, during the transient excitation event, it is possible for certain clutch spring windings to be compressed until they are in direct contact which could lead to introducing an interfacial contact damping component. Even though the amount of structural damping in the coil material might be small, it has a significant effect on transient simulations due to high relative velocities between the different moving parts. Therefore, both hysteresis damping and piecewise damping elements must be included. Thus, a piecewise structural damping element (C_D) that determines the extent of damping present in the springs is added to the multi-staged clutch-damper model (see Figure 9). The clutch-damper torque equation (2) is rewritten in the following constitutive form

\[ T_{cd}(\theta_1(t)) = T_v(\theta_1(t)) + T_h(\theta_1(t)) + H(\theta_1(t), \dot{\theta}_1(t)) + T_r(\theta_1(t), \dot{\theta}_1(t)). \]  

\[ \text{(11)} \]

Here, \( T_v(\theta_1(t), \dot{\theta}_1(t)) \) is the piecewise structural damping torque where \( T_v \) is given by \( C_D \dot{\theta} = \beta K_D(\theta) \dot{\theta} \) where \( \beta \) is an assumed damping (proportionality) factor, and \( K_D(\theta) \) is the nonlinear (piecewise) stiffness that is defined by the following:

\[ K_D(\theta) = K_1 + 0.5(K_2 - K_1)[(\tanh \sigma(\theta(t) - \Theta_2) - \tanh \sigma(\theta(t) + \Theta_1) + 2] + 0.5(K_3 - K_2)[(\tanh \sigma(\theta(t) - \Theta_3) - \tanh \sigma(\theta(t) + \Theta_3) + 2]. \]  

\[ \text{(12)} \]

The refined model of Figure 9 captures the dynamics very well over the entire time span as observed in Figure 10. The measured torque amplitude starts to decrease as the input load is released from the torsion arm tip, and the torsion bar swings upward to unload the energy stored in the torsional system. The torsional vibration goes through the clutch-damper spline and clutch stages, and carries the momentum to create the overshoot seen in the plots. However, the simulation slightly underestimates the overshoot amplitudes in Figure 10.

MODEL EVALUATIONS USING SELECTED METRICS

Three metrics, based on the peak-to-peak impulsive motions that occur across spline or clutch stages, are selected to compare predictions with measurements. These metrics are peak-to-peak (P-P) angular accelerations, P-P impact torque, and the time period \( \lambda \) between the first and second collision impulses. Then metrics are normalized using the relations given in Equation (9). Figures 11 and 12 summarize the simulations for 4 runs of Table 1 for cases A1 and A3, respectively.

It is evident from the analysis of Figures 11 and 12 that most deviations (say about 15%) between measured and predicted metrics are seen in the P-P accelerations metric. Note that acceleration measurements under shock loading are expected to differ by at least 10% (refer to [10] for more details). Nevertheless, these errors decline considerably for the subsequent impact events. Meanwhile, discrepancy between the measured and calculated P-P torque metric is relatively smaller (around ±10% for A1 and less than ±5% for A3). Finally, the time periods metric reveals a good agreement between...
measured and predicted impact times. Overall, the metrics clearly show that predictions are consistent with measurements over a wide loading range.

CONCLUSION

Minimal order nonlinear torsional models are found to be effective, yet simple tools in analyzing the highly nonlinear clutch-damper subsystem dynamics subject to an impulsive excitation. In particular, the 3DOF system model containing a spline compliant contact is superior to the simplified model with a rigid spline contact (SDOF system) model. A study with 3 metrics shows that predictions are generally consistent with measurements over a wide loading range. However, it is observed that the classical clutch hysteresis models using quasi-static measurements are not quite sufficient for accurate simulations under dynamic conditions, though they are still valuable.

As a final point, this study has focused only on the transient oscillations of the clutch-damper subsystem (for a non-rotating system), and hence the proposed models do not include the dynamic effect of a rotating flywheel (with torque fluctuations). Also, this paper has not examined any dynamic interactions among nonlinear powertrain components; such interactions should be studied in future.

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APPENDIX

APPENDIX A - DEFINITIONS/ABBREVIATIONS

SYMBOLS

- b - Spline backlash
- C - Viscous damping coefficient (torsional)
- H - Hysteresis
- J - Inertia
- K - Torsional stiffness
- t - Time
- T - Torque
- x, x, x - Translational displacement, velocity, and acceleration
- β - Damping (proportionality) factor
- θ, θ, θ - Angular displacement, velocity, and acceleration
- θ - Stage length
- δ - Unit step function
- λ - Time period between first and second impacts
- σ - Regularizing factor for smoothening hysteresis
- ω - Circular frequency (rad/s)

SUBSCRIPTS

1, 2, 3 - Inertial or damping elements
12, 23 - Stiffness or damping elements
0 - Initial value
I, II, III - Index for clutch-damper stages
A - Clutch shaft and spline
B - Spline backlash
CD - Clutch-damper
D - Stiffness path
P - Pre-load
P-P - Peak to peak value
v - Viscous damping path

SUPERSCRIPTS

- - Normalized
. - First derivative with respect to time
.. - Second derivative with respect to time
T - Transpose

ABBREVIATIONS

DOF - Degree of freedom
SDOF - Single degree of freedom
P-P - Peak to peak angular velocity

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